CONCERT TICKETS

78 60 101 132 112 144 130 205 187 217 262 294 283 299 279 300

Figure This!
Math Challenges for Families

How **long** do you have to stand in line **??**

Figure This! How long do you think you would have to wait in this line if you hold number 300?

No. **300**

Hint: Estimate the amount of time it would take for one person to buy a ticket. Use this estimate to find the amount of time you will have to wait in line.

Estimation and measurement of time are basic skills for all. Businesses such as banks, fast-food restaurants, ski areas, and airports need efficient ways to minimize time spent waiting in line.

Answer: Assuming that it takes about 30 seconds to buy a ticket, you would wait about 2 1/2 hours.

Figure This!

Get Started:

Pretend you are buying a ticket. How long would it take?

Complete Solution:

If each person takes 30 seconds to buy a ticket, then the 299 people in front will take about 30 x 299 or 8970 seconds about 150 minutes. That's 2 1/2 hours.

Another way to estimate this is to recognize that if it takes 30 seconds for one person to buy a ticket, then 2 people can buy a ticket in 1 minute. Thus, it takes 300 ÷ 2 or about 150 minutes for your turn to arrive. That's about a 2 1/2 hour wait.

Try This:

Go to a fast-food restaurant, supermarket, or someplace where people wait in line. Find the average number of people in line and the average amount of time they have to wait. Do different kinds of lines make a difference in waiting time? For example, is the express line really faster?

Additional Challenges:

1. If you expect 600 people to buy tickets to a rock concert at your school and the box-office opens one-half hour before the show, how many ticket sellers do you need?

2. If there are 300 people ahead of you in line to buy a ticket, about how many feet back are you?

Things to Think About:

- Do lines move faster at movies or at concerts?

- What factors would make the time for purchasing a ticket longer or shorter?

Did You Know That?

- The 1993 *Guinness Book of World Records* reported that the longest line of coins ever created was 34.57 miles long. It had 2,367,234 coins and was in Kuala Lumpur, Malaysia.

- The 1993 World Record for the most valuable line of coins was 1,724,000 quarters. The line was 25.9 miles long in Atlanta, GA.

- On April 21, 1990, an estimated 180,000 people in Maracana Stadium in Rio de Janeiro, Brazil, paid to hear Paul McCartney.

- At the London opening of *The Phantom Menace*, 11,500 tickets were sold in just 30 minutes on June 12, 1999.

- Queuing theory deals with wait times in lines.

Resources:

Book:

Matthews, Peter, ed. *The Guinness Book of World Records 1993*. New York: Bantam Books, 1993.

Website:

"Does this line ever move?" *Informs*
mie.eng.wayne.edu/faculty/chelst/informs

Answers to Additional Challenges:

(1.)
If each ticket purchase takes 30 seconds, it would take 5 hours for one person to sell 600 tickets. Since you only have one-half hour to sell the tickets, you would need 10 ticket sellers.

(2.)
If each person requires about 2 feet of space along the ground, then you are about 300 x 2, or 600 feet back in the line.

Figure This!
Math Challenges for Families

How *fast* does your heart beat?

## How long does it take for your heart to beat 1000 times?

Figure This! If you started counting your heartbeats at midnight on January 1, 2000, when would you count the millionth beat? What about the billionth beat?

Hint: Estimate your heart rate in beats per minute, per hour, and per day.

Estimating and understanding large numbers are useful mathematical skills. Without these skills, it is difficult to comprehend the size of the national debt, for example, or how many miles it is to Mars.

*bah **boom**, bah **boom**, bah **boom***
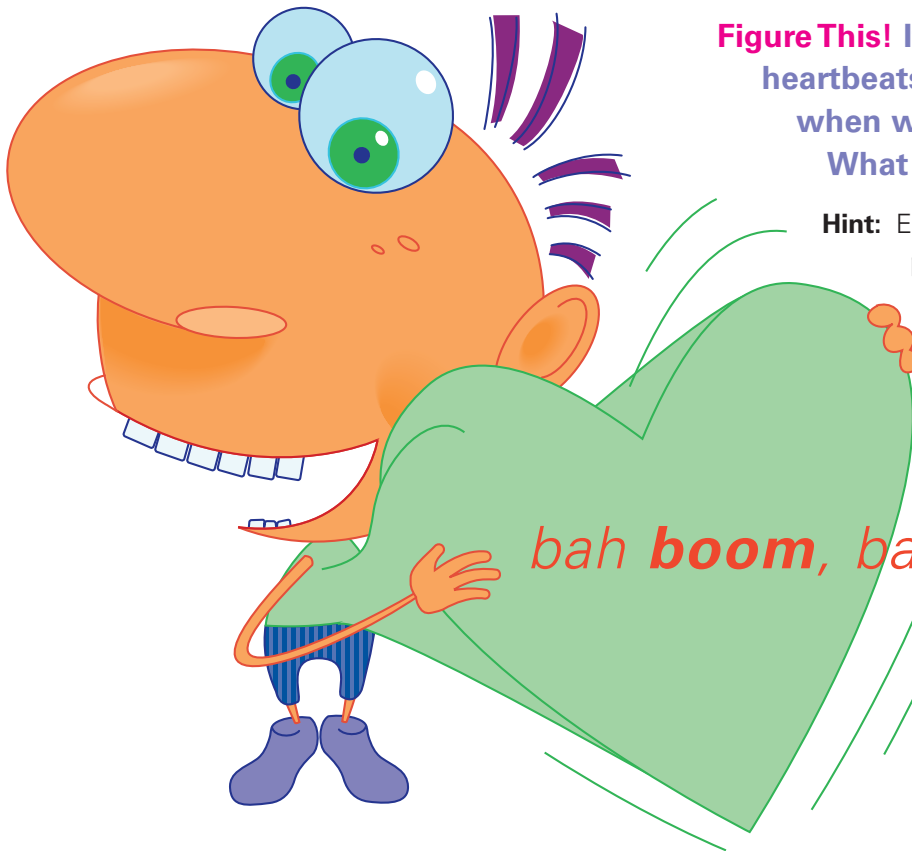
Figure This!

Get Started:

The best place to find your pulse is on your wrist or neck. Count the number of beats in your pulse for 15 seconds and multiply by 4, or count the number of beats for 30 seconds and multiply by 2. Once you know the number of times your heart beats in 1 minute, find the number of beats in 1 hour, and then in 1 day.

Complete Solution:

By estimating, you can often find a reasonable answer more quickly. Round your number of heartbeats to a number that is easy to use for calculation. For example, if your heart beats 72 times per minute, 70 beats per minute would give a reasonable estimate.

Beats per Hour	Beats per Day	Estimated Beats per Day
70 x 60 = 4200	4200 x 24	4000 x 25 = 100,000

Next find how many days it takes for 1 million heartbeats.

1,000,000 ÷ 100,000 = 10. Thus, 1,000,000 beats take 10 days.

If you begin counting on January 1, the millionth beat will be on January 10. For 1 billion heartbeats, the number of days is 1,000,000,000 ÷ 100,000 = 10,000. Since there are 365 days in a year, and 10,000 ÷ 365 is approximately 27.4, it will take about 27.4 years. The billionth beat will be in May 2027.

Try This:

Go to a website and search for large numbers using keywords such as billion, trillion, and large numbers. Try **www.mcn.net/~jimloy/trillion.html**

Additional Challenges:

1. Have you been alive for 1 million minutes? Do you know someone who has been alive for this long?

2. Have you been alive for 1 billion minutes? Do you know someone who has been alive for this long?

3. Have you been alive for 1 million hours? Do you know someone who has been alive for this long?

4. Suppose your heart rate was 72 beats per minute. You rounded it to 70 to make your calculation. If you did not round, how far off would your estimated answers be?

Things to Think About:

- If you had 1 million drops of water, would you be more likely to drink it, take a bath in it, or swim in it? What if you had 1 billion drops of water?

- There are approximately 3 million people in the city of Chicago. How many times larger or smaller is the population of your city than the population of Chicago?

- How much does rounding affect your answers when adding? When multiplying? How close is close enough?

Did You Know That?

- 1 million is 1000 times 1000, or $10^3 \cdot 10^3 = 10^6$, no matter where you live, but the definition of 1 billion depends on where you are in the world.

- In the USA, 1 billion is 1000 times 1 million, or, $10^3 \cdot 10^6 = 10^9$, but in Great Britain and France 1 billion is 1 million times 1 million, or $10^6 \cdot 10^6 = 10^{12}$

Resources:
Books:

- Morrison, Philip. *Powers of Ten.* New York: Scientific American Books, W. H. Freeman, 1982.

- Paulos, John Allen. *Innumeracy.* New York: Hill and Wang, 1988.

- Schwartz, David M. *How Much Is a Million?* New York: William Morrow & Company, 1994.

- Schwartz, David M. *If You Made a Million.* New York: William Morrow & Company, 1994.

- Strauss, Stephen. *The Sizesaurus.* New York: Avon Books, 1997.

Answers to Additional Challenges:

(1.)
Yes. There are 60 minutes in an hour, 24 hours in a day, and 7 days in a week. Since 60 x 24 x 7 = 10,080 there are close to 10,000 minutes in a week. Thus, 1 million minutes is about 100 weeks, or about 2 years, so anyone more than 2 years old has been alive for at least 1 million minutes.

(2.)
No. Since 1 billion is a thousand million, 1 billion minutes is around 1000 x 2 , or 2000 years.

(3.)
There are approximately 24 x 365, or 8760 hours in a year. Because 1,000,000÷8760 is a little more than 114, it is unlikely that you know anyone who has been alive for 1 million hours, about 114 years.

(4.)
For 1 million beats, the answer is off about 15.5 hours. For 1 billion beats, the answer is off about 9 months.

Figure This!
Math Challenges for Families

If you like popcorn,
which one
would you buy

Figure This! Take two identical sheets of paper. [An ordinary sheet of paper measures 8 1/2 inches by 11 inches.] Roll one sheet into a short cylinder and the other into a tall cylinder. Set them both on a flat surface. Does one hold more than the other?

Hint: Place the taller cylinder inside the shorter one. Fill the taller one with dry cereal, rice, or popcorn; then remove it from the shorter cylinder. Which holds more?

Making visual estimates and finding volumes are useful skills.

Designers and engineers use these skills to find economical ways to package and protect items.

Answer:
The shorter cylinder holds more.

FigureThis!

Get Started:

Make a guess and then use the hint.

Complete Solution:

The process described in the "Hint" shows that the shorter cylinder holds more.

- To determine an answer mathematically, find the volume of each cylinder. The volume is the area of the base times the height. In this case, the bases are circular. The area of a circle is $\pi \cdot r \cdot r$ or approximately $3.14 \cdot r \cdot r$. To find the radius, r, use a ruler to estimate the width (or diameter) of the circle. Divide the diameter by 2 to get the radius. Another way to find the radius of a circle is to use the formula:

    Circumference of circle = $2 \cdot \pi \cdot$ radius = $2\pi r$
    Radius = Circumference divided by (2π)

Once you have the radius, the table below shows how to determine the volume of each cylinder. The sheet of paper is 8 1/2 inches by 11 inches.

Cylinder	Base (inches) Circumference	Radius (inches) r	Height (inches) h	Volume (cubic inches) $\pi \cdot r \cdot r \cdot h$
Short	11	11 ÷ (2π) or about 1.75	8.5	$\pi \cdot 1.75 \cdot 1.75 \cdot 8.5$ About 81.8
Tall	8 1/2 = 8.5	8.5 ÷ (2π) or about 1.35	11	$\pi \cdot 1.35 \cdot 1.35 \cdot 11$ About 63.0

The volume of the shorter cylinder is about 82 cubic inches, and the volume of the taller cylinder is about 63 cubic inches.

Try This:

- Go to the grocery store and see what items come in different shaped or different sized cylinders.
- Look in your cupboard or go to the grocery. Find two different shaped containers that hold the same amount. What are the contents of each?

Additional Challenges:

1. For what size paper would the two cylinders hold the same amount?
2. Using any non-square rectangular sheet of paper, does the shorter cylinder always hold more?
3. Another way to describe a cylinder is to rotate an index card about one of its sides. Think about the cylinder traced by a 3 by 5 card as it turns. Which is larger: the volume of the cylinder formed when the card is turned about a short side or a long side?

3 in.

5 in.

4. Suppose you had two equal lengths of wire. Fold the wires to make two rectangles. Do you think the two rectangles will always have the same area?

Things to Think About:

- What items come packaged in cylindrical containers?
- What types of goods are packaged in boxes instead of cylinders? Why do you think companies use boxes?
- If a number is greater than 1, squaring it makes the result greater.

Did You Know That?

- Designing cans and labels is just one aspect of packaging technology. You see the results of this work every time you unwrap a CD, twist open a lipstick container, or open a soft drink.
- Several universities offer degrees in packaging technology. Many of them can be found at **packaging.hp.com/pkgschools.htm**
- Isopermetric figures are figures with the same perimeter. Fencing problems typically fall under this category.

Resources:
Books:

- Lawrence Hall of Science. *Equals Investigations: Flea-Sized Surgeons*. Alsip, IL: Creative Publications, 1994.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and E. Phillips. *Connected Mathematics: Filling and Wrapping*. Palo Alto, CA: Dale Seymour Publications, 1996.

Answers to Additional Challenges:

(1.)
The two cylinders would hold the same amount only for square sheets of paper.

(2.)
Yes. (This can be proven mathematically.)

(3.)
The cylinder with the larger volume is described when the card is turned about its shorter side. This problem compares $\pi \cdot 5 \cdot 5 \cdot 3$ and $\pi \cdot 3 \cdot 3 \cdot 5$.

(4.)
The rectangles may not always have the same area. Consider a piece of wire 16 inches long. You can make a square 4 inches by 4 inches with an area of 16 square inches, or a 1 inch by 7 inches rectangle that has an area of 7 square inches.

Figure This!
Math Challenges for Families
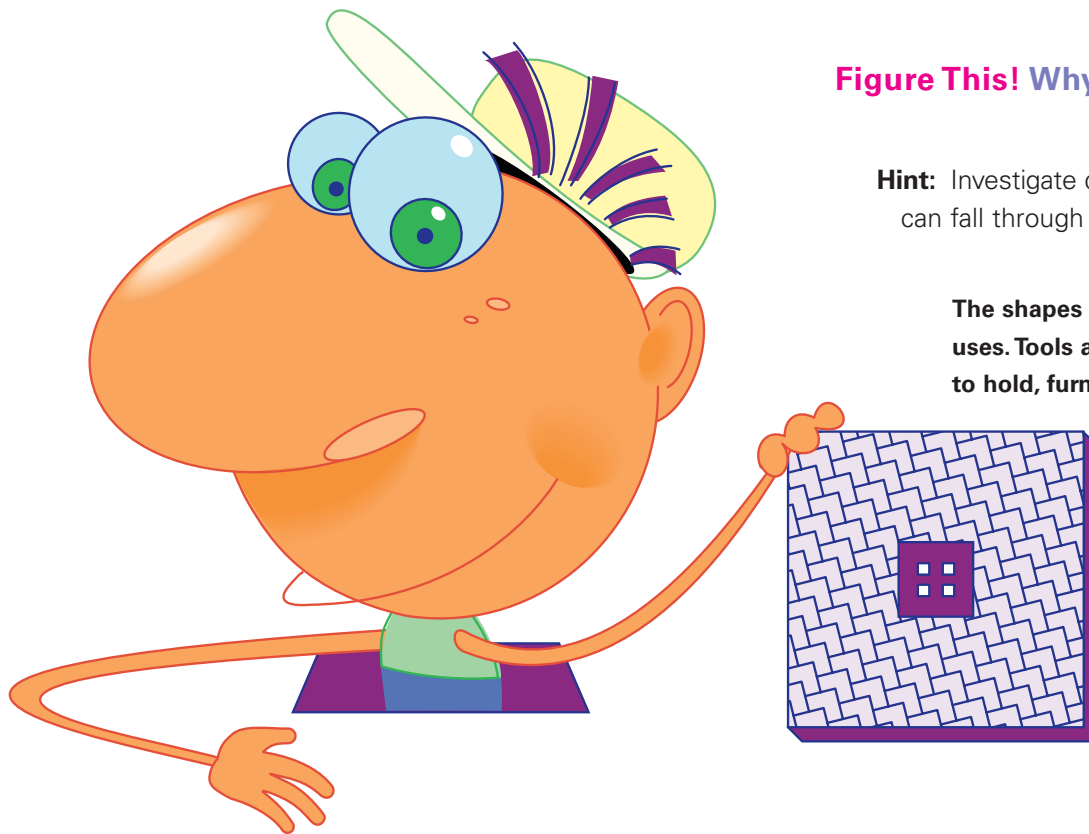
Why aren't manhole covers square? Why?

Figure This! Why are most manhole covers round?

Hint: Investigate different shaped covers to see if they can fall through their corresponding holes.

The shapes of many objects relate directly to their uses. Tools are designed with shapes that are easy to hold, furniture is designed with shapes that are comfortable, and race cars are designed to reduce wind resistance.

Figure This!

Get Started:

Cut a rectangle and a circle out of a 3 by 5 card or light cardboard. See which will easily fall through the hole left in the cardboard.

Complete Solution:

A manhole cover rests on a small lip inside the hole. A circular manhole cover typically will not fall into the hole because its width is the same all around. A rectangular manhole cover, however, could fall through the hole when it is tipped upward. You can see this by drawing a diagonal in a rectangle to create two triangles. Mathematically, the greatest angle of a triangle is opposite the longest side. The greatest angle in each triangle formed by drawing in the diagonal is the right angle at the corner.
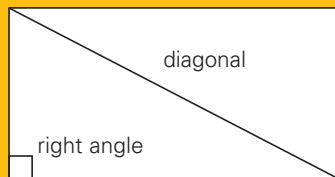
This means that the diagonal of a rectangle is always longer than either of the sides. As a result, rectangular covers can always be dropped through their corresponding holes if the lips are small. Because a square is a rectangle, the same reasoning applies to squares.
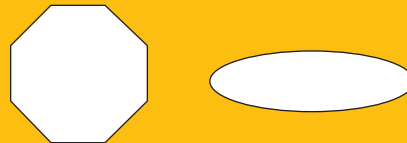
Try This:

• Look around you and find some triangles. Decide which angle is the largest and check to see if it is opposite the longest side. Cut a triangle from a piece of cardboard and see if it can easily fall through the hole formed where you cut it out.

• On a card, draw a triangle with all sides the same length (an equilateral triangle). Next, draw arcs around the triangle, using each corner as the center of a circle and the length of a side of the triangle as the radius. (See the diagram below.)
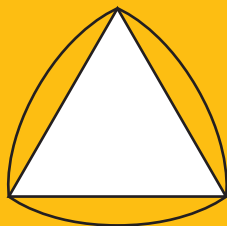
Cut out the shape and see whether it would fall through the hole left in the card. Measure from any point of the shape to the other side. What do you notice about the widths? With the usual small lip of a manhole, would this shape make a good cover?

Additional Challenges:

1. Would the following shapes make good manhole covers?

2. Name a three-dimensional shape that has the same width all around.

Things to Think About:

• A circle is known as a "curve of constant width." Why do you suppose that such curves have this name? Can you think of any other shape that has a constant width? Does the figure you made in the "Try This" section above have a constant width?

• Will any manhole cover that is not a curve of constant width fall into its hole?

Did You Know That?

• A drill bit based on the triangular shape in the "Try This" section cuts a square hole.

• The bases of Pepto-Bismol™ bottles have shapes like the one in the "Try This" section.

• Triangular manhole covers are used in some places in Minnesota.

• Curves of constant width were studied by the Swiss mathematician, Leonhard Euler (1707-1783).

• Reuleaux triangles were named for Franz Reuleaux (1829-1905), a German engineer, mathematician and high school teacher.

Resources:
Books:

• Gardner, Martin. *Mathematics, Its Spirit and Use*. New York: W. H. Freeman, 1978.

• Maletsky, Evan. "Curves of Constant Width." In *Teaching with Student Math Notes, Vol. 2*. Reston, VA: National Council of Teachers of Mathematics, 1989.

• Melnick, Mimi. *Manhole Covers*. Cambridge, MA: MIT Press, 1994.

Answers to Additional Challenges:

(1.) Neither would make a good cover because their widths are not the same all around.

(2.) A sphere.