Math Challenge #26

I WIN!

FigureThis! Math Challenges for Families

"SHE always wins. It's not fair!"

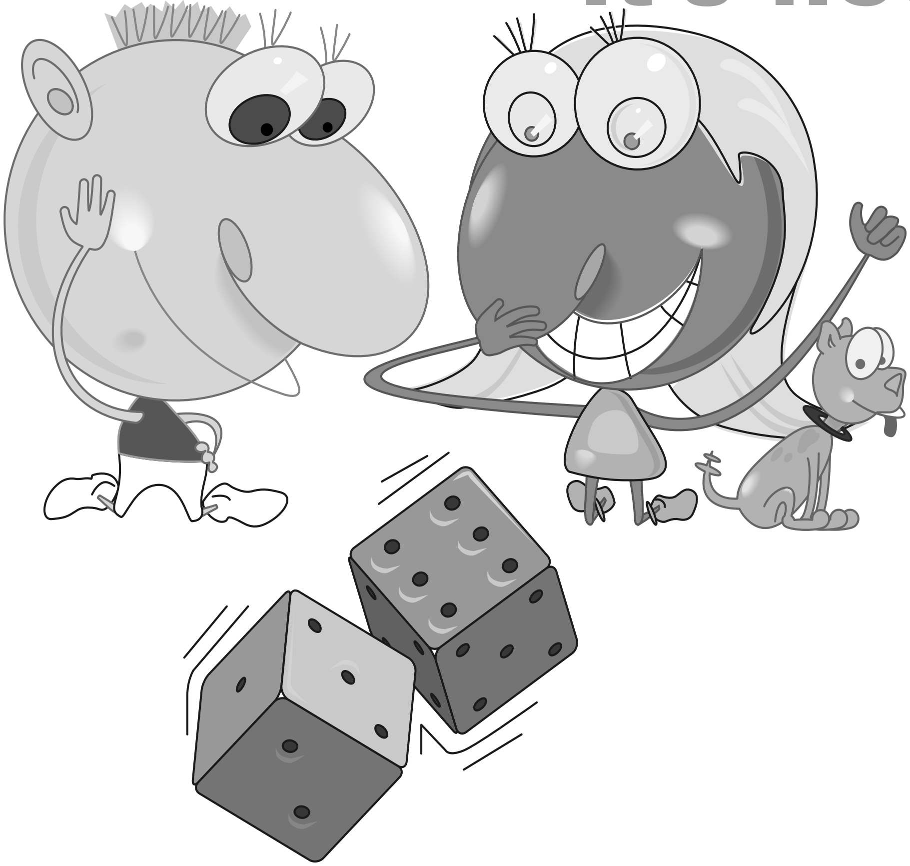


Figure This! Two players each roll an ordinary six-sided die. Of the two numbers showing, the smaller is subtracted from the larger. If the difference is 0, 1, or 2, player A gets 1 point. If the difference is 3, 4, or 5, Player B gets 1 point. The game ends after 12 rounds. The player with the most points wins the game. Is this game fair?

Hint: In a fair game, all players are equally likely to win. Play this game several times and record the results of each roll.

Math can help determine whether or not a game is fair. Math can also determine fairness in other situations, such as assigning seats in the U.S. House of Representatives or settling an estate.

Answer:
This game is not fair. Player A is twice as likely as player B to win the point for each round.

Figure This!

Get Started:

If player A rolls a 1 and player B rolls a 6, the difference is 5. Make a table that shows all the possible differences when rolling two dice.

DIFFERENCE TABLE

PLAYER A \ PLAYER B	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Use the table to determine if one player is more likely to win.

Complete Solution:

Create a table that shows all the possible differences that can result from the roll of two dice.

DIFFERENCE TABLE

PLAYER A \ PLAYER B	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Of the 36 possible outcomes, the differences 0, 1, and 2 appear 24 times, while the differences 3, 4, and 5 appear only 12 times. Player A is twice as likely to win a point on each roll and is therefore much more likely to win the game.

Try This:

- Change the rules of the game in the Challenge. Play it again. Do the new rules make a difference in who wins?

Additional Challenges:

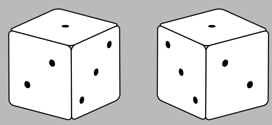
1. Change the rules of the game in the challenge so that the game is fair.
2. Roll two standard six-sided dice and divide the larger number showing by the smaller number. Player A gets 1 point if there is a remainder other than 0. Player B gets 1 point if the remainder is 0. Is this a fair game? If the game is not fair, how would you make it fair?
3. Assign numbers to the faces of two dice so that, on any one roll if you add the numbers on the top faces, each sum from 1 to 12 is equally likely.

Things to Think About:

- Is the game of Monopoly™ fair?
- There is no winning strategy that guarantees a win for the game Rock-Paper-Scissors. Why do you think this is true?
- Could you create a die with an odd number of faces?

Did You Know That?

- The opposite faces of an ordinary six-sided die always add up to 7.
- There are "right-handed" and "left-handed" dice. In Europe and the Americas, dice are typically right-handed. This means that if the 1 is face up, and the 2 is facing you, then the 3 will be on your right.



Right Hand Die   Left Hand Die

In Asia, dice typically have the 3 on the left.

- A branch of mathematics known as probability got its start in 1654, when Blaise Pascal and his friend, the Chevalier de Méré, tried to analyze a game of dice.
- Dice were developed from ancient games played with sheep knucklebones, called *astragali* in Greek, and *tali* in Latin. The bones from sheep's ankles were used to tell fortunes. Called "rolling bones," they were basically rectangular and had numbers on only four sides.
- A die with four faces is a tetrahedron. A die with twelve faces is an dodecahedron. A die with twenty faces is an icosahedron.

Resources:

Books:

- Mohr, Merilyn Simonds. *The Games Treasury*. Shelburne, VT: Chapters Publishing, Ltd., 1993.
- Crisler, Nancy, Patience Fisher, and Gary Froelich. *Discrete Mathematics Through Applications*. New York: W. H. Freeman and Co., 1994.

Website:

- **www.gamecabinet.com/rules/Dice.html**

Answers to Additional Challenges:

(1.)
There are many ways to make this game fair. For example, you might give 1 point to player A for an even difference and 1 point to player B for an odd difference. Or you might give 1 point to player A for differences of 0, 3, 4, or 5, and 1 point to player B for differences of 1 or 2.

(2.)
The game is not fair. Of the 36 possible outcomes, the remainder 0 appears 22 times, the remainder 1 appears 10 times, and the remainder 2 appears 4 times. One way to make this game fair would be to give player A 7 points for a remainder of 0 and player B 11 points for a remainder of 1 or 2.

(3.)
There are several ways to do this. For example, you could label one die with the numbers from 1 to 6 and the other with three 0s and three 6s.

Notes:

Tangent

Math Challenge #27

LIFE EXPECTANCY

Figure This! Math Challenges for Families

Do women live longer than men?

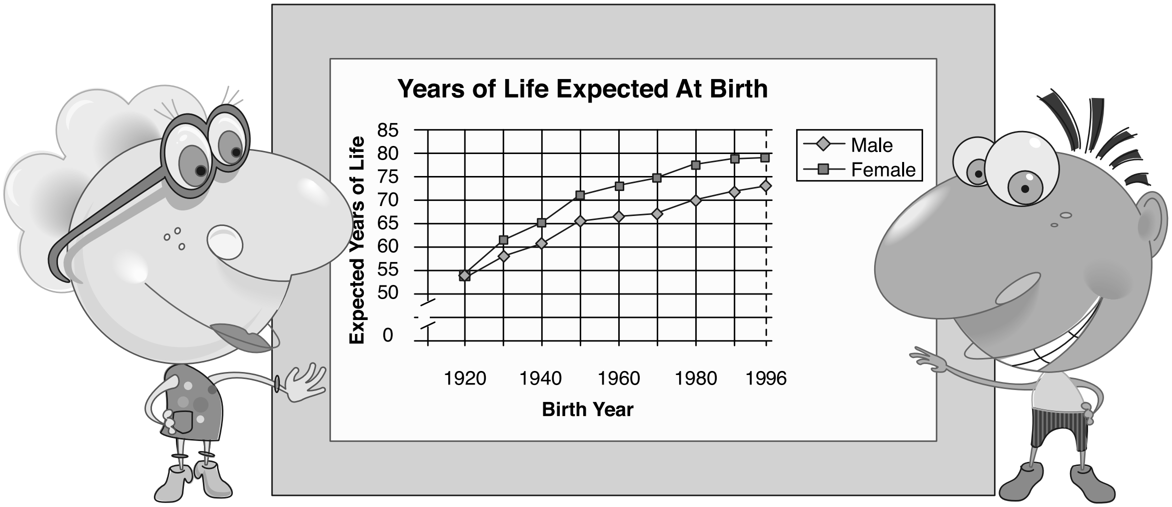


Figure This! This chart shows the life expectancy for persons born in the United States in a given year. Estimate the biggest difference between the life expectancy of men and women in any year from 1920 to 1996.

Hint: A woman born in 1920 had a life expectancy of about 55 years at birth.

Graphs of information over time are useful in identifying patterns and analyzing trends. Market research firms, radio stations, and breakfast-food manufacturers all keep track of trends in order to stay current.

Answer:

In both 1970 and 1980, the difference in life expectancy was between 7 and 8 years.

Figure This!

Get Started:

Think about what the graph means.

- What was the life expectancy for males born in 1940? for females? What was the difference?
- What would it mean if the lines representing males and females crossed?

Complete Solution:

Look at each pair of data points and find the difference in life expectancy for males and females. You can estimate the difference from the graph or measure it with a ruler. You can also use an index card or a sheet of paper and mark the differences to see which is greater. Then identify the years for which the greatest difference occurs. The greatest difference (between 7 and 8 years) appears in both 1970 and 1980.

Try This:

- Look in newspapers and magazines to find a graph describing how something changes over time. What can you observe from the graph?

Additional Challenges:

1. Would the answer in the challenge be different if the scale on the graph were changed?
2. Describe what a graph of male and female life expectancies would look like if the difference were always six years.

Things to think about:

- Do you think the life expectancies of both men and women will continue to increase in the future?
- Why do women tend to live longer than men?
- What factors may have caused life expectancy to increase since 1920?
- How do you think life expectancy is determined?
- What would you expect a graph of life expectancies for another country to look like?

Did You Know That?

- Franklin Delano Roosevelt signed the Social Security Act into law in 1935.
- As of 1999, a woman named Jeanne Calment held the record for longest life. She lived 122 years and 164 days. She died August 4, 1997.

Resources:

Books:

- *The Guinness Book of World Records*. New York: Bantam Books, 1998.
- *The World Almanac and Book of Facts 1999*. Mahwah, NJ: World Almanac Books, 1998.

Website:

- **www.psych.cornell.edu/Darlington/lifespan.htm**

Answers to Additional Challenges:

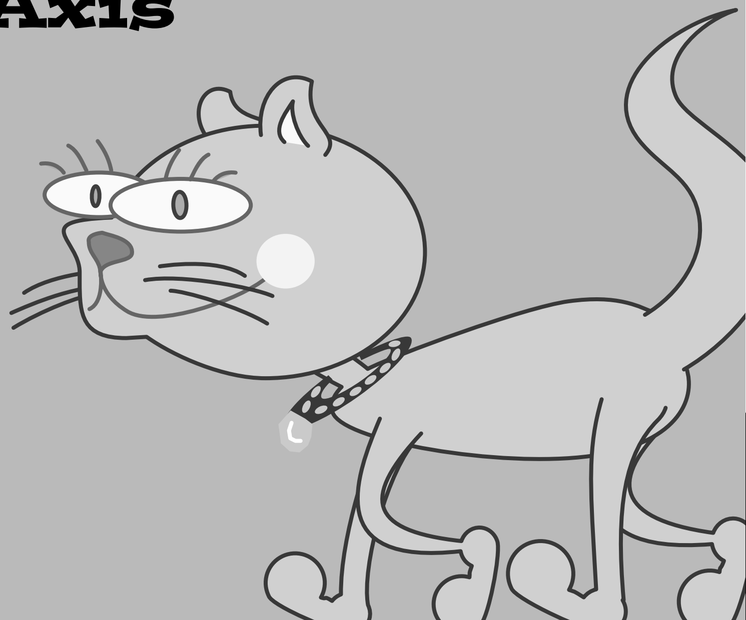
(1.)

No. The result would be the same.

(2.)

The graph for men and the graph for women would have the same shape, with one 6 units above the other.

Axis



Math Challenge #28 PIGEONHOLES

Figure This! Math Challenges for Families

At least two people in a school have the same initials? No way!

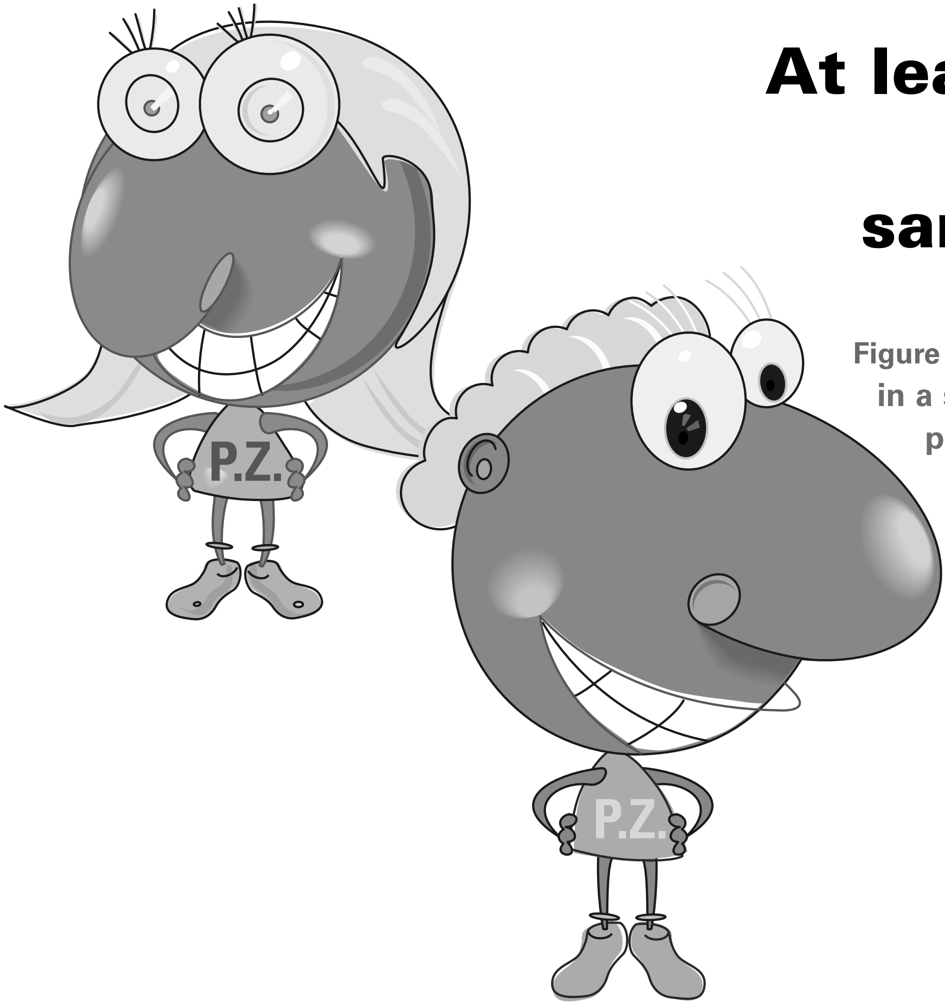


Figure This! How many people would have to be in a school before it contained at least two people with the same first and last initials?

Hint: Consider a simpler problem. How many people would have to enter a room before it contained at least two people with the same first initial?

The "pigeonhole principle" says that if you are putting objects into boxes and you have more objects than boxes, then at least one box will contain more than one object. People apply this principle when they mail announcements, pack shipping crates, or organize files.

Answer:
At least 677 people must be in a school.

Figure This!

Get Started:

How many different possibilities are there for a first initial? for a last initial? for both initials combined?

Complete Solution:

- There are 26 letters in the English alphabet. So there are 26 different possibilities for the first initial. Consider all the possible pairs of two initials. For example, suppose a person has the first initial A. Then the pair of initials could be AA, AB, AC, …, AZ. There are 26 different possibilities. If the first initial is B, the pair of initials could be BA, BB, BC, …, BZ. Again there are 26 different pairs. Continuing in this way and since there are 26 possible first initials, each of which could be paired with 26 last initials, there are 26 x 26, or 676 possible different pairs of initials. If there were 677 people, at least two of them must have the same pair of initials.

- Another way to begin this problem is to think about a situation involving smaller numbers. Suppose you have 11 items (that cannot be broken) to give to 10 people. This means that 1 person will get 2 of the items. The same reasoning can be used to solve the challenge.

Try This:

- Search the web to determine the estimated number of telephones in your state. If you know the number of households in your state, what conclusions can you reach about the number of telephones per household?
- Search the web or look in an encyclopedia to find the average number of hairs on a human head. Considering this information, do you think there are at least two people with the same number of hairs on their heads in your town? What about when your local stadium is filled?
- Search for "pigeonhole principle" on the web.

Additional Challenges:

1. How many people must enter a room to guarantee that at least two of them have the same birthday, regardless of whether or not they were born in a leap year?
2. A drawer contains 11 black socks and 3 gray socks. How many socks must you take out to guarantee that you have at least one pair of the same color?
3. A basketball team has 12 players. The team's jerseys are numbered from 0 to 20. If no two players are assigned the same number, must two players have consecutive numbers?

Things to Think About:

- Why do some states use both letters and numbers on their automotive license plates?
- At any party consisting of 2 or more people, at least 2 of the people must have the same number of friends at the party.
- Given any 17 whole numbers, you can find five of them whose sum is a multiple of 5.

Did You Know That?

- French mathematician Peter Gustav Lejeune Dirichlet (1805–1859) first described the pigeonhole principle.
- The pigeonhole principle was so named because if 10 homing pigeons return to 9 holes, then at least one hole must have two pigeons in it.
- A good approximation of the number of hairs on a human head is about 100,000.

Resources:

Books:

- Engel, Arthur. "The Box Principle." In *Problem-Solving Strategies*. New York: Springer-Verlag, 1991. pp. 39-58.
- Gardner, Martin. *Aha! Insight*. New York: Scientific American, Inc., 1978.
- Larson, Loren C. "Pigeonhole Principle." In *Problem-Solving Through Problems*. New York: Springer-Verlag, 1981. pp. 79-83.

Website:

- **www.optonline.com/comptons/ceo/02055_A.html**

Answers to Additional Challenges:

(1.) 367

(2.) 3

(3.) Yes