Math Challenge #17
PERPLEXING PERCENTAGES

Figure This! Math Challenges for Families

So how much does it cost?

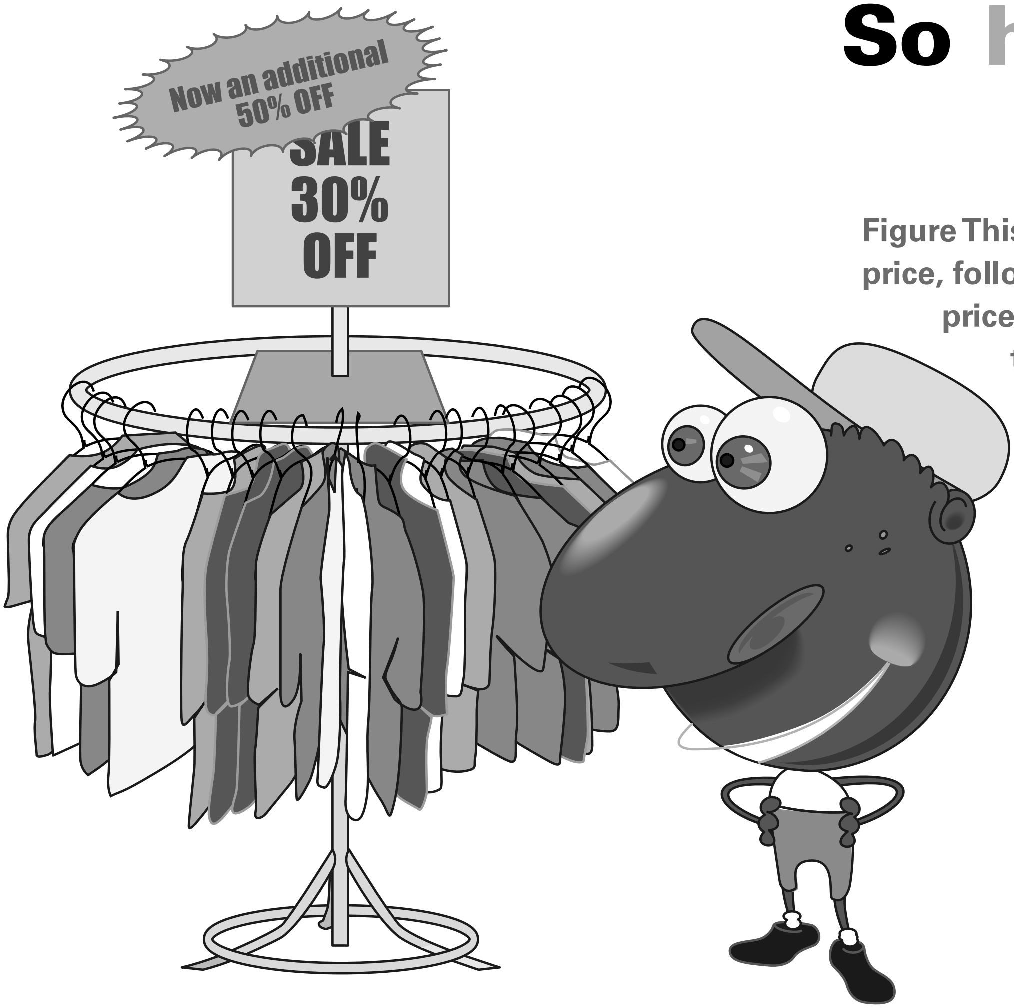


Figure This! Is a discount of 30% off the original price, followed by a discount of 50% off the sale price, the same as a discount of 80% from the original price?

Hint: What would a $100 item cost after these discounts?

Understanding percentages is critical in many everyday and business decisions. Survey results, medical reports, weather information, and interest rates all involve percentages.

Answer:
No. The discount is not 80%.

Figure This!

Get Started:

Choose a price for an item, say $100 as suggested in the hint. Calculate what the sale price would be after a 30% discount. Then find out how much the item would cost at 50% off the sale price.

Complete Solution:

- If an item originally costs $100, the tables below show the different final costs. They are not the same.

Original Price	30% Off	Cost on Sale	50% Off Sale Price	Final Cost
$100	30% • $100 = $30	$100 - $30 = $70	50% • $70 = $35	$70 - $35 = $35

Original Price	80% Off	Final Cost
$100	80% • $100 = $80	$100 - $80 = $20

- For the item on sale at 30% off, you would need to pay 70% of the price. So an additional discount of 50% off the sale price would bring the price to 35% (that is, 50% • 70%) of the original price. Thus, a $100 item would cost $35 after both discounts. An 80% off sale means that you pay 100% – 80%, or 20% of the original cost of the item. Thus, an item that originally cost $100 on sale at 80% off costs 20% • $100 or $20. The costs are not the same.
- You can generalize the problem. If *P* is the original price of an item, with the two discounts, one of 30% followed by another of 50%, you would pay 0.50 • (0.70 • *P*) or 0.35*P*, which is not the same as 0.2*P*.

Try This:

- Look at some of the discounts offered in newspaper or magazine ads. Find examples that use multiple discounts and calculate the actual cost per item.

Additional Challenges:

1. Would you rather become 50% richer and then 50% poorer, or become 50% poorer and then 50% richer?
2. The original price of a washing machine is $500. On the first day of each month, the store will reduce its price by 10% of the previous price. How long will it take before the sale price is half the original price?
3. An ad in a clothing store reads, "Clearance: 60% to 75% off when you take an extra 50% off the previous sale price." The previous sale price on a pair of jeans was $24.99, down from an original $29.99. Is the ad correct for this item?

Things to Think About:

- A discount of 50% is the same as a half-price sale.
- A discount of 25% is the same as paying 75% of the price.
- A cost of 10% more than a price is 110% of the listed price.
- In what situations are percentages more useful than fractions?

Did You Know That?

- The word percent comes from the Latin *per centum*, meaning "per 100."
- Pressing the percent key on some calculators changes the percentage to a decimal.

Resources:

Books:

- Paulos, John Allen. *Innumeracy: Mathematical Illiteracy and Its Consequences*. New York: Hill and Wang, 1988.
- Paulos, John Allen. *A Mathematician Reads the Newspaper*. New York: Basic Books, 1995.

Answers to Additional Challenges:

(1.) :

The result is the same.

(2.) :

In the seventh month, the cost will be less than half the original price.

(3.) :

The final cost is $24.99 – (0.5 • $24.99), or $12.50. The total discount from the original price is $29.99 – $12.50 or $17.49. Since $17.49 is about 58% of $29.99, the ad is not correct.

Math Challenge
#18
TURN TERN

Figure This!
Math Challenges for Families

Are we there yet?

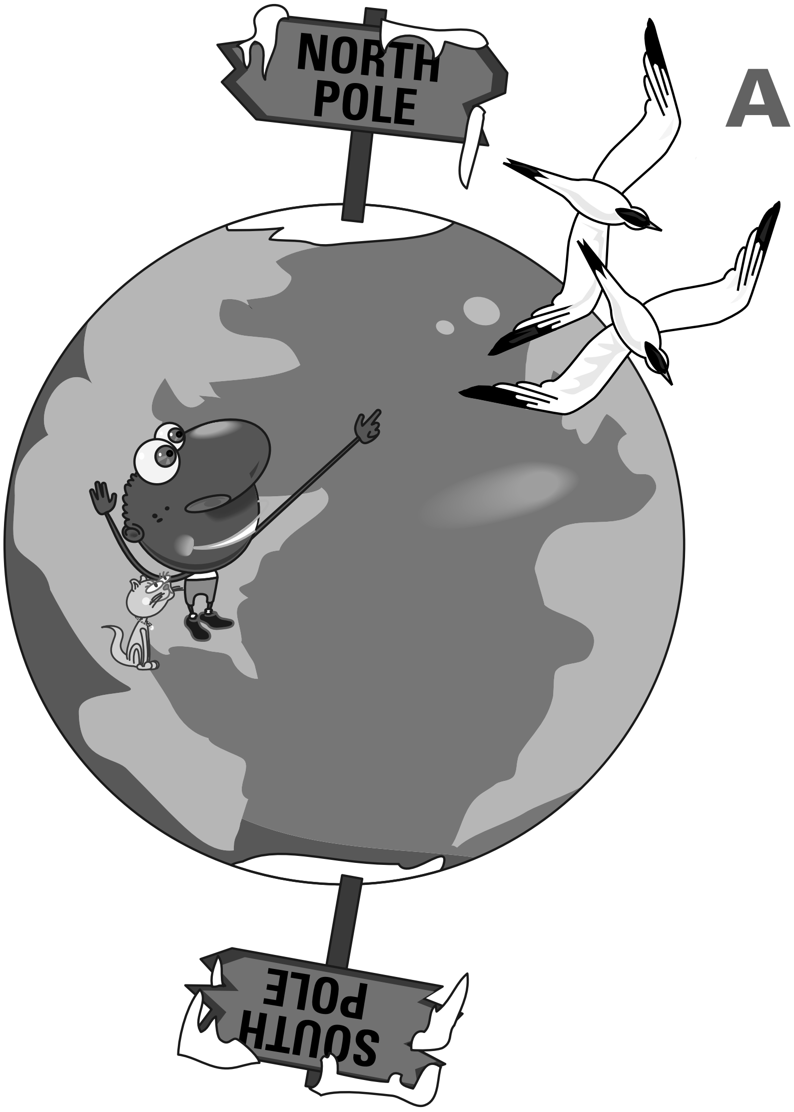


Figure This! **Every year, Arctic terns fly from the Arctic to the Antarctic and back, a distance of about 9000 miles each way. Suppose the birds fly at an average speed of 25 miles per hour for 12 hours a day. How many days of flying would be necessary to make the roundtrip?**

Hint: How many miles would a tern fly in an average day?

The distance traveled by a moving object can be found using its rate and time. Calculating distance is important for railroad companies, airlines, and trucking firms, as well as family travelers on vacation.

Answer:
Flying at 25 mph for 12 hours per day, a tern could make the roundtrip journey in about 60 days.

Figure This!

Get Started:

Find the number of miles a tern flies in one day; then find the number of days required to fly each way.

Complete Solution:

At 25 mph, a tern would travel 100 miles in 4 hours. Flying 12 hours in a day, the bird would cover 300 miles. Therefore, the one-way journey of 9000 miles would require 30 days.

25 mph x 12 hr/day = 300 miles/day
9000 miles ÷ 300 miles/day = 30 days

The roundtrip would take twice that, or about 60 days.

Try This:

- Determine how long it takes you to walk a mile. Estimate how many miles you could walk in a day without getting too tired. Could you walk across the United States in one year?
- Find out about other animal migrations, such as those of robins, swallows, or whales.

Additional Challenges:

1. Monarch butterflies migrate from Canada to Mexico, a distance of about 2500 miles. If it takes them about one month, about how many miles do they fly per day? If they fly for 12 hours per day, what is their average speed in miles per hour?

2 The distance around the earth at the equator is about 25,000 miles. How long would it take an Arctic tern like the ones described in the challenge to fly this distance?

3. The fastest bird is thought to be the peregrine falcon. Its top speed is at least 124 mph. The slowest birds are the American and Eurasian woodcock, which can fly at only 5 mph without stalling. Compare the time it would take a falcon and a woodcock to fly across the United States (approximately 2500 miles).

Things to Think About:

- In 1850, wagon trains usually took from 4 to 6 weeks to travel from Missouri to California, about 2000 miles.
- Some swallows return to San Juan Capistrano, California, at about the same time every year. How do the birds know when and where to go?
- Can a butterfly go faster than you can run?
- How fast is a breeze?

Did You Know That?

- The exact details of the Arctic tern's migration, as with most bird migrations, are unknown. The route is not necessarily direct, and the birds make stops along the way.
- Naturalists catch birds, place identification bands on their legs, then release them. If these birds are caught again, the bands help provide a record of where the birds came from.
- An Arctic tern banded in the Arctic Ocean was captured again three months later—11,000 miles away.
- Observers of the Arctic tern report that they can be found for three to four months in the Arctic region, and for 3 to 4 months in the Antarctic region.
- The blackpole warbler cannot swim, yet it migrates over the open ocean from New England to South America, a nonstop trip of over 4000 miles.
- Birds have been seen at elevations of 30,000 feet — higher than Mount Everest. One bird crashed into an airliner flying at 37,000 feet.
- Carrier pigeons have been clocked at speeds of 35 mph.

Resources:

Book:

- *The Guinness Book of Records*. New York: Guinness Book of Records, 1999.

Website:

- **www.mmm.com/front/monarch/index.html**
- **www.schoolnet.ca/collections/arctic/species/birdsgen.htm**
- **www.randomhouse.com./features/audubon/nas/**

Answers to Additional Challenges:

(1.)
The butterflies travel an average of 83 miles per day. If the fly only 12 hours per day, their average speed would be approximately 7 mph.

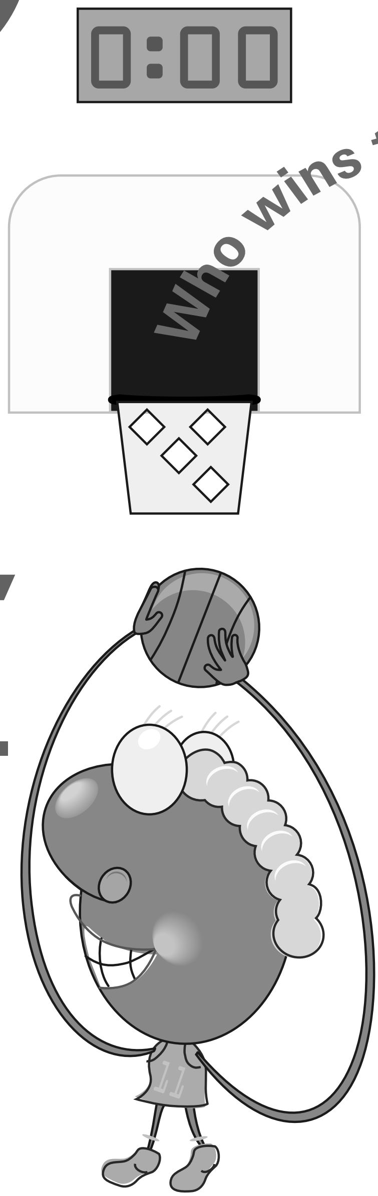
(2.)
At the speed given in the challenge, about 83 days.

(3.)
If a falcon could maintain its top speed of 124 mph, it could fly the distance in about 20 hours. At 5 mph, a woodcock would need about 500 hours of flying time. The difference is 480 hours, or 20 full days.

Math Challenge **#19** TWO POINTS!

FigureThis! **Math Challenges for Families**

Who wins the game???



she shoots, and... she...

Figure This! **Your team is down by one point. Your teammate, who makes free throws about three-fourths of the time, is at the free throw line. She gets a second shot if she makes the first one. Each free throw that she makes is worth 1 point. If there is no time left, what are the chances you win the game with no overtime?**

Hint: Tossing two different coins can be used to represent making or missing a shot. What are the outcomes when you toss two coins? How can you use this to model the free throws?

Probability is a measure of chance. Applications of probability are found in genetics, the insurance industry, lotteries, and medical testing.

Answer:
The probability that your team wins is 9/16 or about 56% of the time.

Figure This!

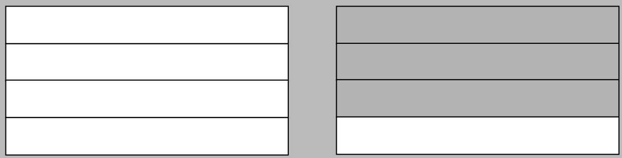
Get Started:

Toss two coins as suggested in the hint. Let getting two heads or a head and a tail represent making the shot; let getting two tails represent missing the shot. Toss the coins once. If you have two tails, she missed the free throw and the game is over. If you have at least one head, she made a point and the game is tied; she gets a second shot. Toss the coins again. Did your team win, or is the game still tied? Do this experiment about 50 times to predict whether your team wins or not.

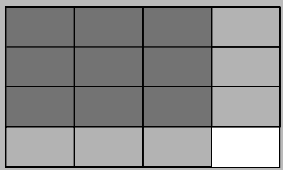
Complete Solution:

There are many ways to answer the question.

- Make a rectangular diagram with four rows of the same size. Shade three of the four rows to represent making the first shot.

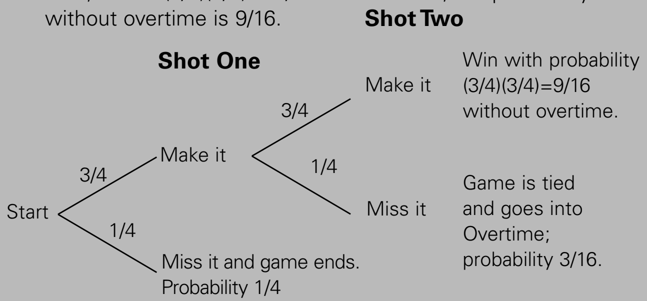


Make four columns of the same size and shade three of them to represent making the second shot. There should now be 16 cells in the grid. The 9 cells that are shaded twice represent success on both shots which means your team wins without any overtime play.



Since 9 of the 16 equally likely outcomes represent wins, the probability of winning is 9/16.

- A different strategy is to draw a tree diagram labeled with all outcomes and their probabilities for each shot. The probability of winning is found by multiplying the probabilities on the appropriate branches of the tree. If your teammate makes the first shot 3/4 of the time, then 3/4 of those times that she makes the first shot, she will make the second shot; that is (3/4)(3/4) = 9/16. In this case, the probability of winning without overtime is 9/16.



[Multiplying probabilities here is correct only if the two shots are independent events. Assume that they are.]

Try This:

Find the probability of getting two heads when you toss a coin twice even if you don't have a penny! You can model what happens with a piece of rectangular paper. Fold a sheet of paper in half lengthwise and mark one half "Heads" to represent obtaining a head on the first toss. Mark the other half "Tails." Fold the paper again. The creases in the paper should now divide it into four equal parts. One part should be labeled "Heads-Heads;" one "Heads-Tails;" one "Tails-Heads;" and one "Tails-Tails." The probability of each outcome is represented by the portion of the paper it occupies.

Additional Challenges:

1. What is the most likely outcome for your team in the game described in the Challenge?
2. How would you change your answer if the player's free-throw percentage was 60%?
3. What free-throw percentage should your teammate have to give your team a 50% chance of winning in this situation?

Things to Think About:

- Near the end of a close game, do National Basketball Association (NBA) players use free-throw percentages to decide whom to foul?
- Probability may be estimated based on the overall pattern after many, many events.
- The probability of an absolutely sure bet is 100% or 1. For any probability less than that, knowing that something "can" happen is no guarantee that it will happen.

Did You Know That?

- When you toss a coin eight times, the chance of getting four heads in a row and then four tails in a row is the same as getting a head and then alternating tails and heads.
- Computer or calculator simulations can be used to estimate the probability of events.
- Probability was invented to decide how to divide the winnings fairly when the players had to leave a game before it was over.

Resources:

Books:

- Gnanadesikan, Mrudulla, Richard L. Scheaffer, and Jim Swift. *The Art and Techniques of Simulation*. White Plains, NY: Dale Seymour Publications, 1987.
- Hopfensperger, P., H. Kranendonk, and R. Scheaffer. *Data-Driven Mathematics: Probability through Data*. White Plains, NY: Dale Seymour Publications, 1999.
- Newman, C. M., T. E. Obremski, and R. L. Scheaffer. *Exploring Probability*. White Plains, NY: Dale Seymour Publications, 1987.

Answers to Additional Challenges:

(1) :
Before she begins to shoot, the probability of winning without overtime is 9/16. The probability of a tie-game is 3/16 and in this case, the game must continue to determine the winner. The probability of losing without overtime is 4/16 or 1/4. The most likely outcome is winning the game without overtime.

(2) :
The most likely outcome is losing the game. The probability of losing without overtime is 40%.

(3) :
To have a 50% chance of winning without overtime, the free-throw percentage would be $\sqrt{0.50}$ or about 0.71 = 71%.

Tangent