Math Challenge #13 THIRTEEN! OH NO!

Figure This! Math Challenges for Families

Are you SUPERSTITIOUS?

Do you avoid the number 13?

Figure This! Is there a Friday the 13th every year?

Hint: If January 1 were on Monday, on what day of the week would January 13 fall? What about February 1 and February 13? Other months?

Reasoning about patterns with numbers and dates helps to develop logical thinking. Matching such patterns determines the annual dates of some national holidays as well as being an operating principle for some machines.

Answer:
There is at least one Friday the 13th every year.

Figure This!

Get Started:

- Make a list. If January 1 is on a Monday, on what day is January 13? On what day is the first day of the next month? The 13th of the next month? Suppose January 1 is on a Tuesday?
- Get a calendar and check to see how the dates fall. How many different calendars are possible? (Remember leap years.)

Complete Solution:

There are 14 different calendars. (Check the website for all calendars.) There are seven possible calendars for non-leap years: one with January 1 on each day of the week. Leap years give seven more calendars, for a total of 14. For example, if January 1 is on a Wednesday, you have the following list:

January 1	Wednesday	April 1	Tuesday
January 13	Monday	April 13	Sunday
February 1	Saturday	May 1	Thursday
February 13	Thursday	May 13	Tuesday
March 1	Saturday	June 1	Sunday
March 13	Thursday	June 13	Friday

Because June 13 is a Friday, you can stop. The chart below shows the total number of Friday the13ths for all 14 calendars.

When January 1st is on	Non-leap Year months in which Friday the 13th will occur	Leap Year months in which Friday the 13th will occur
Monday	April, July	September, December
Tuesday	September, December	June
Wednesday	June	March, November
Thursday	February, March, November	February, August
Friday	August	May
Saturday	May	October
Sunday	January, October	January, April, July

Try This:

- Use an almanac or an encyclopedia to discover how the months were named and why they have different numbers of days.
- If you had been born on February 29, how many birthdays would you have had by this year?
- Thirteen may be a lucky number for the USA. Think of at least one reason why.
- Study a dollar bill to see if you can find "13" objects in common.
- Use an almanac or an encyclopedia to research why the number 13 is considered by some to be unlucky.

Additional Challenges:

1. Why is the year 2000 a leap year when 1900 was not?
2. If your VCR cannot handle the year 2000, to what year can you set it back so that the days will still be the same?

Did You Know That?

- The word triskaidekaphobia means fear of the number 13.
- Some hotels do not have a floor numbered 13 because of people's fears of 13.
- The calendar is based on the movements of the sun and the moon.
- There have been many different calendars in the past. The current calendar began in 1582 when Pope Gregory XIII decreed that the day following October 4 should be October 15 to catch up for the days lost using the previous calendar.
- The calendar correction did not take place in Great Britain and its colonies (including those in North America) until 1752.
- The Chinese New Year falls anywhere from late January to the middle of February. The Chinese Lunar Calendar is based on cycles of the moon, with a complete cycle requiring 60 years (5 cycles of 12 years each).
- The mathematics of modular arithmetic is used to find answers to challenges like this one.

Things to think about:

- Why do you think that we have seven days per week with 52 weeks per year?
- Hotels that rename the 13th floor as the 14th floor still have a 13th floor.
- The words September, October, and November are derived from Latin words *septem*, *octo*, and *novem* meaning seven, eight and nine respectively, but the months are not the seventh, eighth, and ninth months. Why?

Resources:

Book:

- *The World Almanac and Book of Facts 1999.* Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.julian12.com/history.htm**
- **www.stjohndc.org/what/9609ca1.htm**

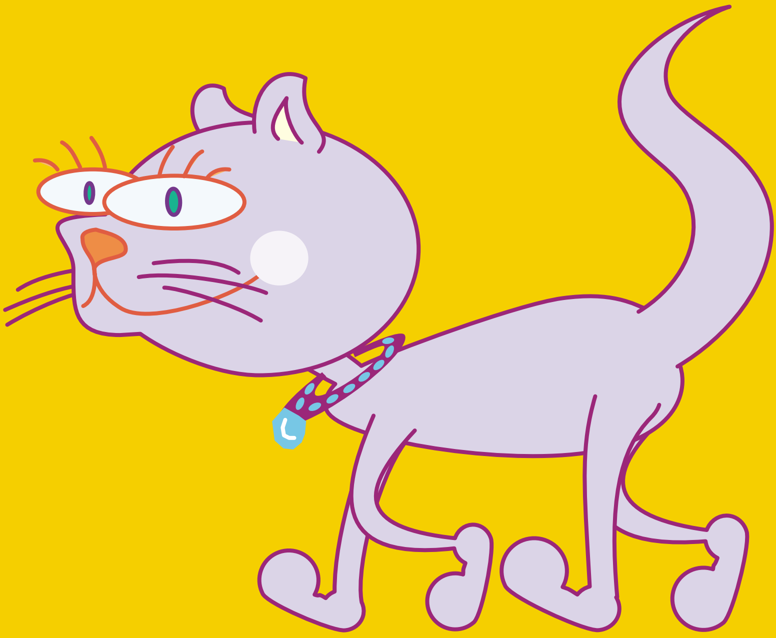
Answers to Additional Challenges:

(1.) Answer:

Leap years typically occur every four years, except at the turn of a century. Only those centuries divisible by 400 are leap years.

(2.) Answer:

1972



Math Challenge #14
BATTLING AVERAGES

Figure This! Math Challenges for Families

Who's on first today?

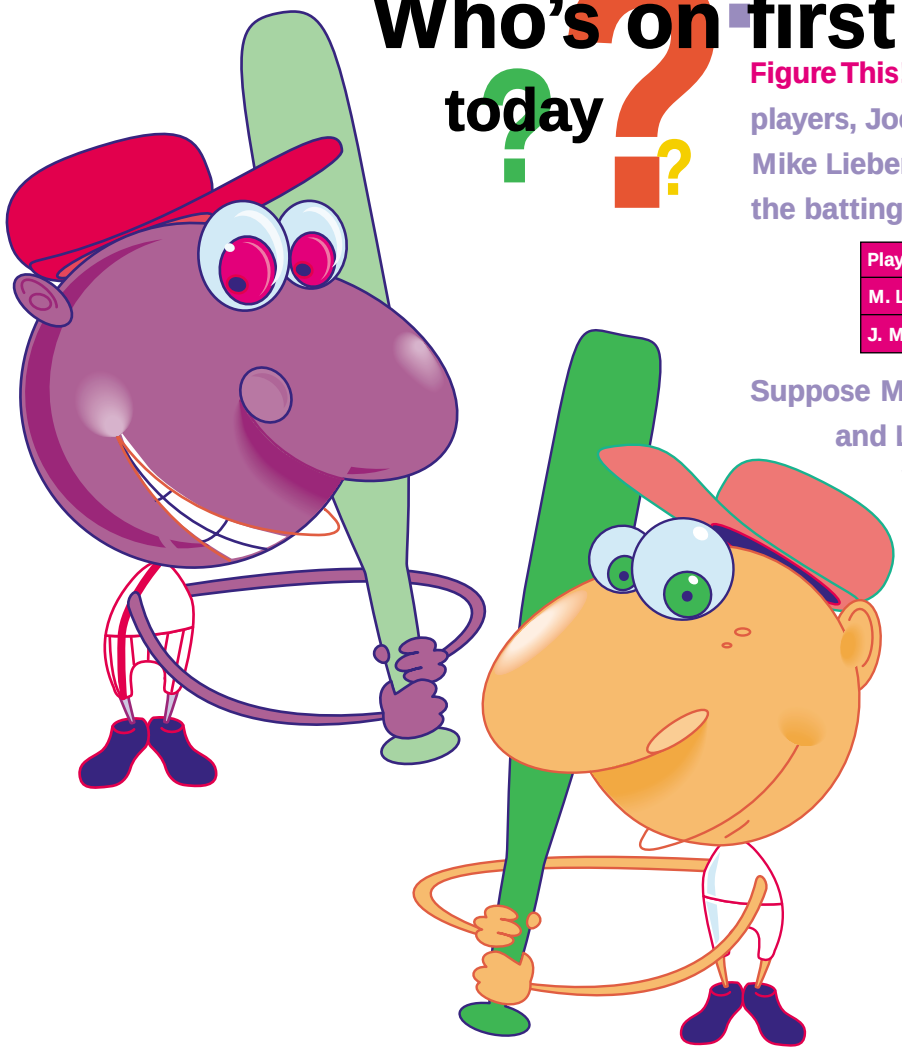


Figure This! In May 1999, two National League baseball players, Joe McEwing of the St. Louis Cardinals and Mike Lieberthal of the Philadelphia Phillies, each had the batting averages shown below.

Player	Team	At Bats	Hits	Batting Average
M. Lieberthal	Phillies	132	45	.341
J. McEwing	Cardinals	132	45	.341

Suppose McEwing then batted .800 (4 hits in 5 at bats), and Lieberthal was perfect (3 hits in 3 at bats). Which player now has the higher batting average? Are you surprised?

Hint: Batting average = $\frac{\text{Number of hits}}{\text{Number of at bats}}$ (rounded to the nearest thousandth)

An average is a tool for helping us understand and compare sets of numbers. Sports, medicine, and insurance are three of the many fields that use averages.

Answer:
McEwing has the higher average.

Figure This!

Get Started:

Make a new table using the updated information.

Complete Solution:

Both players had 45 hits in 132 at bats. Then with the statistics from the next at bats, McEwing's average is 49/137 or about .358 while Lieberthal's batting average is 48/135 or about .356. [≈ is a symbol that indicates "approximately equal to."]

Player	Team	At Bats	Hits	Batting Average	Next At Bats	Next Hits	New Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$	3	3	$\frac{48}{135} \approx .356$
J. McEwing	Cardinals	132	45	$\frac{45}{132} \approx .341$	5	4	$\frac{49}{137} \approx .358$

McEwing has the higher batting average. One way to make sense of this unexpected result is to imagine that McEwing gets 3 hits in his first 3 at bats while Lieberthal also gets 3 hits in 3 at bats. Then the pair is still tied. During McEwing's last 2 at bats, he gets 1 hit. This average of 1 for 2, or .500, is better than his current average, so his batting average goes up.

Try This:

- Check out some other sports. What statistics are collected? In which ones are averages computed?

Additional Challenges:

1. Suppose New York Yankee Chili Davis and Lieberthal have the batting averages shown.

Player	Team	At Bats	Hits	Batting Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$
C. Davis	Yankees	137	47	$\frac{47}{137} \approx .343$

If Davis bats 3 for 3 while Lieberthal has 4 hits in his next 5 at bats, who now has the higher average?

2. What number can be added to both the numerator and the denominator of a fraction so that the new fraction is equal to the original fraction?

Did You Know That?

- Rogers Hornsby of the St. Louis Cardinals had the highest season batting average in modern baseball history. In 1942, Hornsby hit a remarkable .424.
- The word fraction comes from the Latin word *frangere* meaning "to break."
- The ancient Egyptians primarily used fractions whose numerators were 1.
- Batting averages are typically spoken as whole numbers but are actually decimals.

Things to Think About:

- If you add all the numerators and all the denominators in a set of equal fractions, the result is a fraction equal to those in the original set.
- If you got an A on a test and a C on homework, do you have a B average?
- In the challenge, two players began the day with the same batting average. Lieberthal batted 1.000 while McEwing batted .800, yet McEwing ended the day with the higher cumulative batting average. Such unexpected results are called Simpson's Paradoxes, after Thomas Simpson, a mathematician who worked in the 1700s.
- Lou Abbott and Bud Costello had a comedy routine about baseball called "Who's on First?"

Resources:

Book:

- *The World Almanac and Book of Facts 1999*. Mahwah, NJ: World Almanac Books, 1999.

Websites

- Baseball Hall of Fame: **www.baseballhalloffame.org**
- Exploring Data: **curriculum.qed.qld.gov.au/kla/eda/sim_par.htm**
- UCLA: Simpson's Paradox: **www.stat.ucla.edu/~abraverm/Simpson/simpson.html**
- Simpson's Paradox Plaything: **www.stat.ucla.edu/~abraverm/Simpson/simpsonpt.html**
- **www.aentv.com/home/golden/colgate.htm**

Answers to Additional Challenges:

(1.)

Lieberthal's average is better, although not by much.

Player	Team	At Bats	Hits	Batting Average	Next At Bats	Next Hits	New Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$	5	4	$\frac{49}{137} \approx .358$
C. Davis	Yankees	137	47	$\frac{47}{137} \approx .343$	3	3	$\frac{50}{140} \approx .357$

(2.)

The number 0.

Math Challenge #15 BIG TREES

Figure This! Math Challenges for Families

Have you ever seen a tree big enough to drive a car through?

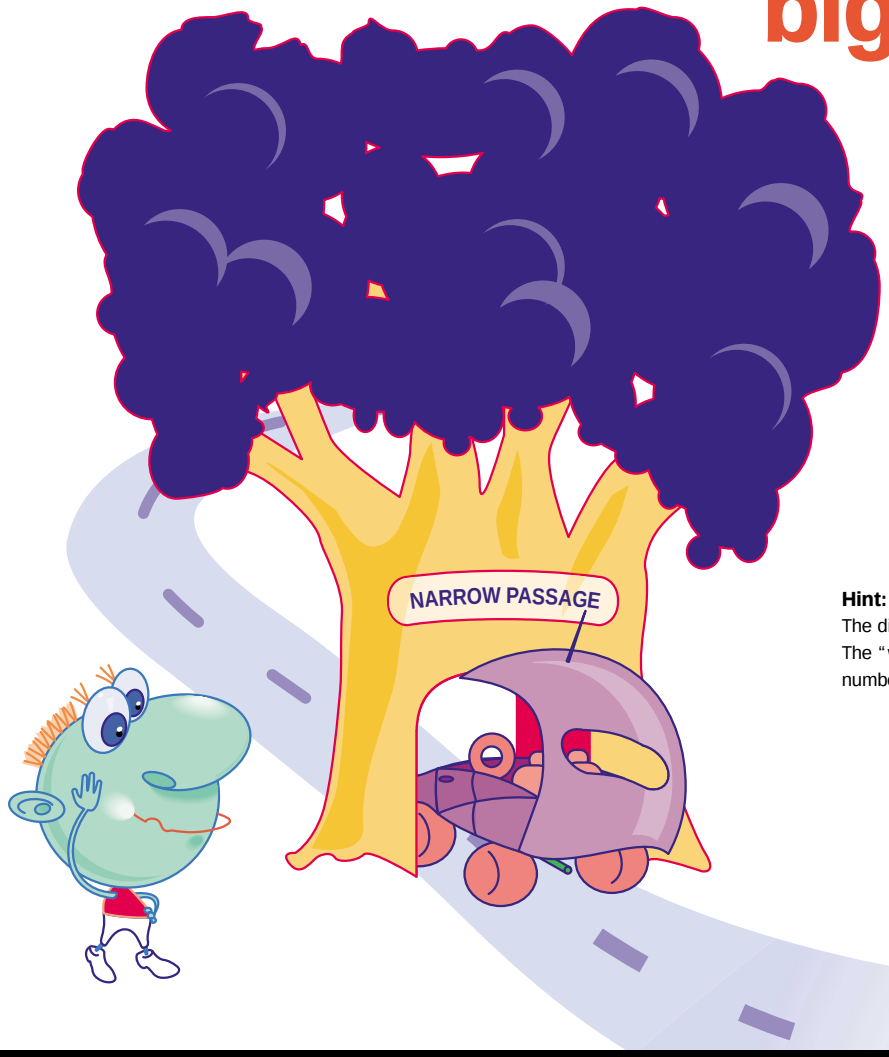


Figure This! Are any of the "National Champion" trees in the table below wide enough for a car to drive through?

Tree	Girth 4.5 ft above ground In inches	Height in Feet	Location
American Beech	279	115	Harwood, MD
Black Willow	400	76	Grand Traverse Co., MI
Coast Douglas Fir	438	329	Coos County, OR
Coast Redwood	867	313	Prairie Creek Redwoods State Park, CA
Giant Sequoia	998	275	Sequoia National Park, CA
Loblolly Pine	188	148	Warren, AR
Pinyon Pine	213	69	Cuba, NM
Sugar Maple	274	65	Kitzmiller, MD
Sugar Pine	442	232	Dorrington, CA
White Oak	382	96	Wye Mills State Park, MD

Hint:
The distance around a tree is its girth. The distance around a circle is its circumference. The "width" of a circle is its diameter. Finding the circumference of a circle involves the number π, about 3.14. The circumference of a circle is π times the diameter.

Measurement is important in many jobs. Carpenters, biologists, foresters, designers, and publishers use measurement formulas in their work.

Answer:
Only the coast redwood and the giant sequoia are clearly big enough. If two-feet "walls" are enough to hold up the tree around a six-foot wide car, then the black willow, the coast Douglas fir, the sugar pine, and the white oak are big enough as well.

Figure This!

Get Started:

What do you need to know about a car before you can answer the question? about the tree? What do you know about circles? How is the distance around a tree related to the width of a tree?

Complete Solution:

To find the diameter of a circle when you know the circumference, you divide the circumference by π (about 3.14). For example, the black willow has a girth of 400 inches and since 400 ÷ 3.14 is about 127, the black willow is about 127 inches wide, enough for a car to drive through and still leave at least 2 ft on each side.

Tree	Girth 4.5 ft above ground In inches*	Diameter (Girth/π) In inches*	Height In Feet
American Beech	279	89	115
Black Willow	400	127	76
Coast Douglas Fir	438	139	329
Coast Redwood	867	276	313
Giant Sequoia	998	318	275
Loblolly Pine	188	60	148
Pinyon Pine	213	68	69
Sugar Maple	274	87	65
Sugar Pine	442	141	232
White Oak	382	122	96

*Measurements rounded to nearest whole number.

Try This:

Find a tree in your yard or a park. Estimate the diameter of the tree.

Additional Challenges:

1. Are any of the National Champion trees taller than a 15-story building?
2. How many people holding hands would it take to go around the giant sequoia?

Things to Think About:

- How do you think foresters estimate the weight of a living tree?
- How do foresters estimate the number of feet of lumber in a tree?
- Why is the girth measured 4.5 feet above the ground?

Did You Know That?

- There are approximately 825 native and naturalized species of trees in the United States.
- The oldest living tree is believed to be a California bristlecone pine tree named Methuselah, estimated to be 4700 years old.
- The world's largest known living tree, the General Sherman giant sequoia in California, weighs as much as 41 blue whales or 740 elephants, about 6167 tons.
- In about 1638, Galileo Galilei (1564-1643) suggested that trees could grow only to be about 300 feet tall because of factors involving form and material.

Resources:

Book:

- *The World Almanac and Book of Facts 1999*. Mahwah NJ: World Almanac Books, 1998.

Website:

- National Register of Big Trees, American Forests **www.amfor.org**

Answers to Additional Challenges:

(1.)

Assuming that a typical story is 12 feet high, 15 stories would be about 180 feet high. Thus, the coast Douglas fir, the coast redwood, the giant sequoia, and the sugar pine would be taller than a 15-story building. The loblolly pine would be about the same height.

(2.)

Assuming that the arm span (with arms outstretched when holding hands) of a typical person is about 60 inches, it would take about 17 people to encircle the tree.

Math Challenge #16 BEANIE BABIES

Figure This! Math Challenges for Families

PROFIT - or - ? LOSS ?

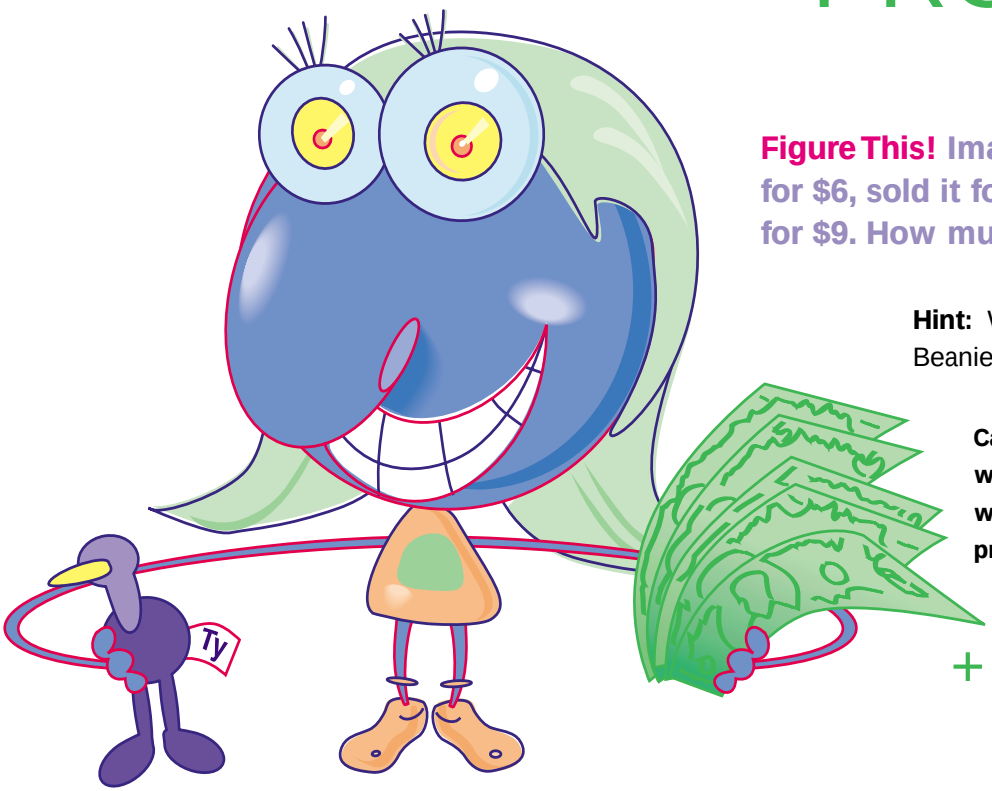


Figure This! Imagine that you bought a Beanie Baby™ for $6, sold it for $7, bought it back for $8, then sold it for $9. How much profit did you make?

Hint: Would it change your calculations if the second Beanie Baby™ were different than the first?

Calculations can often be done in several different ways. As long as the reasoning is correct, the result will be the same. Calculating, predicting, and reporting profit and loss are critical business skills.

+ ? - ? + ? - ? + ? - ? +

Answer: You made $2.

Figure This!

Get Started:

Think of this challenge as two separate problems. How much money did you make on the first sale? on the second?

Complete Solution:

In this situation, profit equals the selling price minus your cost. On the first sale, you made a profit of $1. On the second, you made another $1. The total profit was $2.

Paid	Sold	Profit or Loss
–$6	+$7	+$1
–$8	+$9	+$1
	Total	+$2

Try This:

Find the stock-market listings on television, in the newspaper, or on the Internet. Choose a company or two and pretend to buy their stock. Follow the listing for a week. Would you have gained or lost money on your investment if you did not have to pay someone to buy and sell your stocks?

Additional Challenges:

1. Your class is sponsoring a dance. The expenses include $450 for a DJ, $125 for refreshments, $45 for decorations, and $30 for advertisements. Judging from previous dances, you expect to take in at least $150 on the refreshment stand alone. If you charge $5 per person to attend the dance, how many people must attend so your class can make a profit?

2. Suppose you bought 200 shares of stock at $45 a share. When the price per share went up $5, you sold 100 shares. Several weeks later, the price per share was down $10 from your previous selling price, so you sold the remaining 100 shares. How much money did you make if you don't have to pay someone for buying and selling the stock?

Things to Think About:

- How can people make money by repeatedly buying and selling the same item?
- Can a store always make a larger profit by charging more?
- What factors does a business consider before setting prices?

Did You Know That?

- The Securities and Exchange Commission (SEC), as a part of its job, monitors profits and losses of businesses that are publicly traded.
- The Beanie Baby™ is an invention of H. Ty Warner of Ty, Inc. He wanted to market an inexpensive toy that children could buy themselves. The original nine Beanies—a moose, bear, dolphin, frog, platypus, lobster, whale, dog, and pig—were first made available to the public in 1994, primarily in the Chicago area. According to the 1999 edition of *Toys and Prices*, 1999, a dark blue Peanut the Elephant in mint condition was valued at $4200.
- From October 1, 1981, to September 30, 1982, the American Telephone and Telegraph Co. (AT&T) made a net profit of $7.6 billion.
- As of 1997, the most active common stock on the New York Stock Exchange was Compaq Computer Corporation, with a volume of 1231.6 million shares either bought or sold during a year.

Resources:

Books:

- Brecka, Shawn. *The Beanie Family Album and Collectors Guide*. Norfolk, VA: Antique Trader Books, 1998.
- Korbeck, Sharon. *Toys and Prices*. 6th Edition. Iola, WI: Krause Publications, 2000.
- *The Guinness Book of World Records*. New York: Bantam Books, 1998.
- *The World Almanac and Book of Facts*, 1999. Mahwah, NJ: World Almanac Books, 1998.
- Gardner, Martin (ed.). *Mathematical Puzzles of Sam Loyd.* New York: Dover Publications, Inc. 1959.

Websites:

- **www.ty.com**
- **www.historychannel.com** (search on Toys & Games)

Answers to Additional Challenges:

(1.) more than 100 people.

(2.) None (you broke even).