Math Challenge #69 MISADDRESSED

Figure This! Math Challenges for Families

How could I send the check and not pay the bill?

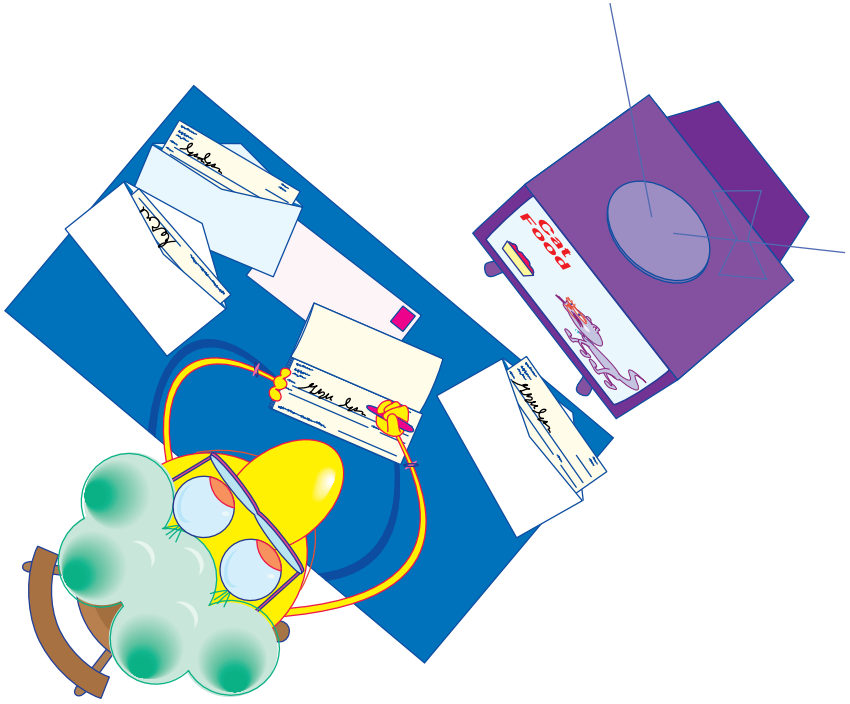


Figure This! While watching television, Tessellation placed three checks in three separate addressed envelopes. If she had paid no attention to which check went in which envelope, what is the chance that each check was in the correct envelope?

Hint: Make a list of all the possible outcomes.

Probability is the mathematics of chance. Estimating the chance of rain, the effectiveness of a treatment, and whether a person will have an accident are important to weather forecasters, doctors in diagnosing illnesses, and insurance companies.

Answer:

The probability is 1/6.

Figure This!

Get Started:

Label the checks *a, b,* and *c,* and label the envelopes *A, B,* and *C.* Assume check *a* belongs in envelope *A.* In how many ways could check *a* be placed in an envelope? Once *a* is placed, how many choices are there for *b*?

Complete Solution:

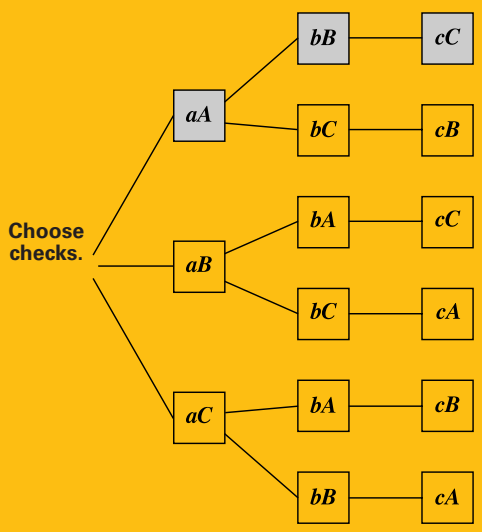
There are several ways to approach this problem.

- The table shows all the possibilities if the checks are labeled *a, b,* and *c,* and the corresponding envelopes *A, B,* and *C.*

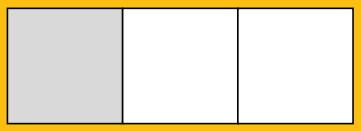
A	B	C
a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a

There are six different possibilities, each one of them equally likely. Only one, however, has all three checks in the matching envelopes. Since exactly 1 case out of 6 has the checks in the correct envelopes, the probability is 1/6.

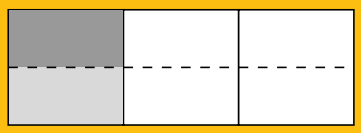
- Another way to solve this problem is draw a tree diagram. In the following diagram, *aA* indicates that check *a* is in envelope *A.* Like the table shown earlier, the tree diagram indicates that there is only 1 correct outcome out of 6 equally likely possibilities. Therefore, the probability of the correct placement is 1/6.



- You also can consider the problem geometrically. Draw a rectangle like the one below to represent all the possibilities in this situation. Because the chance that the first check is placed in the correct envelope is 1 in 3, shade 1/3 of the rectangle to indicate this outcome.



If the first check is placed correctly, there are only two possible envelopes for the second check. This means that the chance that the second check is placed in the correct envelope is 1 in 2. To indicate this outcome, darken 1/2 of the previously shaded area.



If the first two checks are placed correctly, then the final check also must be placed in the correct envelope. No further shading is necessary. The darker shading represents 1/6 of the entire rectangle, so the probability that all three checks will be placed correctly is 1/6.

Try This:

- Label four pieces of paper *a, b, c,* and *d* and four envelopes *A, B, C,* and *D.* Turn all the items over and mix them up. Then, without reading the labels, place a piece of paper into each envelope. Did you place all four in the right envelopes? Predict how many times you would place all four correctly if you repeated the experiment 29 more times. Then test your prediction by doing the experiment again.
- Draw a large circle on a sheet of paper and divide it into four equal-sized sections (quarter circles). Label each section. Bend a small paper clip into a pointer. Place the pointer at the circle's center. Spin the pointer on the circle. In which section did it stop? Spin the pointer a total of 50 times and record the results. If you displayed your results in a circle graph, what would it look like?

Additional Challenges:
(Answers located in back of booklet)

1. If you flip two fair coins, what is the probability that you obtain one head and one tail?
2. If you roll one ordinary six-sided die, what is the probability of obtaining a 7?
3. What is the sum of the probabilities of all the possibilities in an event?

4. Is the probability of getting a passing grade (at least 60%) by guessing on a 20-question true-false test greater than, less than, or equal to the probability of getting at least 60% correct by guessing on a 10-question true-false test?

5. In the challenge, what is the probability of getting at least one check in the correct envelope?

Things to Think About:

- What is the probability that it will rain in your hometown for seven consecutive days?
- If the probability of an outcome is 0.35, would you expect it to occur exactly 35 times out of 100?
- Does a batting average describe the probability of getting a hit?

Did You Know That?

- Mathematician Abraham de Moivre (1667–1754) was a pioneer in the field of probability. According to legend, the aging de Moivre observed that he needed an extra quarter hour of sleep each day. When he needed 24 hours of sleep, he died.
- Antoine-Nicolas Caritat, Marquis de Condorcet (1743–1794), used probability to argue that capital punishment should be abolished. He reasoned that, however great the probability that each guilty verdict was correct, there remained a small chance of error. As the number of guilty verdicts increased, so would the probability that an innocent person would be executed. Condorcet himself was condemned to death during the French revolution.
- The word *probability* appears to receive its first mention in a 1477 commentary on Dante's *Divine Comedy.*

Resources:
Book:

- Billstein, R., and J. Williamson. *Middle Grades Math Thematics, Book 3.* Evanston, IL: McDougal Littell, 1999.
- Eves, H. *An Introduction to the History of Mathematics.* Philadelphia, PA: Saunders Publishing Co., 1983.

Notes:

Math Challenge #70 SODA

FigureThis! Math Challenges for Families

Does it make a difference where you shop?

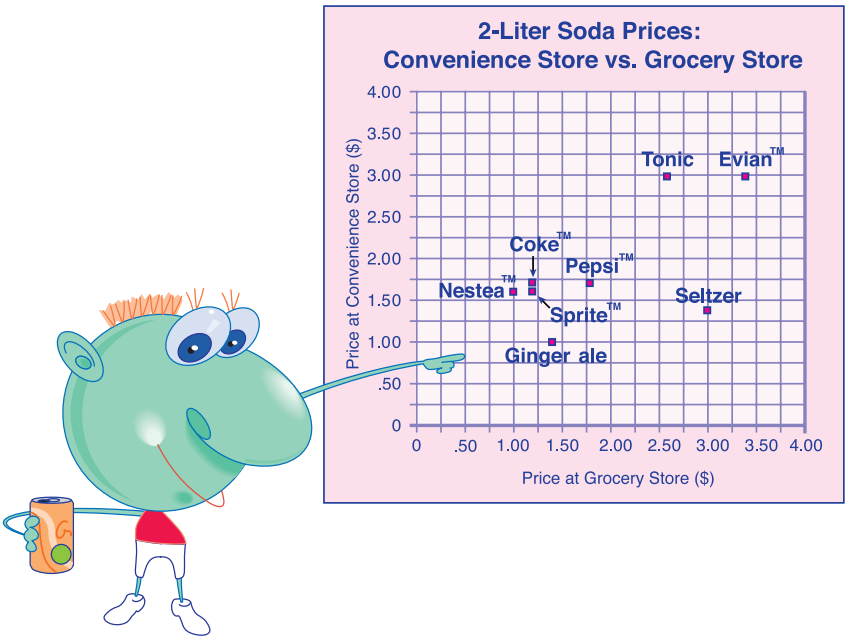


Figure This! Helix checked the prices for two-liter bottles of his favorite soft drinks at two different stores. A graph of the data is shown here. How do the prices compare at the two stores?

Hint: Nestea, the point ($1.00, $1.60), costs $1.00 at the grocery store and $1.60 at the convenience store. What would the graph look like if the two stores' prices were the same for each item?

Graphs are a visual way to organize and display information, and scatter plots are particularly useful to understand how two quantities are related. Marketing experts, bankers, stockbrokers, demographers, and those in the entertainment business use graphs to understand patterns and trends in their fields.

Answer:

Half of the items are cheaper at the grocery store. The convenience store's prices seem lower overall, however, due to the big difference in the prices of seltzer.

Figure This!

Getting Started:

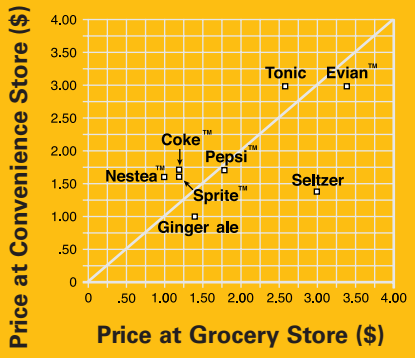
Draw in the line that would show all of the prices equal, the $y = x$ line going from the point (0,0) in the lower left corner to the point (4.00,4.00) in the upper right. What does it mean for a soft drink to be below the line?

Complete solution:

There are several ways to approach this problem.

- Using the hint, draw the line $y = x$ on a copy of the graph. A point below the line indicates that the item's price at the convenience store was less than its price at the grocery store. For example, the point for Ginger ale is below the line. It cost about $1.40 in the grocery but only $1.00 in the convenience store. Four of the eight points are below the line, indicating that the grocery store was more expensive for half the items, while the convenience store was more expensive for half the items.

2-Liter Soda Prices: Convenience Store vs. Grocery



- Another way to examine pricing at the two stores uses the vertical distance from each data point to the line $y = x$. On the scatterplot, this distance represents the difference in prices at the two stores. The table below lists the estimated difference for each item in the column for the store where it costs more. The sums of the differences show that prices in the grocery store are higher overall.

	Approximate Difference at Most Expensive Store	
Soda	**Grocery**	**Convenience**
Nestea™		$0.60
Coke™		$0.50
Sprite™		$0.40
Pepsi™	$0.00	$0.00
Seltzer	$1.60	
Evian™	$0.40	
Tonic		$0.40
Ginger Ale	$0.40	
Total	$2.40	$1.90

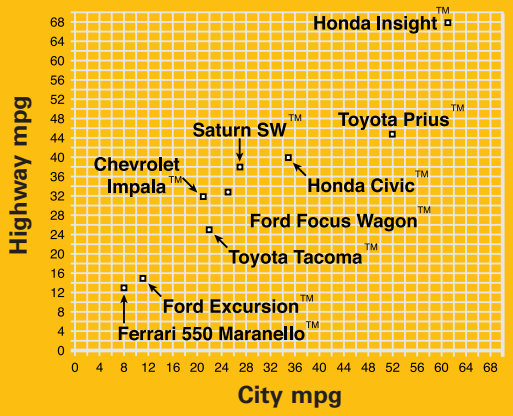
Try This:

- List eight or nine items you regularly buy in a grocery store; then use the grocery ads in a newspaper for two different stores, or check the prices at the stores to compare their prices.
- Find a scatterplot in a newspaper or magazine. What trends or patterns do you see?

Additional Challenges:
(Answers located in back of booklet)

1. The scatterplot shows the estimated mileage per gallon for city driving and for highway driving for the cars as reported in the *USA TODAY* on 2/9/01.

Vehicle miles per gallon



   a. For which car does it look like the data might be wrong?

   b. Which car has the greatest change from estimated mpg in the city to estimated mpg on the highway? the least?

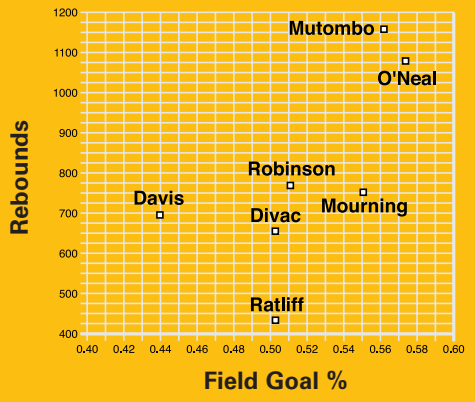
2. Without the Toyota Prius, one equation that describes the general relationship between city and highway mileage is: $H = 5.89 + 1.30C$, where H is the highway mpg and C is the city mpg.

   a. Use this equation to predict the highway mpg for a Ferrari 456M GT/GTA, which gets about 10 miles per gallon in the city.

   b. The actual highway mpg for this car, as reported in the American Council for an Energy Efficient Economy's *Green Book,* is 15. How good was your prediction?

3. The scatterplot below shows some statistics from the 2000 season for seven NBA all-star centers: Antonio Davis, Alonzo Mourning, Theo Ratliff, Shaquille O'Neal, Vlade Divac, David Robinson, and Dikembe Mutombo. Use the information given to rank the players' performances.

2000 Offense and Defense for All-Star Centers



Things to Think About:

- What would the scatterplot in the challenge look like if the two stores had very similar prices?
- What is the leading soft drink, in terms of sales?
- How could your local television station use scatterplots and other graphs?
- How does what you can learn from a table of information compare to what you can learn from a scatterplot?
- Why is the point (0,0) not necessarily a useful place to begin the axes for some graphs?
- Do you think you could tell the difference between two brands of cola in a blind taste test?

Did You Know That?

- Some companies use a *loss leader*—an item advertised at a very low price—to entice consumers into their stores. They hope to make up the loss by selling other goods for higher profits.
- Many of the soft drinks sold in the United States are in liter bottles rather than quarts.
- Several years ago, Coca-Cola™ tried to market their soda in a brown bottle, but consumers did not like it.

Resources:

Books:

- American Council for an Energy Efficient Economy. *Green Book: The Environmental Guide to Cars & Trucks, Model Year 2001.* [This book is available online at **www.greenercars.org**.]
- *The World Almanac and Book of Facts 2001.* Mahwah, NJ: World Almanac Books, 2001.

Website:

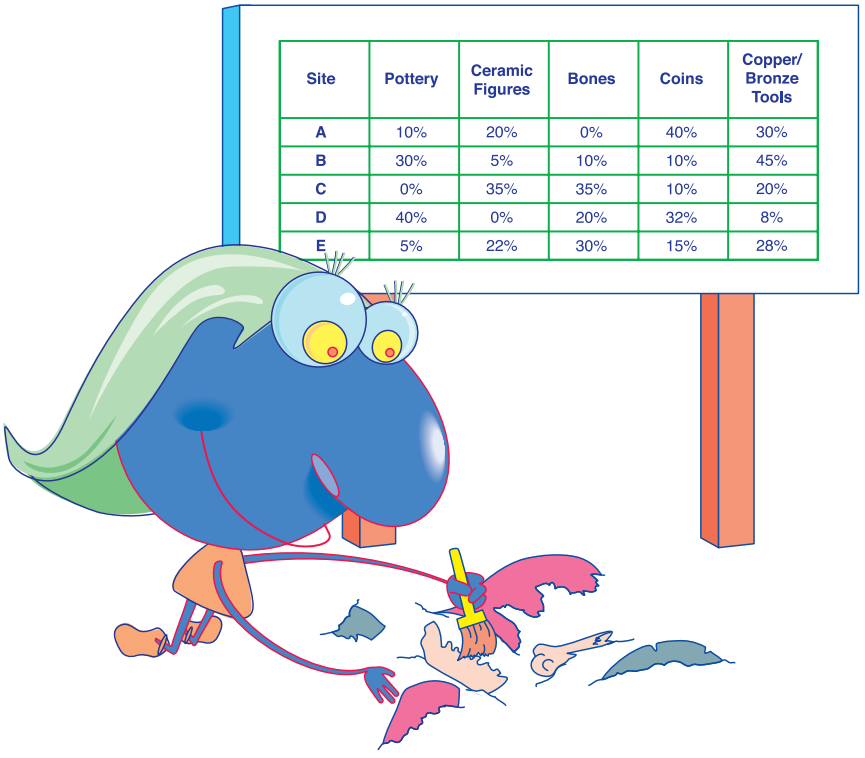
- **www.greenercars.org**

Notes:

Math Challenge #71 ARCHEOLOGY

FigureThis! Math Challenges for Families

Site A! Site B! How alike are we??



Site	Pottery	Ceramic Figures	Bones	Coins	Copper/ Bronze Tools
A	10%	20%	0%	40%	30%
B	30%	5%	10%	10%	45%
C	0%	35%	35%	10%	20%
D	40%	0%	20%	32%	8%
E	5%	22%	30%	15%	28%

FigureThis! **Archeologists noticed differences in the types of evidence found at five dig sites. The table shows the percentages of material found at each site. Using this information, which two sites seem to be most alike?**

Hint: What is the difference in the percentages of pottery found at sites A and B? What about the percentages of bones?

Numerical criteria can be used to compare objects. Determining likenesses or differences between two sets of information is important for geologists searching for oil, doctors deciding different treatments' effects, and marketing experts deciding on comparable advertising strategies.

Answer:

Sites C and E have the most similar percentages.

Figure This!

Get Started:

Pair the sites and then make a table of the differences found by subtracting the smaller percentage from the larger for each category. For example, using sites A and B, the differences are (30% – 10%), (20% – 5%), (10% – 0%), (40% – 10%), and (45% – 30%). The total of these differences is 20% + 15% + 10% + 30% + 15%, or 90%.

Pair of Sites	Pottery	Ceramic Figures	Bones	Coins	Copper/ Bronze Tools	Sum of Differences
A and B	20%	15%	10%	30%	15%	90%

Find the total difference for each pair of sites.

Complete Solution:

Using the hint, find the difference in the percentages of each material at each pair of sites and sum these differences. The resulting information can be arranged in a table as shown. In this table, the cell in row A, column B, corresponds to the sum of the differences for sites A and B, or 90%. (Since there is no difference between a site and itself, the corresponding cells have 0% as entries.)

	A	B	C	D	E
A	0%	90%	100%	100%	64%
B	90%	0%	110%	84%	84%
C	100%	110%	0%	124%	36%
D	100%	84%	124%	0%	104%
E	64%	84%	36%	104%	0%

The pair of sites with the smallest sum (36%) is C and E. This indicates that the percentages of materials at the two sites are the most similar.

Try This:

- Look at the shoes in your house. Identify at least four characteristics that can be used to describe shoes. Categorize the shoes according to these characteristics. Using these criteria, which pairs of shoes are most alike?
- Which of the cereals in your cupboard are most alike in terms of nutritional value?
- Use the web to find other examples of mathematics in the work of archeologists.

Additional Challenges:
(Answers located in back of booklet)

1. Look at the table given in the complete solution to the challenge. If you draw a line from the upper left-hand corner of the table to the lower right-hand corner, the entries in the part above the line match those below the line. Why is this true?
2. Using data collected from about 20 sites, archeologist T. Patrick Culbert of the University of Arizona estimated that in the past, there were approximately 200 people per square kilometer in the southern lowlands of Central America. How does this figure compare to the most recent population density of Minnesota. (In 2000, the state had a population of 4,775,508 and an area of 79,617 square miles. **Note:** 1 square kilometer equals 0.386102 square miles.)
3. The table below shows the prices for bread, milk, eggs, coffee, and orange juice at five neighborhood markets. Which two stores seem to have the most similar prices? Which stores seem to differ the most?

Store	Bread	Milk	Eggs	Coffee	Orange Juice
A	$0.89	$1.90	$1.79	$5.99	$2.39
B	$1.19	$1.65	$1.75	$6.24	$2.49
C	$1.09	$1.99	$1.65	$5.33	$3.24
D	$1.49	$1.75	$2.05	$5.88	$2.75
E	$0.99	$1.89	$2.13	$6.12	$2.33

Things to Think About:

- How can archeologists tell the age of a bone or piece of pottery?
- What do you think the archeologists of the future will find when they excavate your hometown 1000 years from now?
- What causes ancient cities to be destroyed and become buried under land or water?
- How would you compare the climates of several different cities?

Did You Know That?

- Archeologists also explore under water to find buildings and relics from earlier cultures.
- In ancient Rome, it was a mark of status to be buried by a busy road, where passers-by could stop and read the tomb's inscription. This was thought to give the dead person a certain measure of immortality.
- The coins found at archeological sites help historians understand the trade and economic patterns of ancient cultures.

- Archeologists often use dental picks and cotton swabs to scrape away dirt and clay from coins, bones, pottery, and other finds.
- By analyzing differences in embroidery, color, and design, archeologists can sometimes identify the village in which an item of clothing was made.
- Evidence about grid plans for cities, ceremonial avenues, apartment compounds, and plaza structures has led archeologists to think that the several early cities from the Basin of Mexico resemble the ancient Aztec city of Teotihuacan.

Resources:
Books:

- Gardner, M. *Time Travel and Other Mathematical Bewilderments*. New York: W. H. Freeman and Co., 1988.
- Smith, M. *The Mesoamerican Urban Landscape from Teotihuacan to the Aztecs*. Paper presented at the conference, "Archeology of Complex Societies: Centripetal and Centrifugal Forces," October 21, 1995, California State University, San Bernardino.
- Purdy, J. "Ancient Voices Speak Again in Carthage." *Research Reporter*. Vol. 25, No. 2 (Winter 1995). University of Georgia.

Website:

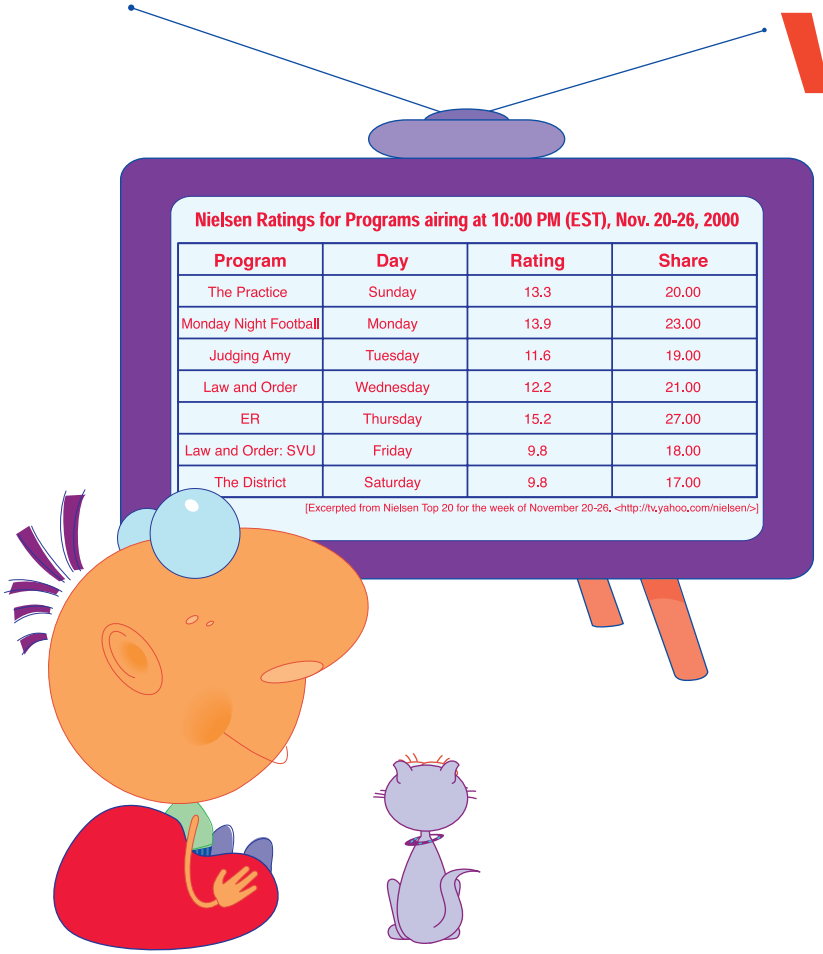
- **www.ovpr.uga.edu/rcd/95w/carthage.html**

Notes:

Math Challenge #72 TV RATINGS

Figure This! Math Challenges for Families

When DO most people watch television??



Nielsen Ratings for Programs airing at 10:00 PM (EST), Nov. 20-26, 2000

Program	Day	Rating	Share
The Practice	Sunday	13.3	20.00
Monday Night Football	Monday	13.9	23.00
Judging Amy	Tuesday	11.6	19.00
Law and Order	Wednesday	12.2	21.00
ER	Thursday	15.2	27.00
Law and Order: SVU	Friday	9.8	18.00
The District	Saturday	9.8	17.00

[Excerpted from Nielsen Top 20 for the week of November 20-26. <http://tv.yahoo.com/nielsen/>]

Figure This! **On which night of the week did the greatest number of US households watch the most popular television shows at 10:00 P.M.?**

Hint: $\frac{\text{Rating}}{\text{Share}} \times 100$ = percent of TV owning households with TVs on watching a particular show at that time. How can you use the Nielsen data to determine the percentage of households watching television on a given night?

Making comparisons involves careful definitions and use of mathematics. Actuaries use comparisons to create insurance rates, biologists use this type of mathematics to compare different species in different eras, as do archaeologists for carbon dating artifacts.

Answer:

Sunday night.

Figure This!

Get Started:

In the Nielsen Company's rankings, *share* describes the percentage of active television sets on, and tuned to a particular program, out of all TV owning households, while *rating* describes the percentage of households who own televisions that are tuned to a particular show. A 15.2 rating means 15.2% of all households owning televisions were tuned to *ER*. A 27.0 share means that 27.0% of all those households with a television on at the time were tuned to *ER*. For *ER*,

$$\frac{\text{Rating}}{\text{Share}} = \frac{15.2}{27.0} \times 100 \approx 56$$

Thus, about 56% of the TV owning households had their TVs tuned to *ER*. What were the percentages for the other shows?

Complete Solution:

Using the relationship between the Nielsen rating and share in Getting Started, calculate the percentages for shows.

Program	Day	Rating	Share	% of TV owning Households With TV on
The Practice	Sunday	13.3	20.0	66.5%
Monday Night Football	Monday	13.9	23.0	60.4%
Judging Amy	Tuesday	11.6	19.0	61.1%
Law and Order	Wednesday	12.2	21.0	58.1%
ER	Thursday	15.2	27.0	56.3%
Law and Order: SVU	Friday	9.8	18.0	54.4%
The District	Saturday	9.8	17.0	57.6%

The highest percentage occurred on Sunday night.

Try This:

- Using a website, TV rating, or a newspaper, find the rating and share of your favorite TV show.
- Look on the web or in the library to determine how the Nielsen ratings are done.
- Check with your friends and neighbors to see if they have been surveyed for TV ratings.

Additional Challenges:
(Answers located in back of booklet)

1. For Thursday November 25, 2000, at 8:00 PM EST, *Friends* had a 9.6 rating and a 15.0 share. There were approximately 102.2 million TV owning households in the US at that time.

   a. What percent of the households had their TV on during *Friends?*

   b. Approximately how many households were watching *Friends?*

2. Companies that advertise on television are usually interested in the number of viewers of a certain age or gender. For example, adults from 18 to 49 are popular with many advertisers. In the United States, this group includes about 129 million people. On November 29, 2000, a *David Blaine Special* received a 7.3 rating and a 10.1 share among this group. If 1 rating point represents 1% of the target group, how many adults ages 18–49 watched this special? What percentage of the group were watching television at the time?

3. To compare the cost of advertising on two or more programs, advertisers use an index called the CPM (cost to reach 1000 households). The CPM is calculated as shown here:

$$\text{CPM} = \frac{\text{Cost per 30 second advertisement}}{\text{thousands of households watching the show}}$$

   In 2000, the cost of 30-second ad on *ER* was $600,000; its rating was 15.2. The cost of a 30-second ad on *Diagnosis: Murder* was $51,000; its rating was 6.3. Compare the CPM for these two shows. (Each rating point represents 1,022,000 households.)

Things to Think About:

- What other types of entertainment are rated?
- Why are both rating and share important in the television industry?
- How is a song's popularity determined?
- Why do advertisers and networks care about the age and gender of viewers?
- How would you collect information on television-viewing habits?

Did You Know That?

- Nielsen Media Research measures audience levels using both electronic meters and diaries. The national sample is a combined sample of the "Nielsen families" using meters and diaries. It consists of about 5000 homes in 210 different television markets. The national Nielsen ratings also include key demographic information such as the number of adults ages 25–54 watching a particular show.
- The highest-rated TV show as of March 2001 was the final episode of *M*A*S*H*, which received a rating of 60.2 and a share of 77 on February 28, 1983.
- As of March 2001, 12 of the 25 highest-rated shows were Super Bowl broadcasts.
- In January 2001, the average number of televisions per US household was 2.4.
- About 60% of US teens have a television in their bedroom.
- In 1999, the automotive industry outspent all other industries for television advertising.

Resources:

Books:

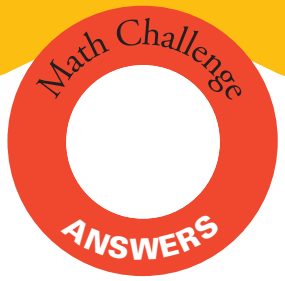
- Hall, Robert W. *Media Math.* Chicago, IL: NTC Business Books, 1991.
- *Consumer Reports,* January 2001.

Websites:

- **www.nielsenmedia.com/whatratingsmean**
- **tv.yahoo.com/nielsen**
- **faculty.washington.edu/baldasty/Feb3.htm**
- **www.infoplease.com/ipsa/A0110473.html**

Notes:

Tangent



FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

Figure This!

Challenge 65

1. The answer may vary. The pattern shows that the factors to go from one complexion type to another are 3, 2.5, and 2. With this pattern, the next factor is 1.5 giving 45 as the recommended SPF.

2. The potato will be done in about 24 more minutes.

3. About 17 minutes.

Challenge 66

1. The ends of one runway should be labeled 26 and 8; the others should be 35 and 17.

2. The ends of one runway should be labeled 13 and 31; the ends of the other should be 4 and 22. The ends of the north-south runway are 18 and 36.

3. 190°.

Challenge 67

1. Probably, since 10° C is 50° F.

2. 30° C.

3. About 67° F.

4. –40°.

5. No.

Challenge 68

1. Graph a.

2a.

	Liked *Titanic*	Disliked *Titanic*	Totals
Liked *Star Wars*	70	43	113
Disliked *Star Wars*	50	37	87
Totals	120	80	200

2b. For the people in this study, there does not seem to be a strong association between liking one movie and liking the other. About 58%—a little more than half (70/120)—of those who liked *Titanic* also liked *Star Wars, Part IV*.

3. The graph reveals little connection between exercise and computer use. Most people in the study frequently use computers, whether they exercise or not.

Challenge 69

1. 1/2.

2. 0.

3. 100% or 1.

4. less than.

5. 4/6 or 2/3

Challenge 70

1a. Toyota Prius; it looks like city mpg and highway mpg were reversed.

1b. Greatest change is either the Chevrolet Impala or the Saturn SW at 11 mpg, least is Toyota Tacoma at 3 mpg.

2a. The equation predicts a highway mpg of about 18.89 or about 19.

2b. The prediction is not far off from the actual mileage.

3. Answers may vary, depending on the relative value assigned to field-goal percentage and rebounds. One approach identifies the highest displayed value in each category and uses it as an "ideal." In this case, the ideal point would be (0.574, 1157). If players are ranked according to this ideal, they fall in the following order: O'Neal, Mutombo, Mourning, Robinson, Divac, Davis, and Ratliff.

Challenge 71

1. The parts are symmetric because the sum of the differences between two sites is the same whether you compare A to B or B to A.

2. The population density of Minnesota is about 60 people per square mile, or about 23 people per square kilometer. This means that the population density in the study area was about 9 times as great.

3. The prices at stores A and E are the most similar, as their sum of differences is only $0.64. The prices at stores C and E are the most different, with a sum of $2.38.

Challenge 72

1a. 64%

1b. 9.8 million

2. About 9.42 million adults ages 18 – 49 watched the show. Of the target group, 72.3% were watching television.

3. The CPM for *ER* was $38.62; the CPM for *Diagnosis: Murder* was $7.92.