Math Challenge #5

UPSIDE DOWN?

Figure This!
Math Challenges for Families

How would you hang this sign

Figure This! What letters, when written in lowercase, can be read the same upside down as right side up?

Hint: Write out each lowercase letter and look at it in different ways.

Symmetry is a basic geometric concept. Understanding how one part of an object mirrors the rest is important in art, design, medicine and other fields.

Answer:

Depending on the way the letters are written or printed, l, o, s, x and z can be read the same upside down and right side up.

Figure This!

Get Started:

Print the lowercase letters of the English alphabet. Turn your paper upside down.

Complete Solution:

Letters that read the same upside down are l, o, s, x, and z.

Try This:

- Distort the letters a bit and see if you can write your name so that it can be read either backward or forward or upside down. For example, the words "upside down" above are distorted so that they can be read when turned halfway around.
- Look around and see if you can find shapes that look the same backward and forward or upside down and right side up.
- Look in the Yellow Pages to find logos of companies that are the same when looked at from different directions.

Additional Challenges:

1. When written in lowercase letters, the name of one professional sports team can be read the same both up right and upside down. What team is it?
2. Create words that are spelled the same backward or forward. Such words are palindromes.
3. Create sentences that read the same backward or forward when punctuation is ignored.
4. What times on a digital clock can be read the same in different directions?
5. Some letters can be rotated 180˚ (or a half circle) to form different letters; for example "d" becomes "p." What other lowercase letters can be rotated to form different letters?

Things to Think About:

- Does a human figure have symmetry?
- How is symmetry used in the design of a pinwheel?
- Is there symmetry in nature?

Try This:

Find the symmetry, if any, in each of the following:

- a plate
- a bowl
- a fork
- a chair

Did You Know That?

- Scott Kim calls anything that can be read in more than one way an "inversion." He used such writing in developing new font systems for computers.
- Graphic artist John Langdon of Philadelphia, has developed writing similar to Kim's that he calls "ambigrams."
- Artist M. C. Escher used many types of symmetries in designing his famous tessellations.
- A circle and a sphere have the most symmetries of any geometric objects.
- A figure with both horizontal and vertical lines of symmetry also has 180˚ rotational symmetry, but not the other way around.

Resources:

Books:

- Ernst, Bruno. *The Magic Mirror of M. C. Escher.* Stradbroke, England: Tarquin Publications, 1985.
- Kim, Scott. Inversions: *A Catalog of Calligraphic Cartwheels*. Petersborough, NH: BYTE Books, 1981.
- Kim, Scott. *Poster: Alphabet Symmetry*. White Plains, NY: Cuisenaire/Dale Seymour Publications. **www.cuisenaire-dsp.com**
- Kim, Scott. *Poster Set: Inversions*. White Plains, NY: Cuisenaire/Dale Seymour Publications. **www.cuisenaire-dsp.com**
- Langdon, John. *Wordplay: Ambigrams and Reflections on the Art of Ambigrams.* New York: Harcourt Brace Jovanovich, 1992.
- McKim, Robert H. *Experiences in Visual Thinking*. Belmont, CA: Brooks/Cole Publishing Company, 1972.
- McKim, Robert. *Thinking Visually*. White Plains, NY Cuisenaire/Dale Seymour Publications, 1997. **www.cuisenaire-dsp.com**

Credits:

Answers to Additional Challenges:

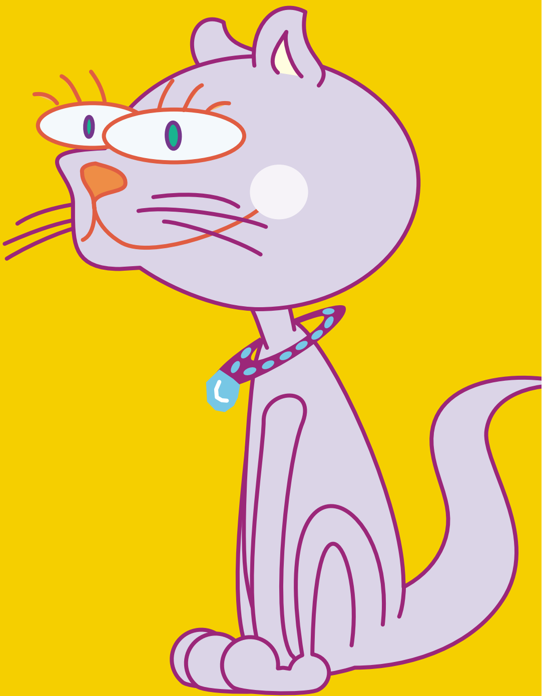
(1.)
Phoenix Suns. The word "suns" reads the same upside down.

(2.)
Words like MOM, DAD, and TOOT qualify. The international distress symbol, SOS, is a set of letters that can be read the same backward, forward, up right, and upside down as well as when turned halfway around.

(3.)
"Madam I'm Adam."

(4.)
Numbers such as 0, 1, 2 and 8 read the same in several ways on a digital clock. Times on a digital clock that read the same backward as forward include 10:01, 11:11, and 12:21

(5.)
The letter "m" becomes "w," "n" becomes "u," and "b" becomes "q."

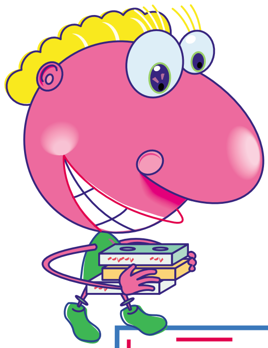


Math Challenge #6

WHICH WAY?

Figure This!

Math Challenges for Families



Oh, which way do I go?

How can you go straight to the store when there are buildings in the way?

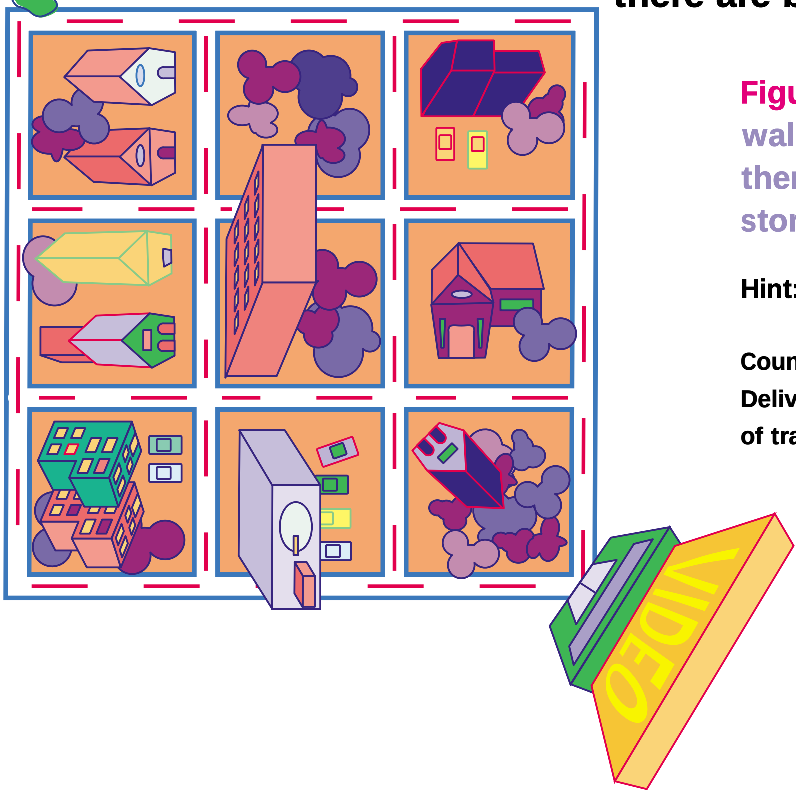


Figure This! **By walking on the side-walk, how many different ways are there to go from home to the video store? No backtracking allowed!**

Hint: Try fewer blocks to start.

Counting is an important mathematical skill. Delivery companies and airlines count the number of travel routes to get from one place to another.

Answer:

There are 20 different ways to get from home to the store.

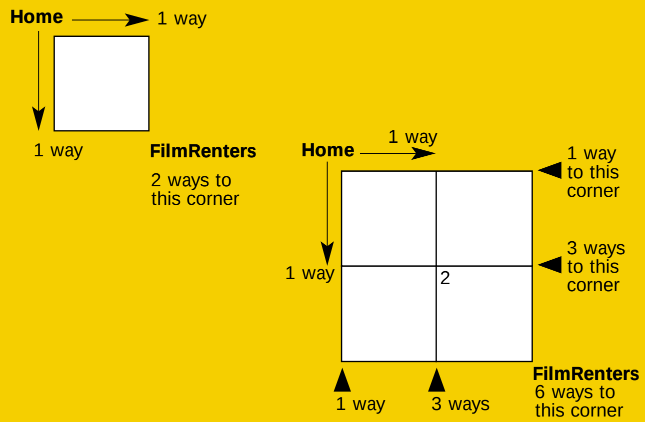
Figure This!

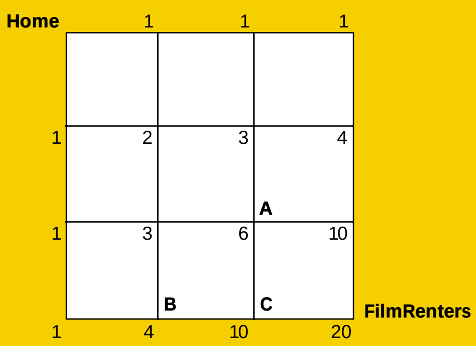
Get Started:

How many ways would there be if your home and the store were on opposite corners of the same block? If the store were two blocks away? Do you have to count both the paths that start south and east separately?

Complete Solution:

Look at a simpler case and count the number of ways to each corner. In the drawings below for example, the arrows have been added to show direction. The numbers indicate how many ways there are to get to each corner.





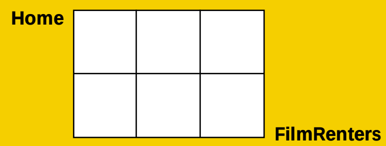
To get to corner C, you must pass through either corner A or B. There are 6 ways to get to corner A, and 4 ways to get to corner B making a total of 10 ways to get to corner C. Thus, there are 20 ways to get from home to FilmRenters.

Try This:

- Find the shortest route from your home to school. Are there different routes of the same length?

Additional Challenges:

1. How long is the shortest route from home to FilmRenters in the Challenge?
2. How many ways are there to go from home to FilmRenters on the map below?

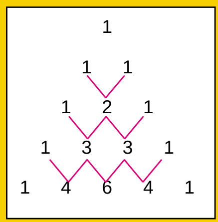


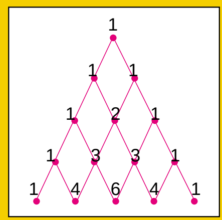
Things to Think About:

- Why is choosing efficient routes important to companies involved in transportation?
- What other jobs involve choosing efficient routes?

Did You Know That?:

- Blaise Pascal was a French mathematician in the 1600s. He worked with a pattern of numbers (Pascal's triangle) to solve many counting problems. Pascal's triangle is formed putting 1s along two "sides" of a triangle, then adding the two numbers above to the right and left to get the next number in the pattern.





- Pascal's triangle can be used to solve the challenge.
- Counting with patterns using Pascal's triangle was in Chu Shih-chieh's *Precious Mirror of Four Elements*, a fourteenth century book in China.
- Combinatorial analysis is a branch of mathematics that deals with counting problems like the one in this challenge.

Resources:

Books:

- Gardner, Martin. "Pascal's Triangle" in *Mathematical Carnival.* Washington, D.C.: Mathematical Association of America, 1989.
- Seymour, Dale, and Margaret Shedd. *Finite Differences*. White Plains, NY: Dale Seymour Publications, 1997. **www.cuisenaire-dsp.com**
- Seymour, Dale. *Visual Patterns in Pascal's Triangle*. White Plains, NY: Dale Seymour Publications, 1986. **www.cuisenaire-dsp.com**

Website:

www.studyweb.com/math

Answers to Additional Challenges:

(1.)

All 20 routes have the same length, 6 blocks.

(2.)

There are 10 ways to go.

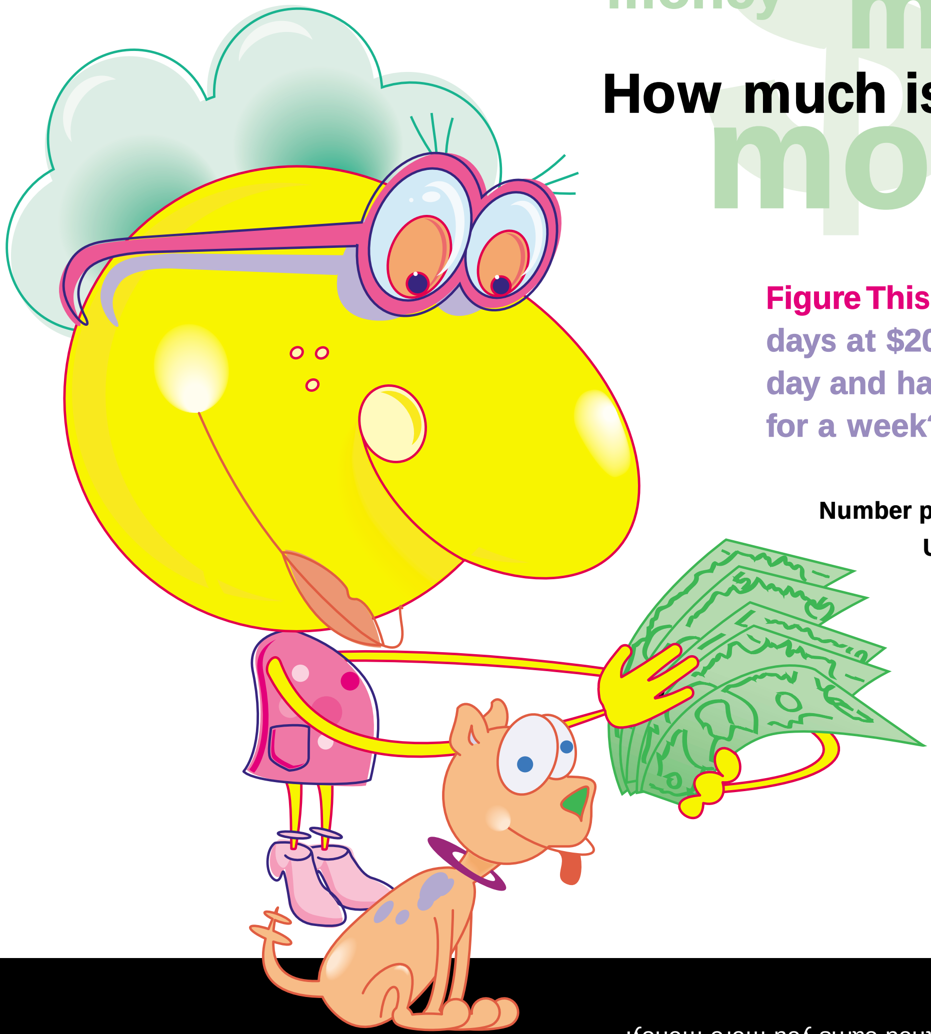
Math Challenge #7

DOUBLE OR NOT

Figure This! Math Challenges for Families

How much is your time worth?

Figure This! Would you rather work seven days at $20 per day or be paid $2 for the first day and have your salary double every day for a week?



Number patterns can change at very different rates. Understanding rates of change is important in banking, biology, and economics.

Answer:

If you are working the entire week, the doubling method earns you more money.

Figure This!

Get Started:

How much would you earn the first day using each method? The second day? What would be your total earnings at the end of the second day?

Complete Solution:

If you are paid $20 per day for seven days, then you earn $20 x 7 or $140. If you are paid $2 the first day and your salary doubles every day for the next six days, then you earn $2 + $4 + $8 + $16 + $32 + $64 + $128, or $254. The second scheme earns you more money by the end of the week.

Try This:

- Take a piece of paper of any size. Fold it in half. How does the thickness of the folded paper compare to the thickness of the unfolded paper? Repeat this process, answering the question each time. How many times can you fold the paper? Does the size of the paper with which you started make any difference? How thick is the folded paper when you can no longer fold it?
- Use a calculator to skip count by 3s. In other words, make the calculator show the sequence 3, 6, 9, 12, …. Can you make it multiply by 3s?

Additional Challenges:

1. If the payment methods described in the “Challenge” were carried out for a month, how much money would you have earned altogether on the 30th day?
2. What process is used to generate each of the following patterns?

- Cup, pint, quart, half-gallon, and gallon
- Penny, dime, dollar, ten dollars, one hundred dollars, and so on.

Things to Think About:

- How does the amount of money in a savings account increase?
- How do banks determine what interest is paid before the principal on loans?

Did You Know That?

- Radioactivity is often described by a half-life, the time required for half the radioactive material to decay.
- Counting by 1s is an example of an arithmetic sequence.
- A sequence like 2, 5, 8, 11, …, in which you add 3 each time, is also an arithmetic sequence.
- A sequence like 2, 6, 18, 54, …, in which you multiply by 3 each time, is a geometric sequence.
- The graph of an arithmetic sequence lies along a straight line.

Resources:

Books:

- Page, D., K. Chval, and P. Wagreich. *Maneuvers with Number Patterns*. White Plains, NY: Cuisenaire/Dale Seymour Publications, 1994. **www.cuisenaire.com**
- Seymour, Dale, and Ed Beardslee. *Critical Thinking Activities in Patterns, Imagery and Logic*. Vernon Hills, NY: ETA 1997. **www.etauniverse.com**

Software:

- “Bounce” (computer software package). Pleasantville, NY: Sunburst Communications, 1999. **www.sunburstdirect.com**

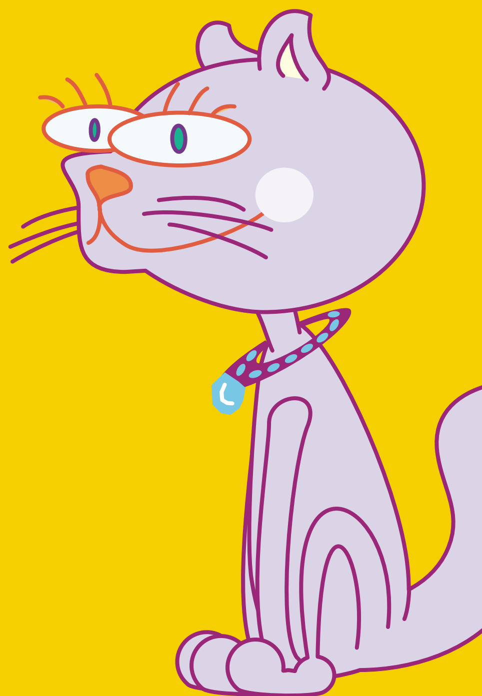
Answers to Additional Challenges:

(1.)

$600 by the first method and $2,147,483,646 by the second.

(2.)

In terms of cups, the pattern is 1 cup, 2 cups, 4 cups, 8 cups, 16 cups. Each measure is twice the previous one. In the sequence of money, each amount is 10 times the previous one giving 1 penny, 10 pennies, 100 pennies, and so on.



Math Challenge #8

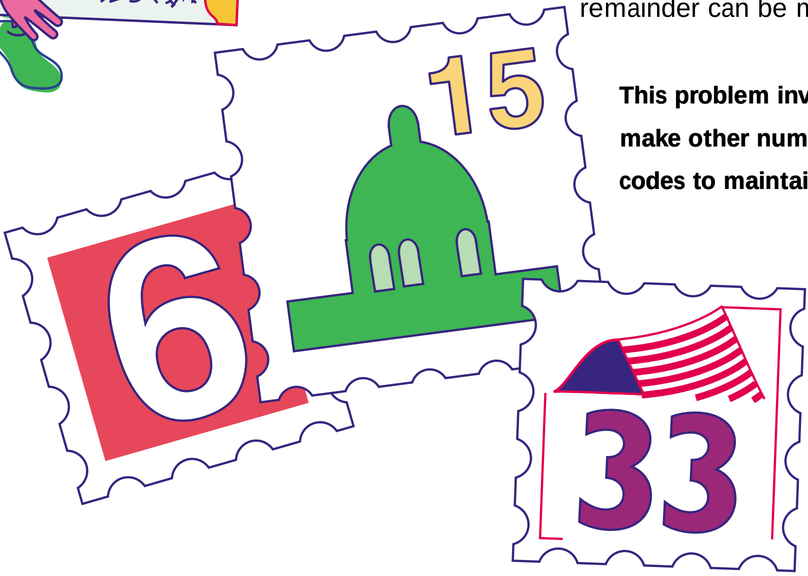
STAMPS

Figure This!
Math Challenges for Families

How can you use OLD stamps?

Figure This! Suppose you found an old roll of 15¢ stamps. Can you use a combination of 33¢ stamps and 15¢ stamps to mail a package for exactly $1.77?

Hint: Use as many 33¢ stamps as you can so that the remainder can be made with 15¢ stamps.



This problem involves using combinations of numbers to make other numbers. Similar processes are used to develop codes to maintain security in banking and computer access.

Answer:

Four 33¢ stamps and three 15¢ stamps equals $1.77 in postage.

Figure This!

Get Started:

What would happen if you tried to use only 33¢ stamps or only 15¢ stamps? What is the closest postage you can get if you use only 33¢ stamps?

Complete Solution:

There are many ways to do the problem.

- Using the hint given above, divide 177 by 33. With five 33¢ stamps, you have $1.65 worth of postage and need 12 more cents. With four 33¢ stamps, you have $1.32 worth of postage and need 45 more cents. Since three 15¢ stamps make 45¢, one solution is four 33¢ stamps and three 15¢ stamps.
- Another strategy is to make a table listing all of the possible combinations of 33¢ and 15¢ stamps that can be used.

Number of 33¢ Stamps	Number of 15¢ Stamps	Value
6	0	$1.98
5	0	$1.65
5	1	$1.80
4	1	$1.47
4	2	$1.62
4	3	$1.77

- Make a combination chart and fill it out until you either reach or pass $1.77 in every row and column. The first row has the values of an increasing number of 33¢ stamps and the first column the values of increasing the number of 15¢ stamps. Remaining cells are the combinations of the two.

		Number Of 33¢ Stamps					
		1	2	3	4	5	6
Number of 15¢ Stamps		$.33	$.66	$.99	$1.32	$1.65	$1.98
1	$.15	$.48	$.81	$1.41	$1.47	$1.80	
2	$.30	$.63	$.96	$1.29	$1.62	$1.95	
3	$.45	$.78	$1.11	$1.44	**$1.77**		
4	$.60	$.93	$1.26	$1.59	$1.92		
5	$.75	$1.08	$1.41	$1.74	$2.07		
6	$.90	$1.23	$1.56	$1.89			
7	$1.05	$1.38	$1.71	$2.04			
8	$1.20	$1.53	$1.86				
9	$1.35	$1.63	$2.01				
10	$1.50	$1.83					
11	$1.65	$1.98					
12	$1.80						

- Another way to approach the problem is to realize that using only 15¢ stamps makes the total end in either 0 or 5. Now look at the number of 33¢ stamps that can be used to get a total that ends in either 2 or 7.

Try This:

- Compare the cheapest way of mailing a package or letter using the U.S. Postal Service, Federal Express, or United Parcel Service.

Additional Challenges:

1. Using only 33¢ and 15¢ stamps, can you make any of the following?

a. $2.77 b. $4.77 c. $17.76

2. Can every whole number greater than 1 be made by adding some combination of twos and threes?

Things to Think About:

- Airmail stamps, which cost more than first-class postage, were once very popular for domestic mail. Why do you think this is no longer true today?

Did You Know That?

- The postage for a first-class stamp on July 6, 1932, was 3¢. It remained at that price until August 1, 1958, when it rose to 4¢.
- The price of a first-class stamp changed twice in 1981, to 18¢ in March and to 20¢ in November.
- More than 400 commemorative 32¢ stamps were issued from January 1, 1995 through December 31, 1998.
- On March 3, 1997, two triangular stamps depicting a clipper ship and a stagecoach were issued.
- Diophantus (ca. 250) is known as the Father of Algebra. Equations of the type in this challenge are called diophantine equations.
- The Euclidean Algorithm can be used in solving this challenge.

Resources:

Books:

- *New York Times World Almanac and Book of Facts 1999*, New York: World Almanac Books, 1999.
- "Media Clips." *Mathematics Teacher 92* (April 1999): 336-338.

Websites:

- U.S. Post Office **www.usps.gov**
- National Postal Museum **www.si.edu/postal**
- Foreign Postal Sites **www.upu.int/web/An**

Answers to Additional Challenges:

(1.)

These Challenges encourage solutions based on the reasoning in the original challenge.

a. No. There is no way to make the additional dollar of postage from combinations of 33¢ and 15¢ stamps. (Note that 100 is not a multiple of 3 as are 33 and 15. Also, there are no other combinations that work.)

b. Yes. You already know how to make $1.77 in postage. Make the additional $3.00 by using five 33¢ stamps and nine 15¢ stamps, or twenty 15¢ stamps. Either twelve 15¢ stamps and nine 33¢ stamps or twenty-three 15¢ stamps and four 33¢ stamps make $4.77 in postage.

c. Yes. You already know how to make $1.77 and $3.00 in postage. Make increments of $3.00 using twenty 15¢ stamps. Five sets of $3.00 plus the set for $1.77 equals $16.77. Since $17.76 is 99¢ more than $16.77, you need three more 33¢ stamps. The $17.76 in postage can be made using thirty-two 33¢ stamps and forty-eight 15¢ stamps, or one hundred three 15¢ stamps and seven 33¢ stamps.

(2.)

Yes.