



FigureThis!
Math Challenges for Families

Do movies make \$ \$money?



Figure This! The theater box office receipts for the movie *Ratio in Magicville* for the past four weeks were \$15,000, \$12,000, \$12,000, and \$10,000, respectively. The theater owner pays the movie distributor the following percentages of the box-office receipts: 70% for each of the first two weeks, 60% for the third week, and 50% for the fourth week. Other operating expenses are \$4500 per week. Did the theater make or lose money?

Hint: How much did the theater have to pay the distributor the first week to show the movie?

Profit and loss calculations are essential to financing and operating any business. Business owners, shippers, accountants, and bankers all carefully examine income and expenses when making financial decisions.

After four weeks, the theater showed a loss of \$100.

Answer:

Figure This!

Get Started:

Make a table to keep track of the profit (or loss) each week.

Week	Box-Office Receipts	Percentage to Distributor	Amount to Distributor	Other Expenses	Profit (or Loss)
1	\$15,000				
2	\$12,000				
3	\$12,000				
4	\$10,000				
					Total

Complete Solution:

For each week, subtract the distributor's share and other expenses from the theater box-office receipts. The result is the weekly profit (or loss). See the table. After four weeks, the theater has a loss of \$100.

Week	Box-Office Receipts, R	Percentage p , to Distributor	Amount, $A = Rp$ to Distributor	Other Expenses, E	Profit or (Loss) $R - A - E$
1	\$15,000	70%	\$10,500	\$4500	\$0
2	\$12,000	70%	\$8400	\$4500	-\$900
3	\$12,000	60%	\$7200	\$4500	\$300
4	\$10,000	50%	\$5000	\$4500	\$500
					Total -\$100

Try This:

- Theater owners generally consider that it takes 10,000 ticket buyers to support one movie screen. Based on this rule, determine if your town has too many screens.
- Check your local newspaper for a report on box-office receipts, or consult an appropriate website such as www.film.com/reviews/rev_box/index.jhtml.
- Identify the movie that has had the longest run at your local theaters.
- Do some research in publications such as *Entertainment Weekly* to identify the box-office receipts made by your favorite movies.

Additional Challenges:

(Answers located in back of booklet)

- Suppose the theater in the challenge extends the contract for *Ratio in Magicville*. For the next two weeks, the distributor's percentages will be 40% and 30%, respectively. If the theater hopes to earn a profit of at least \$1000 over the next two weeks, how much will have to be taken in at the box office?

- For additional challenge 1, what would have to be the box office receipts for the profit to be \$500 for week 5 and \$500 for week 6?
- A theater's typical profit for popcorn is around 85% of the price paid by the patron. If you paid \$4.00 for a bag of popcorn, about how much did it cost the theater owner?
- A theater charges \$8.00 for admission. At the concession stand, a 32-ounce soda which costs \$0.30 to produce is sold for \$3.00. During the first four weeks of *Ratio in Magicville*, 40% of moviegoers bought sodas. About how much did the theater earn on soda?

Things to Think About:

- Why have many movie theater chains filed for bankruptcy in recent years?
- What is the most popular concession item sold at movies?
- Is it fair to compare box office receipts for movies such as *Titanic* and *Gone with the Wind*?
- What kind of movies have the longest box-office runs?

Did You Know That?

- Theater owners generally do not like long movies (three hours or more) because there's time for only one nightly showing per screen. This means fewer opportunities for concession sales and ticket sales.
- The bulbs for a movie projector cost about \$750 each.
- Movie theaters make 95% of their money on weekends.
- Only 20% of movies are highly profitable.
- In 1999, the United States had about 37,000 movie screens.
- As of May, 2001, the current leading money-maker was *Titanic*, which has grossed over \$600.8 million.

Resources:

Newspaper:

- Dallas Morning News*. Nov. 26, 2000.

Websites:

- www.howstuffworks.com/movie-distribution3.htm
- www.hollywood.com/nato
- faculty.washington.edu/baldasty/200.htm
- www.film.com
- www.imdb.com
- www.film.com/reviews/rev_box/index.jhtml



FigureThis!
Math Challenges for Families

Does he have a chance?

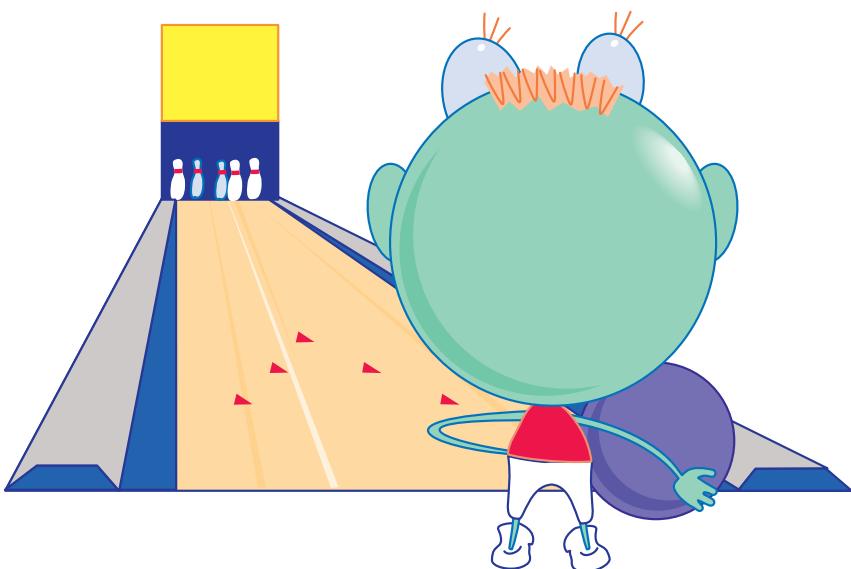


Figure This! Helix is about to roll his final game in a bowling tournament. His previous scores were 134, 99, 109, 117, and 101. To win the tournament, his average score must be at least 114. Considering his past performance, estimate his chances of winning.

Hint: What is the least score Helix needs to win the tournament?

Probabilities, or measures of chance, can be estimated considering the results of past events. People who calculate insurance premiums, batting averages, and medical treatment risks all use past events to estimate probabilities.

About 20%.
Answer:

FigureThis!

Get Started:

How many games will Helix bowl in the tournament? To reach an average of at least 114, what is the lowest possible sum of his scores?

Complete Solution:

- Helix will bowl a total of six games. To reach an average of at least 114, his total score must be at least 6×114 , or 684. The sum of his scores after five games is 560. Subtracting 560 from 684 leaves 124. Therefore, any score 124 or higher will result in a win for Helix. Now, consider that Helix has scored higher than 124 in only one of the previous five games. Based on these performances, you might estimate that his chances of winning are $1/5$, or 20%.
- Another way to figure the average is to subtract 114 from each score and add the differences. In order, his scores are $+20, -15, -5, +3, -13$ from 114.

$$\text{The total is } +20 + (-15) + (-5) + 3 + (-13) = -10$$

He must score at least 10 above the desired average of 114 to compensate, or 124, again having $1/5$ chance, or 20%.

Try This:

- Toss a thumbtack into the air 200 times and record how it lands, point up or point down. Use your results to estimate the probability that a thumbtack will land with its point up.
- Look at the test scores you presently have in one of your classes and the grade scale for your school. Determine if it is possible to have a B average in the class after your next test.

Additional Challenges:

(Answers located in back of booklet)

1. The average of 5 test scores is 84%. If the highest grade is 100%, what is the lowest possible score for one of these tests?
2. In diving contests, the highest and lowest scores from the individual judges are dropped before calculation of the final average. One diver received the following scores: 9.6, 9.4, 9.7, 9.8, 9.8, and 9.9. In this case, does the practice of dropping the highest and lowest scores help or hurt the final average?
3. In a basketball contest, 80% of a player's free throws had been made after 130 shots. If the player makes the next 5 shots, what is the percentage made?

Things to Think About:

- How are handicaps determined for bowlers?
- An average (or mean) may be affected by one very high or one very low number.

- Is the average always the best way to describe the "center" of a set of data?
- In many cases, the results from a small number of experiments will not provide a very accurate estimate of the probabilities involved.

Did You Know That?

- The highest individual bowling score for three consecutive games is 900 by Jeremy Sonnenfeld of Lincoln, Nebraska, on February 2, 1997. These are perfect games of rolling only strikes (knocking all ten pins down in one roll); the record cannot be broken.
- During World War II, John Kerrich, a prisoner of war, tossed a coin 10,000 times, recording 5067 heads.
- In 1777, Georges Louis Leclerc, Comte de Buffon, described the first example of a geometrical probability. He used a needle and a lined sheet of paper to approximate a value for pi (π).
- Probability was used to show that the selection of draftees for the Vietnam War was biased against certain groups in the country, in that they were overrepresented among the forces sent to Vietnam.

Resources:

Books:

- Eves, Howard. *An Introduction to the History of Mathematics*. Philadelphia, PA: Saunders Publishing Co., 1983.
- *The Guinness Book of World Records*. New York: Bantam Books, 1999.

Notes:



FigureThis!

Math Challenges for Families

Can you turn a **frown** into a **smile**?

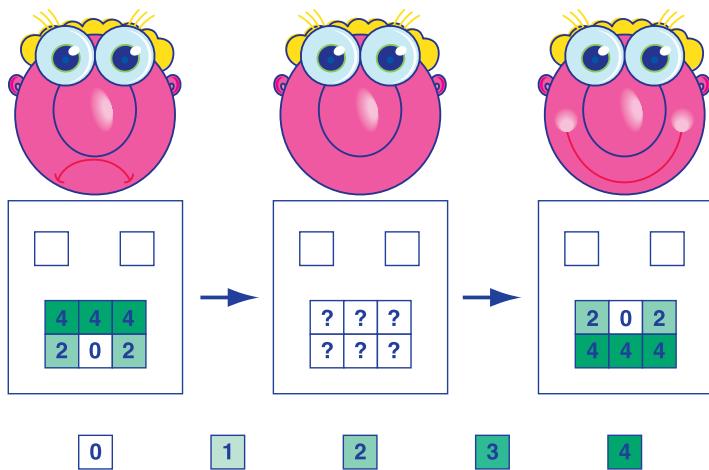
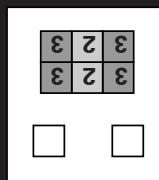
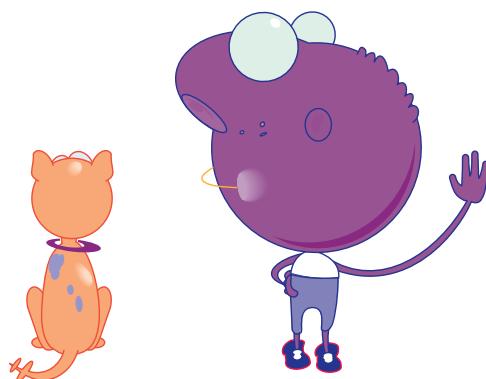


Figure This! In computer graphics, morphing can be used to gradually change one image to another. A frown can be "morphed" to a smile by changing the shades of the squares. As you morph the frown on the left to the smile on the right, what shades should be in the image in the middle?

Hint: Use only the numbers 0, 1, 2, 3, and 4 to represent the five different shades. What shade is halfway between 0 and 4?

Pictures or sounds can be "digitized" using a collection of numbers. Digitized images or sounds can be morphed or manipulated by arithmetic. Special effects in movies, pictures on computers, and sounds on some CDs use morphing.



One answer is shown here.

Answer:

FigureThis!

Get Started:

Replace each shade in the graphics by the corresponding number. Look at the grids of numbers and decide what "halfway" might mean.

Complete Solution:

Replaces each shade in the frown and the smile by the corresponding number, as shown here:

4	4	4
2	0	2

?	?	?
?	?	?

2	0	2
4	4	4

To find the appropriate numbers for each square in the middle image, average the numbers in the matching squares of the frown and the smile. The grid on the right below shows the appropriate numbers while the grid on the left shows the corresponding shades.

3	2	3
3	2	3

3	2	3
3	2	3

To create the illusion of a smooth transition, an actual morphing uses many more shades and has many more steps.

Try This:

- Design a set of flip cards so that an image changes shape gradually from one card to the next and so that when you "flip" through the cards, the image appears to move.
- Design grid patterns to represent the letters A and B. If you morphed the A to the B, what would the half-way image look like?
- Search the web for sites that feature morphed images.
- Ask your art or computer teacher if your school has software capable of morphing images.

Additional Challenges:

(Answers located in back of booklet)

1. Draw the grid that represents an image which is 70% transformed from the frown to the smile.
2. The town of Belgrade is halfway between Bozeman and Butte and in a line with them. Bozeman is located at mile marker 200, while Belgrade is at mile marker 155. At what mile marker is Butte?
3. A boat is sailing in a line from Miami to Bermuda. Miami is about 1000 miles south and 100 miles west of New York City. Bermuda is about 400 miles south and 300 miles east of New York City. What is the boat's location, in relation to Miami, when it has completed 30% of its trip?

Things to Think About:

- As noted in the solution to the challenge, an actual morphing uses many more shades (or colors) and has many more steps. How would you make an image appear to morph more slowly at the beginning and faster at the end?

- How do police "identi-kits" use morphing?

- Most computer images consist of colors, not just shades of black and white. How would you assign numbers to different colors?

Did You Know That?

- The musical equivalent to morphing is the *segue*.
- The dimensions of a typical computer screen are approximately 800 by 600 squares, or pixels. Each of these almost one-half million pixels may display one of thousands of different colors and shades.
- One step in the morphing of a full-screen image can be completed in less than 0.1 seconds.
- At a typical movie theater, viewers are shown 24 different frames (or images) per second.
- To create the movie *Toy Story II*, animators at Pixar Studios drew about 4000 key storyboard sketches by hand. To create frames for these key sketches, they used computers to manipulate elements on the screen. After that, they relied on software to fill screens using variations of morphing.
- In a style of painting known as *pointillism*, painters apply dots of paint to the canvas to create images. The French artist Georges Seurat (1859–1891) is perhaps the best-known artist who used this style.
- *Time Magazine* in the fall of 1993 published an article entitled "The New Face of America: How Immigrants Are Shaping the World's First Multicultural Society." The cover of that issue featured a "morphed" image of a woman using various racial and ethnic features over the caption "The New Face of America."

Resources:

Magazine:

- Porter, T., and G. Susman. "Creating Lifelike Characters in Pixar Movies." *Communications of the ACM* 43 (No. 1, 2000): 25–29.
- Liss, Steve. "The New Face of America: How Immigrants Are Shaping the World's First Multicultural Society." *Time* 142 (Fall 1993): 3–7.

Websites:

- hera.itc.it:3003/list_project.html#spotit
- www.chilhavisto.rai.it/CIV/English/mele.htm
- odyssey.ucc.ie/www/misc/model1.html
- www.publish.csiro.au/cyberscience/helix/TH60/TH60A1.htm
- www.pixar.com
- www.toystory2.com
- www.siggraph.org
- www.awn.com



FigureThis!
Math Challenges for Families

Is there room for all these gifts?



Figure This! Exponent has won the grand prize in a radio sweepstakes. His gifts will be delivered over 10 days. On the first day, he gets a kitten. On the second day, he gets two hummingbirds and a kitten. On the third day, he receives three rings, two hummingbirds, and a kitten. This pattern continues for seven more days! The remaining gifts, in order of appearance, include books, boxes of candy, bracelets, bouquets, cologne, CDs, and puppies.

After 10 days, which gift did he receive most?

Hint: How many of each gift did Exponent receive on each day? Make a table to organize this information.

Organizing information and making decisions based on the information are necessary to solve problems. Census takers, librarians, managers, and demographers have methods to organize large quantities of information.

The most numerous gifts are boxes of candy and bracelets (30 each).

Answer:

FigureThis!

Get Started:

One way to approach this problem is to list the gifts received on each day. This information could be organized in a table like the one below.

Day	1	2	3	4	5	6	7	8	9	10	Total
Kitten	1	1	1								
H-Bird		2	2								
Ring			3								
Book											
Candy											
Bracelet											
Bouquet											
Perfume											
CD											
Puppy											

Complete Solution:

Use the pattern described in the challenge to complete the table of gifts received on each day. Then calculate the total number of each type of gift. As shown below, Exponent receives a total of 30 boxes of candy and 30 bracelets.

Day	1	2	3	4	5	6	7	8	9	10	Total
Kitten	1	1	1	1	1	1	1	1	1	1	10
H-Bird		2	2	2	2	2	2	2	2	2	18
Ring			3	3	3	3	3	3	3	3	24
Book				4	4	4	4	4	4	4	28
Candy					5	5	5	5	5	5	30
Bracelet						6	6	6	6	6	30
Bouquet							7	7	7	7	28
Perfume								8	8	8	24
CD									9	9	18
Puppy										10	10

Try This:

- Build a model staircase using sugar cubes (or other small blocks). How many cubes does it take to build a 12-step staircase?
- Ask your local librarian what kinds of books are most popular in the library and how the decision was made.

Additional Challenges:

(Answers located in back of booklet)

1. What is the total number of gifts received in the challenge?
2. Find the solution to this English nursery rhyme.

As I was going to St. Ives,

I met a man with 7 wives.

Each wife had 7 sacks.

Each sack had 7 cats.

Each cat had seven kits.

Kits, cats, sacks, and wives,

How many were going to St. Ives?

3. If each of the 25 people in a room shakes the hand of each of the other people in the room and no person shakes any other person's hand twice, how many handshakes take place?
4. What would happen if the order of the gifts in the challenge were reversed? In other words, suppose Exponent received a puppy on the first day, two CDs and a puppy on the second day, and so on. Would your solution be the same?

Things to Think About:

- How could you determine the total value of the gifts described in the challenge?
- How many gifts would be received if the pattern continued for 365 days?
- How does the counting in this challenge compare to the counting in Challenge #7 "Double or Not"?

Did You Know That?

- The numbers of gifts received on successive days form this pattern: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. These are triangular numbers. The total handshakes for increasing numbers of people also are triangular numbers, as are the total possibilities for double-dip ice cream cones for increasing numbers of flavors.
- A sequence like 4, 7, 10, 13, ..., in which you add 3 each time is an example of an arithmetic sequence.
- A sequence like 1, 5, 25, 125, ..., in which you multiply by 5 each time is an example of a geometric sequence.

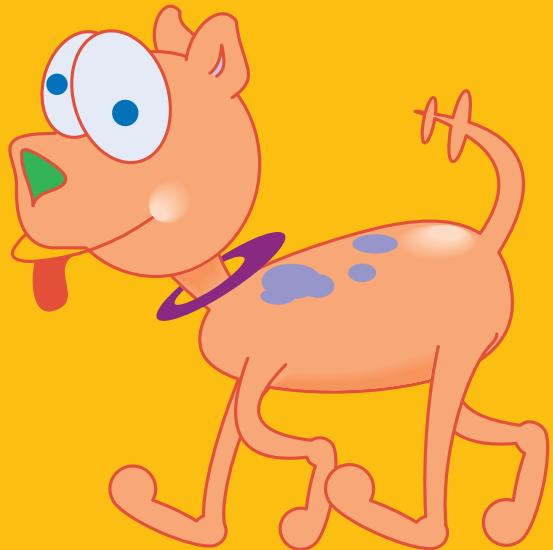
Resources:

Books:

- Billstein, R., S. Libeskind, and J. Lott. *A Problem Solving Approach to Mathematics for Elementary Teachers*. Reading, MA: Addison Wesley Longman Publishing Co., 2000.
- Lobel, Arnold. *Book of Mother Goose*. New York: Random House, 1986.

Notes:

Tangent





FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the
Additional Challenges section
of each Challenge.

Challenge 73

- If the rectangle also is a square, then it has four lines of symmetry. If not, then it has two, neither of which is a diagonal.
- Find two lines of symmetry. They will intersect in the center.
- A triangle with at least two sides of the same length (an isosceles triangle).

Challenge 74

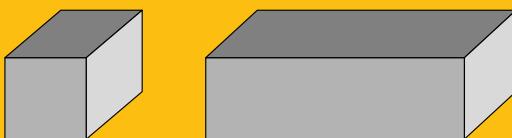
- Polygon will win this race. Although they will be tied at the 45-meter mark, Polygon will run the last 5 meters faster than Exponent.
- At their previous speeds, Polygon will run 500 meters (10 laps) in the same time that Exponent runs 450 meters (9 laps). Therefore, after 10 laps, Polygon will be 1 lap ahead of Exponent.
- They will meet about 26.3 meters from Polygon's starting point (or 23.7 meters from Exponent's starting point).

Challenge 75

- The one with the glide ratio of 14%.
- The ratios 0.4 and 1/2.5 are both the same as 2/5.
- Yes.
- The glider will land at a horizontal distance of about 350 feet from the base of the cliff. Its actual flight path is about 353 feet long.

Challenge 76

- No
- 20 maximum
- Answers will vary. One possible solution is shown here:



Challenge 77

- Answers may vary. For example, box-office receipts of \$9000 in week 5 and \$6600 in week 6 will yield a profit of \$1020.
- About \$8333 for week 5 and about \$7142 for week 6.
- \$0.60.
- About \$6615.

Challenge 78

- 20%.
- It helps. The average of the six scores is 9.7. After dropping the high and low scores, the average is 9.725.
- About 81%.

Challenge 79

- To create an image that is 70% of the way to the smile, you must identify the number that is 70% (or 0.7) of the way from each number on the smile to its corresponding number on the frown. For example, the number 4 on the frown corresponds to 2 on the smile. The difference of 4 and 2 is 2. Find 70% of this difference, $0.7(2) = 1.4$. Since $4 - 1.4 = 2.6$, the corresponding square on the image should be 3 (rounding to the nearest whole number).

3	1	3
3	3	3

3	1	3
3	3	3

- 110.
- It will be 180 miles north and 120 miles east of Miami.

Challenge 80

- 220.
- 1 (The man with the wives was going the other way.)
- 300.
- Yes.