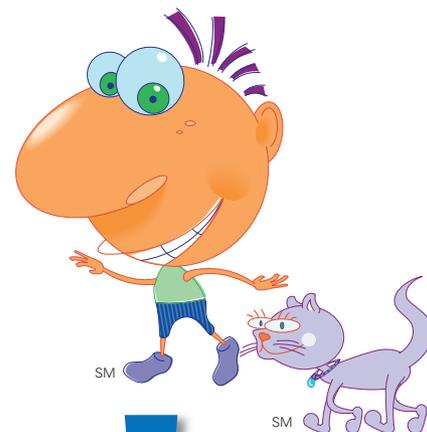
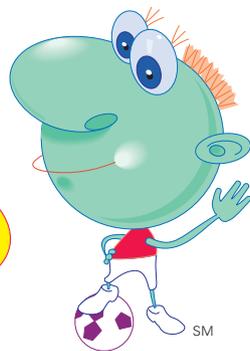


Figure This!
Math Challenges for Families SM



Take a Challenge!

Set IV: Challenges 49 - 64





Axis

Thank you for your interest in the **Figure This! Math Challenges for Families**. Enclosed please find Challenges 49 – 64. For information about other challenges, go to www.figurethis.org.

The **Figure This!** Challenges are family-friendly mathematics problems that demonstrate what middle-school students should be learning and emphasize the importance of high-quality math education for all students. This campaign was developed by the National Action Council for Minorities in Engineering, the National Council of Teachers of Mathematics, and Widmeyer Communications, through a grant from The National Science Foundation and the US Department of Education.

Figure This!

Math Challenges for Families

Tessellation



Polygon

Ratio

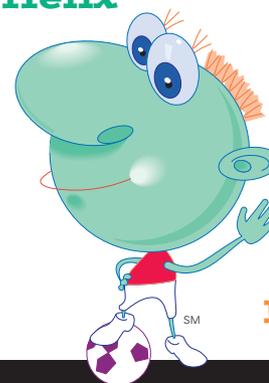


Tangent



Perimeter

Helix



Exponent



We encourage you to call toll free **1-877-GO-SOLVE** or visit our website at www.figurethis.org where you can find these and other challenges, along with additional information, math resources, and tips for parents.

Figure This!

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Figure This!

Math Challenges for Families

WHAT'S *missing*?

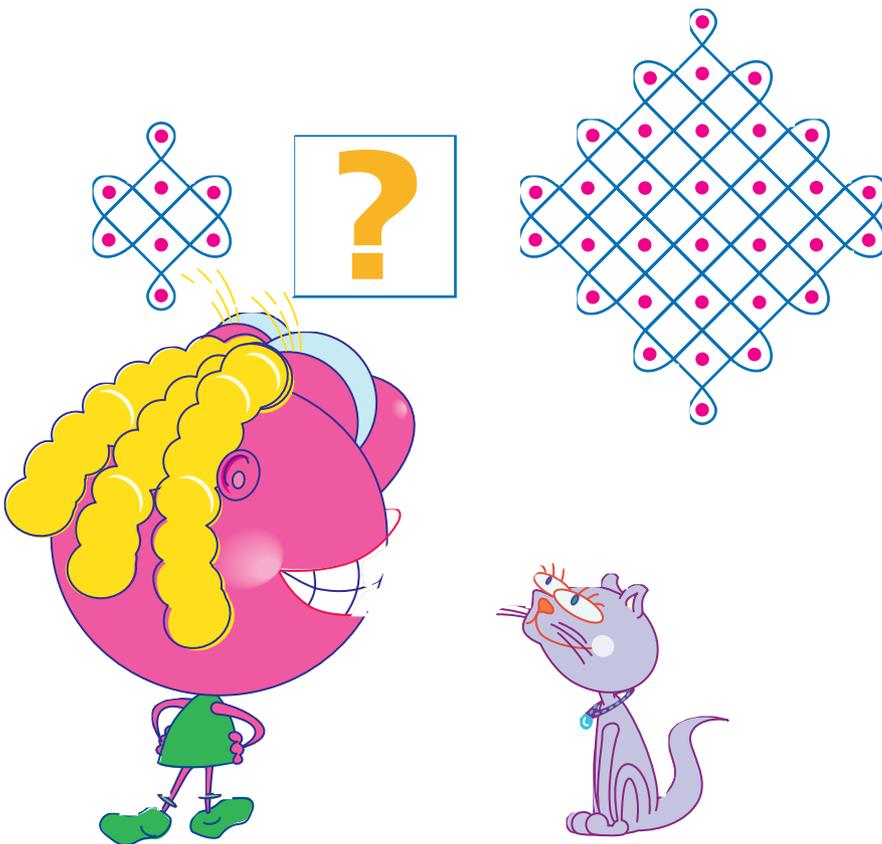
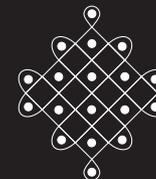


Figure This! The designs shown here are typical of a *sona*, an African sand tracing. If the missing design is part of a pattern of increasing size, what might the missing design look like?

Hint: Look for patterns in the tracing, as well as in the number and arrangement of dots in each figure.

Most people throughout history have created designs and patterns to express their cultures. Many such designs also feature an underlying mathematical pattern. Studying mathematical designs and patterns helps archaeologists and anthropologists understand ancient cultures.



Answer: The missing design could be:

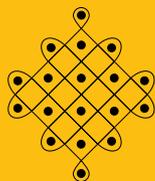
Figure This!

Get Started:

How are the dots arranged? Are they in rows? How are the loops arranged? What are the relationships among the dots, the loops, and the squares? Can you determine the number of dots in the missing figure?

Complete Solution:

- One way to approach this problem is to examine the pattern of dots. In the first figure, there are four rows of horizontal dots in the pattern 1, 3, 3, 1. In the third figure, there are eight rows of dots in the pattern 1, 3, 5, 7, 7, 5, 3, 1. It seems logical, therefore, that the missing figure would include six rows of dots in the pattern 1, 3, 5, 5, 3, 1.



- Another way to look at the pattern is to count the “loops” in the design. The first figure has two loops per side, while the third figure has four loops per side. In this case, it seems reasonable for the missing figure to have three loops on each side.
- A third way to consider the problem involves counting squares. The first design is a two-by-two square; the third is a six-by-six square. Therefore, the second might be a four-by-four square.

Try This:

- The sona patterns were drawn by tracing in the sand. Trace the outline of the designs in the Challenge without lifting your finger.
- Arrange five rows of dots in the pattern 1, 3, 5, 3, 1. Draw a design that encloses each dot in its own “cell,” with no two dots in the same cell.
- Construct your own sequence of patterns of dots and designs. Remove one figure from the sequence. Challenge a family member to draw the missing pattern.
- Research Native American culture looking for patterns and designs in clothing and pottery.

Additional Challenges:

(Answers located in back of booklet)

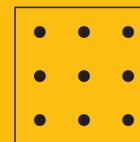
1. If the design shown in the challenge were expanded to a fourth figure, how would the dots be arranged?
2. The diagram below shows three figures in a pattern.



- a. If the pattern continues, what do you think the next figure will look like?
 - b. The number of dots, S , in each figure can be found using the formula $S = \frac{n(n+1)}{2}$ where n is the number of the figure. How is this formula related to the pattern?
3. Without lifting your pencil, draw four straight line segments on the pattern below so that every dot lies on at least one of the line segments.



4. Draw two triangles in the square to create nine cells, so that each cell contains exactly one dot.



Things to Think About:

- How are fruits or cans stacked in grocery displays?
- What products use repeating designs in their advertisements or company logos?
- Hopscotch patterns are typically passed from adult to child or child to child. What hopscotch designs do you know?
- Are there other ways to describe the pattern in Additional Challenges 2?

Did You Know That?

- The sand patterns in the Challenge were created by the Tchokwe tribe of northeastern Angola. Members of this tribe used these designs for telling stories.
- Anthropologists Dorothy Washburn and Donald Crowe have studied linking of geometric patterns to cultural identification.
- Some African cultures, such as the Ndebele, make decorations that use line symmetry. These patterns contain no curves, in contrast to the Angolan patterns in the Challenge.
- Jhane Barnes is a designer who uses mathematics to form patterns to weave into cloth for her line of clothing.
- Many entrance examinations for higher education, including the Dental Aptitude Test (DAT), include questions involving the recognition and extension of patterns.

Figure This!

Resources:

Books:

- Gerdes, P. *Geometry from Africa: Mathematical and Educational Explorations*. Washington, DC: The Mathematical Association of America, 1999.
- Nelson, D., G. Joseph, and J. Williams. *Multicultural Mathematics: Teaching Mathematics from a Global Perspective*. Oxford: Oxford University Press, 1993.
- Washburn, D., and D. Crowe. *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis*. Seattle, WA: University of Washington Press, 1988.
- Zaslavsky, C. *Africa Counts: Number and Pattern in African Culture*. Brooklyn, NY: Lawrence Hill Books, 1990.

Website:

- www.jhanebarnes.com

Notes:

Tangent





Figure This!

Math Challenges for Families

Do dogs age *faster* than people???

Large Dog's Age in Years	0	1	2	3	4	5	6	7
Equivalent Human Age	0	12	19	26				

<http://petcare.umn.edu/Fun/DogAge.html>

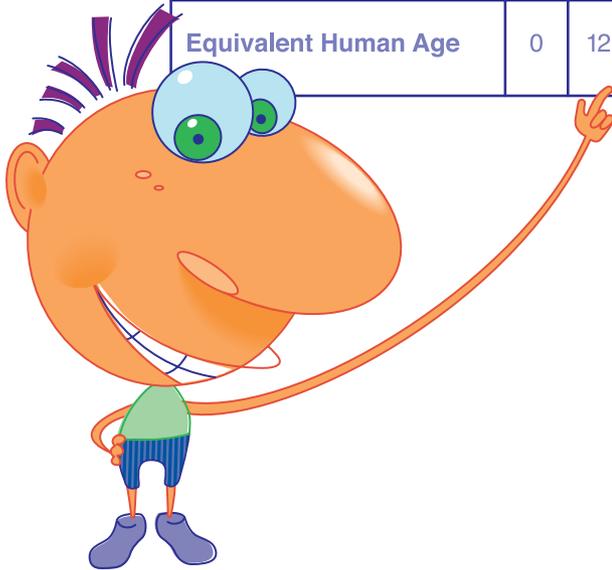


Figure This! Ratio's dad is 35 years old. When will his dad and his newborn Irish setter puppy be the same age in human years?

Hint: The growth rate is different the first year. After the first year, how many human years does a dog age each year?

Information is often converted from one system of measurement to another. Statisticians, scientists, and engineers all use conversion formulas in their work.

Answer: In 5 years, when Ratio's dad is 40, the dog's "human age" also will be 40.

Figure This!

Get Started:

Add a row to the table that shows Ratio's dad's age with each passing year.

Large Dog's Age in Years	0	1	2	3	4	5	6	7
Equivalent Human Age	0	12	19	26				
Ratio's Dad's Age in Years	35	36	37	38				

Complete Solution:

- After the first year, the "human age" of a large dog such as an Irish setter increases by 7 each year. Use this information to complete the table. When the dog is 5 years old, its human age will be 40. In 5 years, Ratio's dad also will be 40.

Large Dog's Age in Years	0	1	2	3	4	5
Equivalent Human Age	0	12	19	26	33	40
Ratio's Dad's Age in Years	35	36	37	38	39	40

Try This:

- Find the ages of some neighbors and friends, along with the ages of their dogs or cats. How many of them are "older" than their pets? [Note: The age table given in the Challenge is for large dogs. See the Additional Challenges for age tables for small dogs and for cats.]
- Estimate the age (in months) of a large dog whose "human age" is about the same as your own.

Additional Challenges:

(Answers located in back of booklet)

- Find a formula that shows the relationship between the human age H of a large dog and the dog's actual age d .
- The table below shows the relationship between actual age and "human age" for small dogs.

Small Dog's Age in Years	0	1	2	3	4	5	6
Equivalent Human Age	0	15	25	30	35		

- Based on this information, what type of dogs appear to age faster: large or small?
 - When Ratio's dad was 35, if he had adopted a small newborn dog, when would the two have been the same age in human years?
 - If a small dog and a large dog were born on the same day, when will the small dog be 10 human years younger than the large dog?
- How would a formula for the human age of a small dog differ from the one for large dogs?

- The following table shows the relationship between actual age and "human age" for cats.

Cat's Age in Years	0	1	2	3	4	5	6
Equivalent Human Age	0	16	24	28			

The formula for the human age of a cat is $H = 16 + 4c$, for $c \geq 2$. When a cat is 36 in human years, how old is it in cat years?

Things to Think About:

- Given the actual age of a small dog, how could you find the actual age of a cat with the same "human age?"
- How many actual years does it take for a small dog to reach "retirement age?"
- In the Challenge, you related the years in an animal's life to a human age. How could you relate the years in a human's life to an animal's age? For example, if you were 13, what age would you be in cat years?
- How did biologists decide that a 1-year-old small dog is comparable to a 15-year-old human?
- Why do puppies grow faster in their first year of life than in later years? Is this also true for human babies?
- Do larger animals typically live longer than smaller ones?
- Which animals live the longest?

Did You Know That?

- An Asian elephant has an average life span of about 40 years. In 1998, the average life span of a US resident was about 77. The average life span of an opossum is 1 year.
- One familiar conversion formula describes the relationship between degrees Celsius and degrees Fahrenheit: $F = 1.8C + 32$.
- For an age older than 1, the graph of equivalent human age versus a large dog's age in years lies along a straight line.
- According to *The Guinness Book of Records*, the world's oldest dog was an Australian cattle-dog named Bluey. He died at the age of 29 years and 5 months in 1939. The world's oldest cat, a sphinx, was Granpa Rexs Allen, adopted from the Humane Society of Travis County (Texas), who died in 1998 at the age of 34 years, 2 months.

Resources:

Books:

- Burrill, G., M. Clifford, and R. Scheaffer. *Exploring Symbols: Data Driven Mathematics*. White Plains, NY: Dale Seymour Publications, 1987.
- Roizen, M. *Real Age: Are You As Young As You Could Be?* Scranton, PA: Harper Collins, 2000.



Figure This!

Math Challenges for Families

Can you D E C O D E a bar code?

Post Office Conversion Code		
11000 = 0	01001 = 4	10010 = 8
00011 = 1	01010 = 5	10100 = 9
00101 = 2	01100 = 6	
00110 = 3	10001 = 7	

Figure This! The U. S. Post Office uses short and long bars to represent ZIP Codes. A short bar is 0, and a long bar is 1. The first bar on the left and the last 16 bars on the right are not part of the ZIP Code. What is the 9 digit ZIP Code on Tesselation’s envelope?

Hint: Group the bars into sets of five, ignoring the first bar and the last 16 bars that are not part of the ZIP Code. For example, what number is represented by the following bars?



Bar-code systems were devised by scientists and mathematicians to speed the input and flow of data. Their use has enabled many businesses to operate more efficiently, including banks, delivery services, and supermarkets.

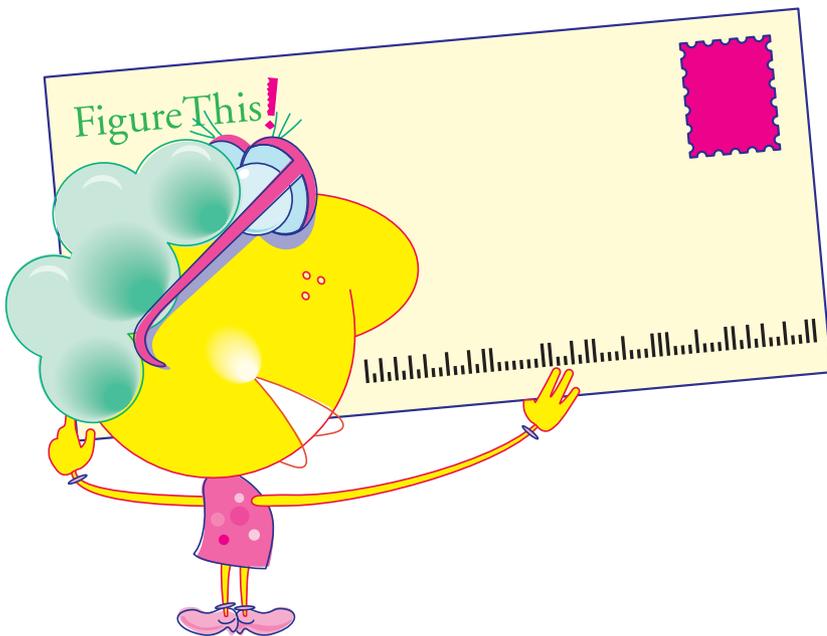


Figure This!

Get Started:

Convert the long and short bars to 0s and 1s. Then match each group of five to the code chart shown in the Challenge.

Complete Solution:

The first long bar in the US Postal Service bar code is a “start” bar. The next five bars on the envelope represent the sequence 01010 which represents the number 5 on the conversion chart. The following diagram shows the remaining groups of bars. The bars in the box are not a part of the ZIP Code.



The corresponding ZIP Code is 59801-2717.

Try This:

- Use the method described in the Challenge to decode the bar code on a piece of mail delivered to your home.
- The UPC code on supermarket items is another type of bar code. Look in your refrigerator or cupboard for several different products made by the same company. Check their UPC codes. Do you notice similar patterns in their codes?

Additional Challenges:

(Answers located in back of booklet)

1. How many different bar codes are possible using two long bars and three short bars?
2. If no restrictions are placed on the digits, how many different ZIP+4 Codes are possible?
3. The ZIP Code for Honolulu, Hawaii is 96826. Draw the portion of a US Postal Service bar code that would represent this ZIP Code.

Things to Think About:

- The abbreviation UPC stands for Universal Product Code. The figure below shows a typical UPC code. How does this type of bar code differ from the US Postal Service bar code?



- How does the US Postal Service handle pieces of mail that do not have bar codes on them?
- Before UPC codes were invented, how did cashiers know the price of an item?
- UPC labels are never printed in red ink. Why not?

Did You Know That?

- The ZIP in ZIP Code stands for Zone Improvement Plan.
- The bar-code system used by the US Postal Service is called Postnet.
- After the ZIP Code bars, the next two groups of five bars represent the delivery code for the mail on the primary street address. The last set of bars in the code represents a check digit that helps a computer determine if the first digits have been read correctly. The check digit at the end of a US Postal Service code is calculated by adding the 11 previous digits in the code, then identifying the smallest digit that can be added to this sum to obtain a multiple of 10. For example, the check digit for the postal code in the Challenge is 3, since
$$5 + 9 + 8 + 0 + 1 + 2 + 7 + 1 + 7 + 3 + 4 = 47$$
and the smallest positive number that can be added to 47 to produce a multiple of 10 is 3.
- The bar code found on most supermarket items is UPC type A. Such codes consist of 12 digits. When space does not allow 12 digits, a UPC type E code—which has only 8 digits—is used.
- The US grocery industry formally adopted the UPC system in 1973.
- The mission of the Uniform Code Council, based in Dayton, Ohio, is to take a global leadership role in establishing and promoting multi-industry standards for product identification and related electronic communication.
- Bar codes are used by libraries, airlines, and blood banks.

Resources:

Book:

- *Designing Letter Mail*. Publication 25. Washington, DC: US Postal Service, August 1995.
- Harmon, C. *Lines of Communication, Bar Code and Data Collection Technology for the 90s*. Peterborough, NH: Helmers Publishing, 1994.
- Masunaga, D. “Zips and Strips.” *Student Math Notes*. Reston, VA: National Council of Teachers of Mathematics, January 1994.
- Palmer, R. *The Bar Code Book*. Peterborough, NH: Helmers Publishing, 1995.

Websites:

- www.architext.com
- www.beakman.com/upc/barcode.html
- www.usps.gov/ncsc/lookups/lookup_zip+4.html
- www.uc-council.org/main/ID_Numbers_and_Bar_Codes.html



Figure This!

Math Challenges for Families

HOW MANY fish IN THE POND???



Figure This! To estimate the number of fish in a pond, scientists captured 150 fish, marked them, and then let them go. The next day, they captured 170 fish from the pond. Of these, 20 had been marked the day before. About how many fish are in the pond?

Hint: What fraction of the fish caught on the second day were marked?

Capture-recapture is a statistical method used to estimate the size of a population. Fish and wildlife management experts, demographers, and scientists use this and other techniques to find the number of people or animals in a region.

Answer: Based on the samples caught by the scientists, there are about 1275 fish in the pond.

Figure This!

Get Started:

How do you think the fraction of marked fish caught on the second day is related to the fraction of marked fish in the entire pond?

Complete Solution:

To use the capture-recapture method to estimate a population, scientists assume the proportions of the sample of the marked fish to the total fish in the sample is the same as the marked fish to the total fish in the pond.

$$\frac{\text{Marked Fish in Recapture Group}}{\text{Total Fish in Recapture Group}} = \frac{\text{Marked Fish in Pond}}{\text{Total Fish in Pond}}$$

There were 20 marked out of 170 of the sample. There were 150 marked out of the total fish in the pond. Thus, the following relationship is approximately true. Substitute the information given in the Challenge, then solve the equation:

$$\frac{20}{170} = \frac{150}{\text{Total Fish in Pond}}$$
$$20 \cdot (\text{Total Fish in Pond}) = 150 \cdot 170$$
$$\text{Total Fish in Pond} = \frac{25,500}{20}$$
$$\text{Total Fish in Pond} = 1275$$

This does not mean that there are exactly 1275 fish in the pond, but it is likely to be a reasonable estimate.

Try This:

- For the following experiment, you can use different flavors of goldfish crackers, or you could use a bag of dry beans that you can mark with a nontoxic marker.

Take two boxes of different flavored goldfish crackers and a large paper bag.

- Pour one flavor of goldfish crackers into the paper bag.
- Remove a large handful of crackers from the bag and count them. Replace the crackers taken out with the same number of a different flavored cracker. Mix well.
- Remove another large handful of crackers from the bag. Record the total number of crackers in this handful and the number of these that are of the second flavor.
- Use the method in the challenge to estimate the total number of crackers in the bag.
- Replace the handful of crackers in step three. Mix well. Repeat steps three and four to get 5 estimates in all.
- Find the average of the 5 estimates.
- How does the average compare to the total number of crackers in the bag?

Additional Challenges:

(Answers located in back of booklet)

- A biologist set several large traps in a meadow. On the first day, she caught 50 mice. She marked each mouse with non-toxic paint, then released it. The next day she caught 170 mice. Of these, 20 had been marked. About how many mice do you think live in the meadow?
- Two scientists are trying to estimate the trout population in a mile-long stretch of river. On their first day they catch 150 trout. Each fish is tagged, then released. On their next visit, they catch 340 trout. About how many were marked if their estimate of the trout population in the river is about 2550 trout?
- Four students are using the capture-recapture method to estimate the number of fish in an experimental pond. Their professor marked 150 fish from the pond, then asked each student to net a sample of fish. Their results are shown in the table below.

Student	Captured	Marked
A	180	30
B	160	20
C	205	38
D	110	16

Which student will report the largest estimate for the population? Which will report the smallest?

Things to Think About:

- When using the capture-recapture method to estimate population size, scientists make several assumptions. For example, they assume that each member of the population has the same chance of being captured and that the marks or tags won't fall off or become unrecognizable. What other assumptions do you think would be reasonable when using this method?
- What sort of problems might occur when using the capture-recapture method?
- How do you think the size of the samples in a capture-recapture experiment affects the accuracy of the estimate?
- What other populations might be estimated using a capture-recapture model?
- How would you mark a fish for a capture-recapture experiment?

Did You Know That?

- The foundations for the capture-recapture method were established in 1812 by French mathematician Pierre Laplace, considered by some to be the father of probability.
- In 1896, Carl George Johannes Petersen, a Danish fisheries scientist, published the results of a study in which he estimated the size of a fish population using the capture-recapture method and brass tags he invented.

- Biologists use a number of different ways to mark individuals in a capture-recapture study. Birds are typically fitted with leg bands. Turtles receive non-corroding metal or plastic tags. Large mammals, such as elk or bear, often are fitted with ear tags or radio collars. Migrating salmon are sometimes equipped with tiny microchips inserted under the skin.
- The US Census Bureau has considered using the principles of the capture-recapture method to help count the homeless population in large urban areas.
- To estimate deer populations, wildlife biologists take a picture from an airplane and then count the number of deer in the photograph.
- In Iowa, the rooster pheasant population is estimated by rural mail carriers, who count the cackles that roosters make early in the day.

Resources:

- Your state department of natural resources.

Book:

- Landwehr, J., J. Swift, and A. Watkins. *Exploring Surveys and Information from Samples*. Palo Alto, CA: Dale Seymour Publications, 1987.

Website:

- www2.pitt.edu/~yuc2/cr/history.htm

Notes:

Tangent



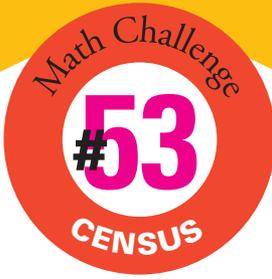


Figure This!

Math Challenges for Families

How many PEOPLE live in the United States?

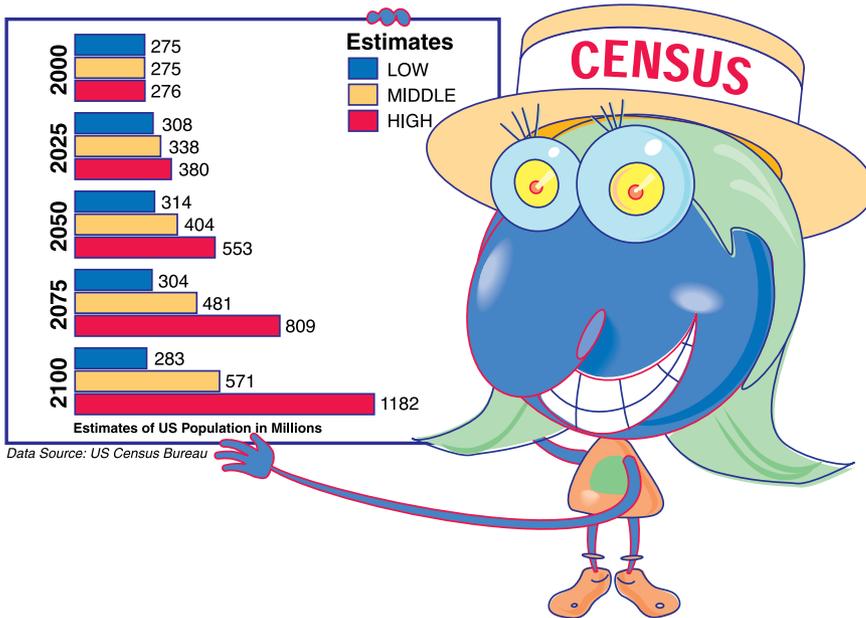


Figure This! The US Census Bureau estimates that the US population may double in the next 100 years. Other experts have suggested that the population may increase by 400% over the same period. Does Polygon's graph support these statements?

Hint: How many people would there be in 2100 if the population in 2000 doubled?

Information is often presented graphically. Politicians, business people, city managers, and government officials read and interpret graphs to predict population growth, plan for schools, and allocate funding for transportation and health programs.

Based on the low estimate, the US population will not double by 2100. Based on the middle and high estimates, it will more than double. None of the estimates shows a 400% increase.

Answer:

Figure This!

Get Started:

What was the estimated US population in 2000? What does it mean to double a value? What is a 100% increase? A 200% increase?

Complete Solution:

The estimated US population in 2000 was about 275 million people. Note that a 100% increase is the same as doubling which gives 550 million people. An increase of 400% would mean the 2000 population plus 4 times the population.

$275,000,000 + (4 \cdot 275,000,000)$ or $5 \cdot 275,000,000$, which is 1,375,000,000 people.

Comparing the doubled 2000 population and projected 2100 population, the low estimate is the only one that is not at least doubled. None of the estimates in the graph supports a 400% increase.

	Low Estimate	Middle Estimate	High Estimate
2000	275,000,000	275,000,000	276,000,000
2100	283,000,000	571,000,000	1,182,000,000
Double the 2000 population	550,000,000	550,000,000	552,000,000
400% increase of the 2000 population	1,375,000,000	1,375,000,000	1,380,000,000

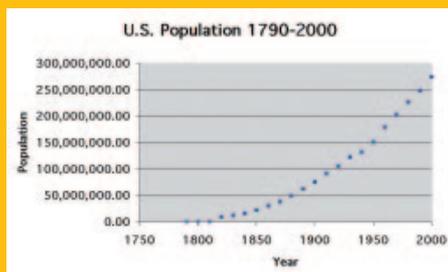
Try This:

- Look in an atlas, almanac, or website for information about the population of your state. Has the population doubled in the past 100 years?
- Look in a newspaper or magazine to find a percentage greater than 100%. How is this value used and what does it mean?
- Find out what questions your family had to answer for the 2000 Census.

Additional Challenges:

(Answers located in back of booklet)

1. The US population has doubled several times over the past 200 years. What does the graph below tell you about the amount of time it has taken for the population to double?



2. Can a 500% increase be found simply by multiplying by 5?
3. The US Census Bureau estimates that about 1 of every 30 people in Prince Georges County, Maryland was uncouncted in the 1990 census. If the reported population was 665,071, estimate the actual population.

Things to Think About:

- Why is the spread between the low, middle, and high estimates so small in the year 2000 and so large in the year 2100?
- What factors will affect US population growth in the next 100 years? What factors will affect global population growth? Will these factors be the same?
- The low estimate on the graph predicts a decrease in the US population between 2050 and 2100. What might cause such a decrease?

Did You Know That?

- The United States conducted its first census in 1790. Marshals of US judicial districts visited every home, asking for the name of the head of the family and the number of persons in each household. The reported population was 3.9 million.
- In the late 1990s, the fastest-growing states were concentrated in the South and West.
- By order of the US Constitution, a census must be conducted every 10 years. This has traditionally been done in the years ending in 0.
- Every two years, the Census Bureau conducts a sampling focused on a particular area, such as transportation or housing, to identify trends.
- The results of the US Census determine each state's number of members in the House of Representatives.

Resources:

Books:

- *The World Almanac 2000*. Mahwah, NJ: World Almanac Books, 1999.
- Maloney, B., and D. Stevens. "Census 2000: Answers Behind the Questions," *The Washington Post*, April 26, 2000.
- Nasser, H. "Census Begins Controversial Use of Statistical Procedure," *USA Today*, May 19, 2000.
- Nasser, H. "Census Predicts Ethnic Face of Nation in 100 Years," *USA Today*, January 13, 2000.

Websites:

- fisher.lib.virginia.edu/census/background
- www.gov.state.md.us/gov/press/2000/may/html/census.html
- www.census.gov



Figure This!

Math Challenges for Families

What's a FAIR SHARE??

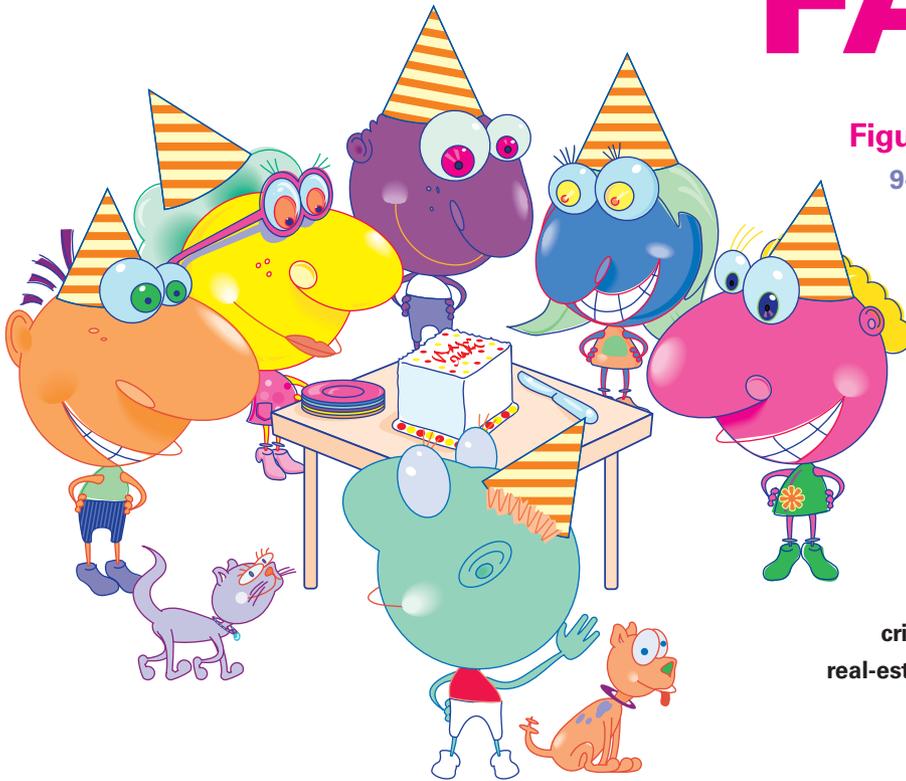
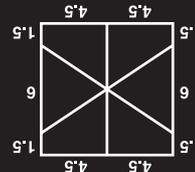
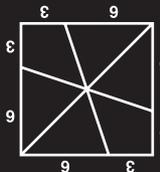


Figure This! Ratio and five friends want to share a 9-inch square chocolate cake with marshmallow icing. How can Ratio cut the cake so that each person receives an equal share of both cake and icing?

Hint: If six people will share the whole cake, then three people will share half the cake. Don't forget the icing on the sides!

Properties of three-dimensional shapes can be understood by thinking about their two-dimensional faces. Dividing geometric shapes according to certain criteria is critical to the work of city planners, graphic designers, and real-estate developers.



Two possible solutions are shown here.
Answer:

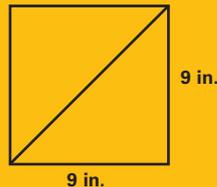
Figure This!

Get Started:

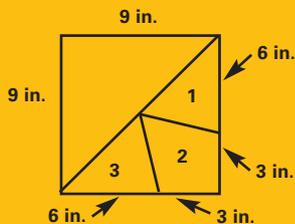
Draw the top of the cake. Divide the square in half, then try to divide one half into three equal shares. How can you make sure that the amount of icing on each piece is the same? How can you make sure that the volume of each piece is the same?

Complete Solution:

- There are many ways to approach this problem. Assuming that all pieces of the cake have the same height, the size (or volume) of each piece depends on the area of its top. Since icing is on both the top and the sides, however, each piece must also have an equal share of the perimeter of the square. If six people share the whole cake, then three people will share half the cake. One way to divide the cake in half is shown here.



Each half of the cake is 18 inches on its two outer edges. So each person should receive $18/3$, or 6 inches, of the cake's outer edge. One way to make this division is shown below.



To check the areas of the three pieces, use the formula for the area of a triangle:

$$\text{Area} = (1/2) \text{ base} \cdot \text{height}$$

The height of triangles 1 and 3 in the diagram is half the width of the cake, or 4.5 inches. Since the length of each base is 6 inches, the area of each of these two triangles is:

$$A = (1/2) \cdot 6 \cdot 4.5 = 13.5 \text{ sq. in.}$$

The area of shape 2 is half the area of the entire cake, minus the area of triangles 1 and 3:

$$\text{Area of half} = (1/2) \cdot 9 \cdot 9 = 81/2 = 40.5, \text{ or } 40.5 \text{ sq. in.}$$

$$\text{Area}_{\text{Triangle 1}} = \text{Area}_{\text{Triangle 3}} = (1/2) \cdot 6 \cdot 4.5 = 13.5, \text{ or } 13.5 \text{ sq.in.}$$

$$\text{Area}_{\text{Region 2}} \text{ is Area of half of the cake} - \text{Area}_{\text{Triangle 1}} - \text{Area}_{\text{Triangle 3}}$$

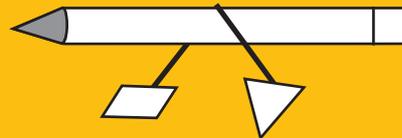
$$\text{Area}_{\text{Region 2}} = (1/2) \cdot 9 \cdot 9 - 13.5 - 13.5 = 13.5, \text{ or } 13.5 \text{ sq.in.}$$

Therefore, these three pieces represent equal shares. The other half of the cake can be divided similarly.

- There are other ways to cut the cake into six equal shares. For example, you could start by dividing the cake into two rectangles of the same size and shape.

Try This:

- Cut a rectangular piece of heavy cardboard into two pieces with different shapes that you think have the same area. One way to determine if the areas are roughly the same is to compare the weights of the two pieces. Assuming that the thickness of the cardboard is constant, if the weights are the same, then the areas must be the same. You can compare weights by taping each piece to one end of a string, marking the midpoint of the string, then placing a pencil under the midpoint, as shown in the following diagram. If the string does not move, the pieces balance.



Additional Challenges:

(Answers located in back of booklet)

- How would you share a 9-inch square cake among 5 people so that each person receives an equal share of both cake and icing?
- In how many different ways can you divide a rectangle into two pieces of the same size and shape?
- Using one cut, how can you divide the triangle below into two pieces with equal areas?



Things to Think About:

- Would the way in which you solved the Challenge have changed if the cake had been rectangular but not square?
- Would you prefer a piece of cake with a regular shape or an irregular shape?
- How do you cut an apple in half?
- Which half of the sandwich below would you choose? Why?



- What factors should be considered when dividing waterfront property into lots?

Figure This!

- Would it have been easier to share a round cake in the Challenge?
- In what other situations is it important to divide something into equal parts?

Did You Know That?

- According to the *1999 Guinness Book of Records*, the world's largest cake was prepared by EarthGrains Bakery to celebrate the centennial of Fort Payne, Alabama in 1989. Shaped like the state, it weighed a little more than 58 tons and used 16,209 pounds of frosting.
- If a cake is square and of the same height everywhere, then you can divide it into five shares of equal area using the idea behind the following drawing.



The square is separated into four triangles and rearranged as shown. If the sum of the bottoms of the triangles on the right is cut into five equal parts, then by connecting the tops of the triangles to the cuts on the bottom, there are 5 equal servings.

- The Italian mathematician Francesco Bonaventura Cavalieri (1598–1647) developed a theorem that states if two three-dimensional solids have the same height and all cross-sections of the solid parallel to the base and at equal distances from the bases have equal areas, then the solids have the same volume.

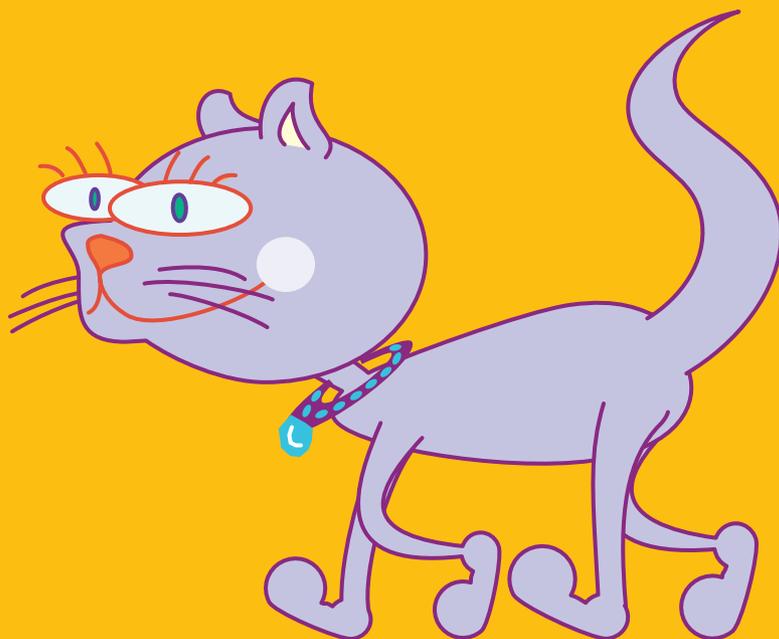
Resources:

Books:

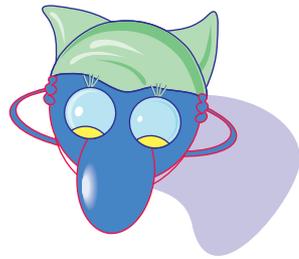
- *The Guinness Book of Records 1999*. New York, NY: Bantam Books, 1999.
- James, R. *Mathematics Dictionary*. New York, NY: Chapman & Hall, 1992.

Notes:

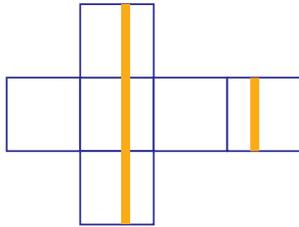
Axis



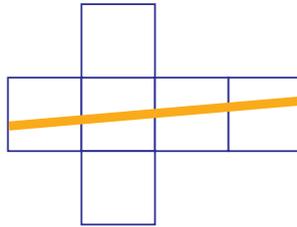
How do you print a ribbon on a box?



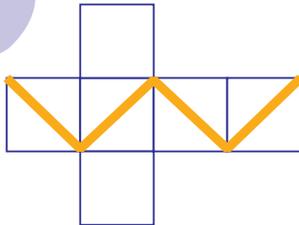
1.



2.



3.



4.

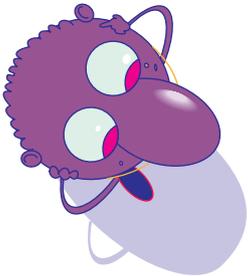
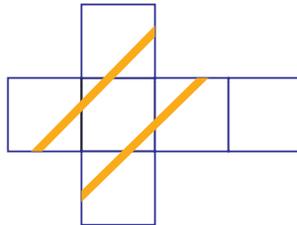


Figure This! Which of the patterns can be folded into a box with an orange ribbon printed continuously all the way around it?

Hint: Think about folding each pattern to make a box. Do the ends of the orange ribbon meet?

Two-dimensional patterns are often used to design three-dimensional objects. Visualizing three-dimensional objects from two-dimensional patterns is important for architects, artists, designers, model manufacturers, and doctors.



Figure This!

Math Challenges for Families

How much **water** do you **waste?**



Figure This! A faucet drips every 2 seconds. In 1 week, how much water goes to waste — enough to fill a glass, a sink, or a tub?

Hint: One teaspoon holds about 20 drops. There are 96 teaspoons in a pint, and 8 pints in a gallon.

Mathematics can be used to describe, estimate, and measure environmental factors and to communicate this information to the public. This information is used by those interested in the environment, consumers, and governmental agencies.

Answer: In 1 week, this leaky faucet could more than fill a kitchen sink, but not a standard tub.

Figure This!

Get Started:

How much water drips in an hour? How long would it take for the drips to fill a 1-gallon container?

Complete Solution:

- One drop every 2 seconds is equivalent to 30 drops per minute, or about 1 1/2 teaspoons per minute. This corresponds to 90 teaspoons per hour (about 15 oz. or a bit less than 2 cups.). As shown in the following equation, the faucet would leak about 20 gallons per week.

$$\frac{1 \text{ drop}}{2 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{7 \text{ day}}{1 \text{ wk}} \cdot \frac{1 \text{ tsp}}{20 \text{ drops}} \cdot \frac{1 \text{ pt}}{96 \text{ tsp}} \cdot \frac{1 \text{ gal}}{8 \text{ pts}} \approx 20 \text{ gal/wk}$$

- Another way to approach this problem is to create a table like the one shown below:

1 drop per 2 seconds	60 seconds per minute	30 drops per minute
30 drops per minute	60 minutes per hour	1800 drops per hour
1800 drops per hour	24 hours per day	43,200 drops per day
43,200 drops per day	7 days per week	302,400 drops per week
302,400 drops per week	20 drops per teaspoon	15,120 teaspoons per week
15,120 teaspoons per week	96 teaspoons per pint	157.5 pints per week
157.5 pints per week	8 pints per gallon	About 20 gallons per week

Try This:

- The following table shows the amount of water used by a typical American for some basic tasks. Use this information to estimate how much water you use in one day.

USE	AVERAGE AMOUNT (in liters)	AVERAGE AMOUNT (in gallons)
taking a bath	110	29
taking a shower	75	20
flushing a toilet	22	6
washing hands, face	7	2
getting a drink	1	0.3
brushing teeth	1	0.3
doing dishes (one meal)	30	8
cooking (one meal)	18	5

Table from Billstein, R., et al. *A Problem Solving Approach to Mathematics for Elementary Teachers*. Menlo Park: Benjamin Cummings Publishing Co. 1981.

Based on the information given in the table above, how much water does your family use in one day?

- Call your local water company or the city government to find the cost of water per gallon in your community.

Additional Challenges:

(Answers located in back of booklet)

1. If a faucet drips once every 2 seconds, how much water would it waste in 1 year? Could you swim in it?
2. There are about 101,018,000 households in the United States. If 1 of every 5 households has a leaking faucet that drips once every 2 seconds, how much water is wasted by leaking faucets each year?
3. Which is more expensive, gas at \$1.56 per gallon or at \$0.42 per liter?

Things to Think About:

- What could you do to conserve water at home?
- Where does your community get its water? Is this a renewable source?
- What happens to the water that goes down your drain?
- How much water does it take to wash a car?
- According to the US Environmental Protection Agency, 21% of watersheds in the nation had serious pollution problems in 1997.

Did You Know That?

- Approximately 74% of the earth's surface is covered with water. However, only 3% of this is fresh water. Of the fresh water, only about 1% is contained in lakes, rivers, and wetlands.
- If one in every five households has one dripping faucet, for one year, this is equivalent to nearly four hours of flow of Niagara Falls.
- A cloudburst has about 113 drops per square foot per second; a moderate rain has about 46; and a drizzle has about 14.
- The city of Los Angeles offers its residents free low-flush toilets to help them conserve water and save money. The toilets use only 1.6 gallons per flush.
- In some rural areas, people obtain their water from private wells. In others, water is trucked to a central cistern for distribution.

Resources:

Books:

- *The World Almanac 2000*. Mahweh, NJ: World Almanac Books, 1999.
- *The Old Farmer's Almanac 2000*. Dublin, NH: Yankee Publishing, 1999.

Websites:

- www.theplumber.com/h_index.html
- www.stemnet.nf.ca/CITE/water.htm#General
- freespace.virgin.net/john.cletheroe/usa_can/ny/niagara.htm
- www.monolake.org/socalwater/ultralow.htm



Figure This!

Math Challenges for Families

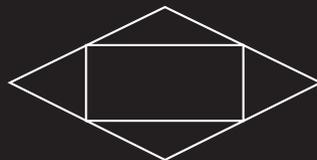
How can you **FOLD** a sheet of paper to make an envelope?



Figure This! Suppose you wanted to make a business-size envelope from a sheet of paper, with as little waste as possible. What shape would you use for the pattern?

Hint: Unfold a business envelope along its seams, then examine the pattern.

Manufacturers design containers for efficiency. When certain shapes fit together without overlaps or gaps, they form a tessellation, used to minimize waste in making containers. Engineers use tessellations to design heat shields for space shuttles.



One common pattern for an envelope is shown. Other shapes also are used.

Answer:

Figure This!

Get Started:

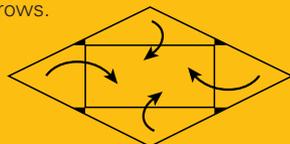
Unfold an envelope at the seams. If you wanted to make many envelopes from this pattern out of a large piece of paper, would there be much waste? Would a different shape reduce the waste?

Complete Solution:

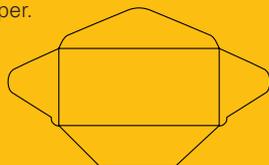
To cover a sheet of paper with many copies of the same shape so that there are no holes or overlaps, the copies have to fit together. A rhombus, with four sides of equal length, as pictured below in bold will do this.



To create tabs for gluing an envelope together, many manufacturers cut notches in the edges of each pattern, as shown in the following diagram. This is the only waste paper in the design. The pattern is then folded in the directions indicated by the arrows.



Other patterns also are used, such as the one shown below, but these may waste slightly more paper.



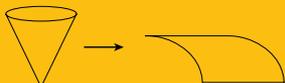
Try This:

- Find several envelopes of various sizes and unfold them at the seams. Can the resulting patterns be cut from a large sheet of paper with little waste?
- Design a pattern that folds into a square envelope.
- Take apart other types of packages or containers, such as paper bags, boxes, or fast food containers. Consider how the design of each minimizes waste in materials, provides strength, and allows for ease of production.

Additional Challenges:

(Answers located in back of booklet)

1. A Baskin-Robbins™ ice cream cone paper holder is unglued and flattened. Will the cone wrapper minimize waste of the material used to make it?



2. Can any four-sided polygon be used to cover a flat surface without gaps or overlaps?

3. What shape is the pattern for the cardboard roll inside a roll of toilet tissue?

Things to Think About:

- Which snack crackers come in shapes that will cover a flat surface with no gaps or overlaps?
- What cookie cutter shapes allow you to make cookies with little wasted dough?
- What other shapes tessellate the plane?

Did You Know That?

- An exhibit at the US Postal Museum chronicles the development of machines that make envelopes.
- Michigan State University offers both bachelors and masters-degree programs in package design.
- The shape of the pattern for a Pillsbury crescent roll will tessellate a plane.
- Paving stones and floor tiles are designed to cover a flat surface with no gaps or overlaps.
- The topic of tiling is an important area of investigation for geometers. There are still unanswered questions in this field, such as finding all the different shapes that can be used to tessellate a plane.
- The Alhambra, a fourteenth century palace in Granada, Spain, contains many decorative majolica tilings and stucco designs that are tessellations. Dutch artist M.C. Escher's designs were influenced by the Alhambra tilings.

Resources:

Books:

- Grünbaum, B., and G. Shephard. *Tilings and Patterns*. New York: W. H. Freeman and Co., 1987.
- Minnesota Educational Computing Corporation. *Tesselmania!* Minneapolis, MN: MECC, 1994.
- Schattschneider, D. *Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M. C. Escher*. New York: W. H. Freeman and Co., 1990.
- Laycock, M. *Bucky for Beginner: Synergetic Geometry*. Hayward, CA: Activity Resources Co., 1984.

Website:

- web1.si.edu/postal/exhibits/cards4.html
- forum.swathmore.edu/sum95/suzanne/tess.intro.html
- web.inter.nl.net/hcc/Hans.Kuiper
- www.camosun.bc.ca/~jbritton/jbsymteslk.htm



Figure This!

Math Challenges for Families

my, my,
little fish —
how you've

GROWN!

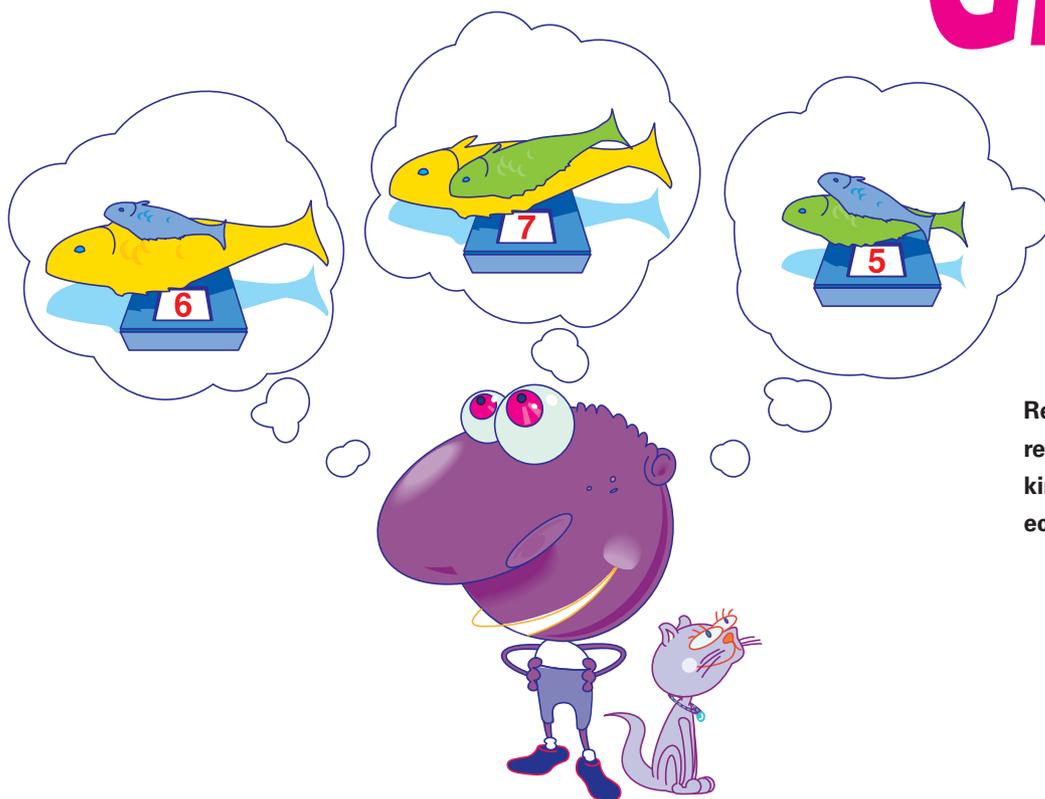


Figure This! How much does each fish weigh?

Hint: How much do the fish weigh all together?

Reasoning about quantities and how they are related is essential in the study of algebra. This kind of reasoning is used by engineers, scientists, economists, statisticians, and psychologists.

Answer: The little fish weighs 2 pounds, the medium-sized fish weighs 3 pounds, and the big fish weighs 4 pounds.

Figure This!

Get Started:

What happens if you combine the weights on each scale?

Complete Solution:

- There are many ways to solve this problem. One is to add the weights of the fish on all three scales:

$$2 \text{ big fish} + 2 \text{ medium fish} + 2 \text{ little fish} = 18 \text{ pounds.}$$

This means that:

$$1 \text{ big fish} + 1 \text{ medium fish} + 1 \text{ little fish} = 9 \text{ pounds}$$

But,

$$1 \text{ big fish} + 1 \text{ medium fish} = 7 \text{ pounds}$$

Subtract and find,

$$\text{the little fish weighs } 2 \text{ pounds.}$$

If the little fish weighs 2 pounds, and

$$1 \text{ little fish and } 1 \text{ medium fish} = 5 \text{ pounds}$$

then, 1 medium fish weighs 3 pounds.

$$\text{Also, } 1 \text{ little fish and } 1 \text{ big fish} = 6 \text{ pounds}$$

So, 1 big fish weighs 4 pounds.

- If you put the fish with weights 6 and 7 together on the scales, then

$$1 \text{ little fish} + 1 \text{ medium fish} + 2 \text{ big fish} = 13 \text{ pounds}$$

But,

$$1 \text{ little fish} + 1 \text{ medium fish} = 5 \text{ pounds.}$$

Subtract and find,

$$2 \text{ big fish} = 8 \text{ pounds}$$

So, 1 big fish must weigh 4 pounds.

Consider the weights in the two balloons showing 6 and 7. The weights differ by one pound and the big fish is on both scales, so the medium fish weighs 1 pound more than the little one. The third scale shows that the combined weights of the little and medium fish is 5 pounds. Since the medium fish weighs 1 pound more than the little one, the little one must weigh 2 pounds and the medium one 3 pounds.

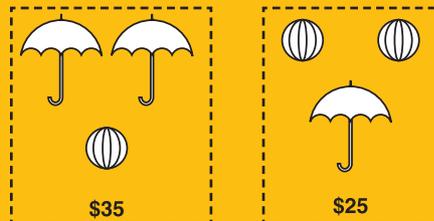
Try This:

- Look in your cupboard. What combinations of cereal, milk, bread, and peanut butter could you eat to get the minimum daily requirements of vitamins and minerals?
- Check the menu at your favorite fast food restaurant. If you have \$10, what can you buy to use as much of your \$10 as possible?

Additional Challenges:

(Answers located in back of booklet)

1. How much does the umbrella cost? How much is a beach ball?



2. In the equations shown below, the oval, the triangle, and the rectangle each represent a value. How much is each one worth?

$$\begin{array}{r} \text{Oval} + \text{Triangle} = 17 \\ \text{Oval} + \text{Triangle} + \text{Rectangle} = 26 \\ \text{Rectangle} - \text{Oval} = 4 \end{array}$$

3. Solve:

$$\text{The Number of Apples} + \text{The Number of Bananas} = 170$$

$$\text{The Number of Apples} - \text{The Number of Bananas} = 90$$

Things to Think About:

- If three grapefruit cost \$1.00, why does one cost \$0.34?
- How can pictures help you solve math problems?
- Is it easier to use words or symbols when describing mathematical relationships?
- Why do people use letters, such as FBI or NASA, to identify organizations?
- In math and science, certain letters and symbols are commonly used to refer to specific quantities, such as d for distance. What are some other letters and symbols that are used?

Did You Know That?

- Another way to write the problem in the Challenge, where x is the weight of the little fish, y is the weight of the medium-sized fish, and z is the weight of the big fish, is:

$$x + y = 5$$

$$y + z = 7$$

$$x + z = 6$$

- As a young woman, Mary Fairfax Somerville (1780–1872) discovered algebra while reading a ladies' fashion magazine. In its pages, she saw a question with "strange looking lines mixed with letters, chiefly X's and Y's" (Perl, 1978). She continued to study mathematics until she was 92.

- The National Council of Teachers of Mathematics (NCTM) recommends that algebraic thinking start in pre-kindergarten using concrete, pictorial, verbal, and symbolic representations.
- Spreadsheet formulas require algebra.
- Some formulas are found in popular reading materials. For example, the formula for Body Mass Index (BMI) is found in *The Old Farmers' Almanac* (2000). (See Challenge 21)

Resources:

Books:

- Greenes, C., and C. Findell. *Groundworks: Algebra Puzzles and Problems, Grades 4-7*. Chicago, IL: Creative Publications, 1998.
- Kindt, M., K. Abels, M. Abels, M. Myers, and M. Pligge. "Comparing Quantities." In *Mathematics in Context*. Chicago, IL: Encyclopaedia Britannica Educational Corp., 1998.
- Kroner, L. *In the Balance: Algebra Logic Puzzles*. Alsip, IL: Creative Publications, 1997.
- Perl, T. *Math Equals: Biographies of Women Mathematicians + Related Activities*. Menlo Park, CA: Addison-Wesley, 1978.
- *The Old Farmer's Almanac 2000*. Dublin, NH: Yankee Publishing Incorporated, 1999.

Notes:

Axis

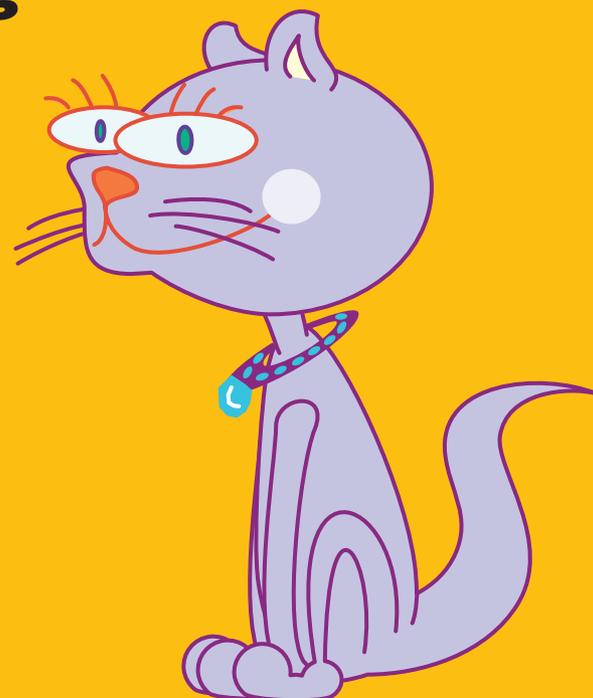




Figure This!

Math Challenges for Families

"How do I cover thee? Let me *count* the ways."



Figure This! Tatami mats help define standard sizes of Japanese rooms. Floors are completely covered by these mats which are about 3 feet by 6 feet. In how many different ways can the mats be arranged to cover a 6 foot by 15 foot floor?

Hint: Sketch some arrangements of tatami mats that could cover the floor.

Arrangements of geometric shapes influence living space, storage space, commercial displays, and the natural environment. The study of shapes is important to architects, pipefitters, construction workers, and biologists.

Answer: There are eight different ways, each using five mats.

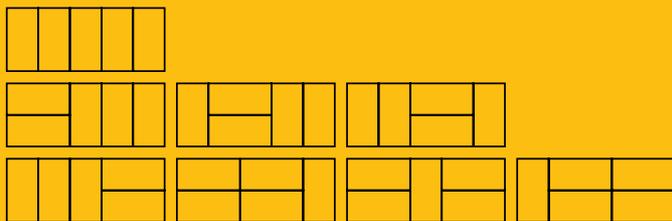
Figure This!

Get Started:

How many mats are needed to cover the floor if they are all lined up in the same way? What are the dimensions of the area covered by two mats? How does that affect their placement?

Complete Solution:

The eight possible arrangements are shown below.



Try This:

- The ratio of the lengths of the sides of a tatami mat is 2 to 1. Dominos have this same ratio. Use dominos to model different patterns for room sizes.
- Look up tatami mats and their use in Japanese culture in an encyclopedia or on the web.
- In an encyclopedia or on the web, look up bees and how their honeycombs are built.

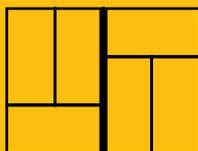
Additional Challenges:

(Answers located in back of booklet)

- a. Build a table as below to see how many ways you can use tatami mats to cover the floor of the rooms with sizes listed.

Room Size	6x3	6x6	6x9	6x12	6x15	6x18	6x21	6x24
No. of Ways								

- Describe the sequence found in part a.
- Japanese homebuilders do not like patterns where four corners come together, believing that is a sign of bad luck. How many of the arrangements that you found in the Challenge have places where four corners meet?
 - When constructing homes from brick, builders try to avoid “fault lines,” seams that run the height or length of a wall. For example, the arrangement of bricks shown below has a vertical fault line.



Find a possible fault-free design using more than one brick. In other words, find a rectangle that contains no fault lines, either vertical or horizontal.

Things to Think About:

- In Japan, the size of a room is often described in terms of tatami mats—for example, a four-mat room or a six-mat room. What are some possible dimensions for a ten-mat room?
- Why do you think the size of tatami mats has helped set the standards for room size in Japan?
- Why do bricklayers try to avoid fault lines?

Did You Know That?

- A tatami mat consists of a thin layer of tightly woven rushes, on top of a coarser mat of straw bound with cords. The upper mat is sewn to the lower one with twine. The mat is firm but not hard.
- In the United States, building dimensions often are dictated by the size of a standard sheet of plywood: 4 feet by 8 feet.
- The sequence in Additional Challenge 1b is a Fibonacci sequence. Fibonacci was the nickname for the Italian mathematician Leonardo of Pisa. In his book *Liber abaci*, published in 1202, Fibonacci described the following problem:
How many pairs of rabbits can be produced from a single pair in a year, if every month each pair bears a new pair which, from the second month on, also becomes productive?

The resulting sequence of the pairs at the end of each month, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ... is known as the Fibonacci sequence.

Resources:

Books:

- Gardner, M. *New Mathematical Diversions*. Washington, DC: Mathematical Association of America, 1995.
- Garland, T. *Fascinating Fibonacci: Mystery and Magic in Numbers*. Palo Alto, CA: Creative Publications, 1987.

Websites:

- www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Fibonacci.html
- www.swifty.com/apase/charlotte/@A4.html



Figure This!

Math Challenges for Families

Bet I can guess your **color**!

MAGIC NUMBERS

Take the number of your birth month.
Add 32.
Add the difference between 12 and
the number of your birth month.
Divide by 2.
Add 3.
The result is your special number.

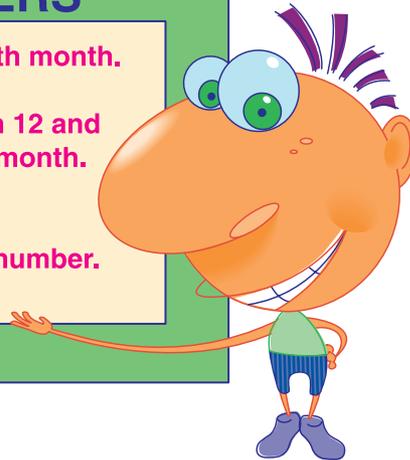
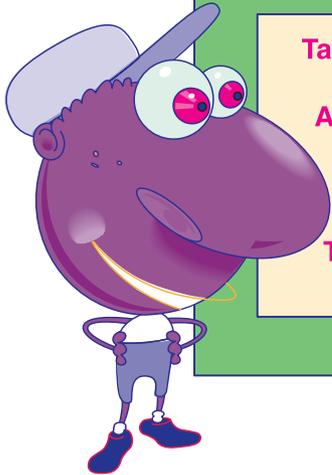


Figure This! Find your special number.

If $a = 1$, $b = 2$, and so on, what letter corresponds to your special number?

Write the name of a color that begins with this letter. Helix bets you chose yellow. Why?

Hint: Find the special numbers for a few of your family members. How do the numbers compare?

In algebra, symbols are used to generalize arithmetic procedures. Many people use algebraic procedures everyday in working with spreadsheets, monitoring dosages of medicine, and applying formulas.

Answer: The special number is 25, which corresponds to the letter Y. Yellow is the most common color that begins with Y.

Figure This!

Get Started:

Let B represent the number of your birth month. What happens as you go through the steps?

Complete Solution:

If B represents the number of the birth month, each step can be written as shown in the table below.

Step	Algebraic Representation
Take the number of your birth month.	B
Add 32.	$B + 32$
Add the difference of 12 and the number of your birth month.	$B + 32 + (12 - B)$
Divide by 2.	$\frac{B + 32 + (12 - B)}{2}$
Add 3.	$\frac{B + 32 + (12 - B)}{2} + 3$

The final step can be simplified as follows:

$$\begin{aligned}\frac{B + 32 + (12 - B)}{2} + 3 &= \frac{(B - B) + (32 + 12)}{2} + 3 \\ &= \frac{0 + 44}{2} + 3 \\ &= 22 + 3 \\ &= 25\end{aligned}$$

Therefore, no matter what number you choose for B , it is added and then subtracted and does not affect the answer. The answer is always 25. The 25th letter of the alphabet is y . The most obvious color beginning with y is yellow.

Try This:

- Create a number trick of your own. Try the trick on a friend; then explain how the trick works.
- Computer spreadsheets, interest-bearing savings accounts, and many medicine dosages involve algebraic formulas. Ask your friends and family members to describe how they use formulas in their daily lives.

Additional Challenges:

(Answers located in back of booklet)

1. Write a three-digit number, then make a six-digit number by writing the three digits again. Divide the six-digit number by 7, then the result by 11, and then that result by 13. The answer is your original three-digit number. Explain how this trick works.
2. Write a three-digit number in which each digit is different. Reverse the digits. Subtract the larger number from the smaller. If you tell me the first digit of the difference, I'll tell you the result. Why does this trick work?

3. Take any number and add 15 to it. Now multiply that sum by 4. Next subtract 8 and divide the difference by 4. Subtract 12 from the quotient. If you tell me the result, I will tell you the number with which you started. How does this trick work?

Things to think about:

- Number tricks typically depend on performing an arithmetic operation, then "undoing" it. What are some pairs of operations that "do" and "undo"?
- Is it easier to follow a procedure explained in words or using algebra?
- Algebra is often referred to as a form of shorthand for arithmetic.
- Why do you think the letter x is used so often in algebra?

Did You Know That?

- Algebra is required for admission to most colleges.
- The word *algebra* comes from the Arabic *al-jabr*, which means "the reduction."
- If n is a whole number, $2n$ is always an even number and $2n + 1$ is always odd.
- The Greek mathematician Diophantus (about 250 AD) is sometimes called the father of algebra.

Resources:

Books:

- Boyer, C. *A History of Mathematics*. Princeton, NJ: Princeton University Press, 1985.
- Greenes, C., and C. Findell. *Groundworks: Algebra Puzzles and Problems*, Grades 6, 7, 8. Chicago, IL: Creative Publications, 1998.
- Perl, T. *Math Equals: Biographies of Women Mathematicians + Related Activities*. Menlo Park, CA: Addison-Wesley, 1978.
- Townsend, C. *World's Most Baffling Puzzles*. New York: Sterling, 1992.



Figure This!

Math Challenges for Families

IS THE STATUE OF LIBERTY'S NOSE TOO long???



Figure This! The arm of the Statue of Liberty is 42 feet long. How long is her nose?

Hint: How long is your nose? How long is your arm?

Similarity and scaling underlie design and model building. Architects, clothing designers, toy makers, and civil engineers all use scaling in their work.

Answer: The actual length of her nose is about 4 feet, 6 inches.

Figure This!

Get Started:

Measure the lengths of your nose and your arm in the same units. About how many times longer than your nose is your arm? What might this tell you about the dimensions of the Statue of Liberty?

Complete Solution:

- There are several ways to think about this problem. One way is to assume that approximately the same relationship holds between your dimensions and those of the Statue of Liberty. If your nose is 3" long and the length of your arm is 24" long, then the length of your nose is $\frac{3}{24}$, or $\frac{1}{8}$, of the length of your arm. If your nose is $\frac{1}{8}$ of the length of your arm, then the nose of the Statue of Liberty might be $\frac{1}{8}$ the length of her arm. In general, the length of her nose would be:

$$\frac{\text{Length of your nose}}{\text{Length of your arm}} = \frac{\text{Length of nose of Statue of Liberty}}{\text{Length of Arm of Statue of Liberty}}$$

- Another approach is to assume that the Statue of Liberty is similar in scale to an average person. In this case, the lengths of corresponding body parts should have the same ratio. (To estimate the lengths for an "average person," take a group of people, measure them, and find the average measures. Answers will vary depending on measurement errors and what was used as an average.) Using this method, the relationship would be:

$$\frac{\text{Length of nose of Statue of Liberty}}{\text{Length of typical nose}} = \frac{\text{Length of arm of Statue of Liberty}}{\text{Length of typical arm}}$$

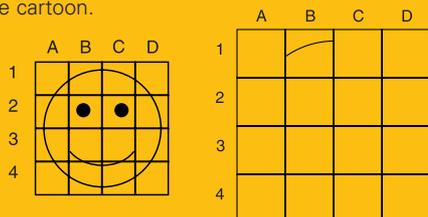
For the example above:

$$\frac{\text{Length of nose of Statue of Liberty}}{3''} = \frac{42'}{24''}$$

Length of nose of Statue of Liberty = $3'' \left(\frac{42'}{24''} \right)$, or $5 \frac{1}{4}'$

Try This:

- Measure the lengths of a friend's nose and arm. Find the ratio of these measurements. How does this ratio compare to yours?
- Measure the length, width, and height of a Hot Wheels™ car and a real car. Does the Hot Wheels™ appear to be a scale model?
- You can use similarity and scale to enlarge a picture. Select a simple cartoon. Draw a square around it, then use horizontal and vertical lines to divide the square into 16 smaller squares, as shown in the first diagram. Now draw a larger square, also divided into 16 smaller squares. In each cell in the larger square, draw the corresponding part of the figure in the cartoon.



For example, in the large grid, cell B1 is the reproduction of cell B1 in the small grid. Try this method to enlarge a cartoon.

Additional Challenges:

(Answers located in back of booklet)

1. An HO gauge model train is scaled $\frac{1}{8}$ inch to a foot. If an HO caboose is 4 inches long, what is the length of the real caboose?
2. The average height of a Lilliputian (from Jonathan Swift's *Gulliver's Travels*) is slightly less than 6 inches. The height of a Lilliputian is about what fraction of your height?
3. When translating a textbook, 15 pages of English resulted in 17 pages of Spanish. Based on this information, about how many pages of English would be necessary for a 187 page Spanish book?

Things to Think About:

- If a T-shirt shrinks, is it a scale model of its original shape before it was washed?
- Are enlargements of pictures always scale models of the original?
- Architects use scale models to help determine weight loads and structural strength.
- Do different-sized boxes of the same breakfast cereal have the same length-to-width ratios?

Did You Know That?

- The movies *Honey I Shrank the Kids* and *Honey We Shrank Ourselves* involve scale models to make people look small.
- Some maps are scale models of actual areas.
- In 2000, the National Building Museum contained a scale model of the White House.
- Sometimes people are posed in oversized furniture to make them look small.
- The relationship:

$$\frac{a}{b} = \frac{c}{d}, \text{ where none of } a, b, c, \text{ or } d \text{ is } 0,$$

also can be written in any of the following ways:

$$\frac{a}{c} = \frac{b}{d} \qquad \frac{d}{b} = \frac{c}{a} \qquad \frac{b}{a} = \frac{d}{c}$$

- The Statue of Liberty was originally called Liberty Enlightening the World. It was completed in Paris in 1884, then shipped to New York.
- If you double all the dimensions of a quart carton, the new container will hold 2 gallons (8 quarts).
- The index finger of the Statue of Liberty is 8 feet long.



Figure This!

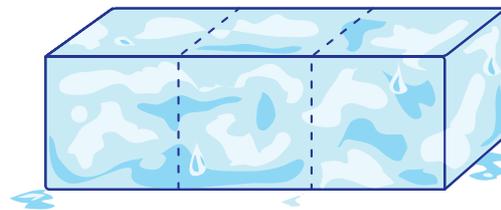
Math Challenges for Families

When should you buy **block** ice or **Crushed** ice?

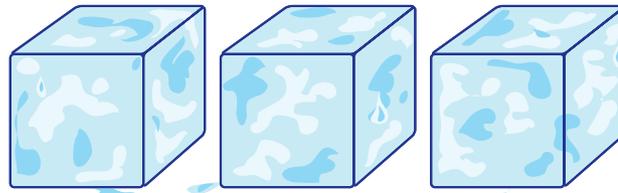
Figure This! Which typically melts faster, a single block of ice or the same block cut into three cubes?

Hint: Compare the exposed areas.

Surface area is a critical factor in heating and cooling. Architects, interior decorators, chemists, and environmental engineers use surface area and volume in their work.



Block of Ice



Cubes of Ice

Answer: The three cubes will melt faster.

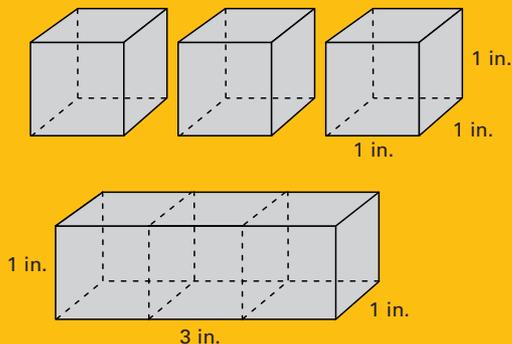
Figure This!

Get Started:

Heat enters an object through its exposed surface. The more surface area exposed, the faster it melts. Ice will melt faster if its exposed surface area is bigger. Compare the surface area of the cubes and the block.

Complete Solution:

Suppose that each of the smaller cubes is 1 in. high. The area of one face of the cube is length times width: $1 \times 1 = 1$ or 1 sq. in.



On each cube there are six faces, each with an area of 1 sq. in. The total surface area for each of the cubes is 6 sq. in. The surface area of the three cubes is 3×6 , or 18 sq. in. The big block of ice has four faces, each with an area of 3×1 for a total of 2 sq. in. Thus, the surface area of the block of ice is $12 + 2$ or 14 sq. in. The three cubes broken apart will melt more quickly since the surface area of 18 sq. in. is exposed while the block of ice has only 14 sq. in. exposed.

Think what happens when you cut the block into cubes. There is more exposed surface area so the cubes melt faster.

Try This:

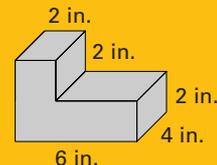
- Ask an adult for two Alka-Seltzer™ tablets and two glasses of water. Carefully cut one of the tablets into four pieces. Which tablet do you think will dissolve faster: the whole one, or the one cut into pieces? Use the two glasses of water to test your prediction.

Additional Challenges:

(Answers located in back of booklet)

1. Imagine that you left two identical bottles of cold water in the sun. One bottle contains 1 liter of water; the other contains 2 liters. After 15 minutes, which will be cooler?

2. Find the surface area of this box.



3. Do all boxes with the same surface area have the same volume?
4. Do all boxes with the same volume have the same surface area?

Things to Think About:

- To minimize melting, the best shape to make ice from water is a sphere.
- Does surface area affect how architects design buildings?
- Why are some pills crushed before they are given to a patient?
- When people fall into cold water, the risk of hypothermia (dangerously lowered body temperature) is greater than that of drowning. If you are alone and wearing a life preserver, try the Heat Escape Lessening Posture (HELP; see fig. 1) Keep your arms close to the sides of your chest, cross your legs and pull them up as far as you can.

If you are in the water with two or more people, try the huddle position shown in Figure 2. Stay close together and keep still to keep colder water out. The huddle can help small children survive longer.



Figure 1



Figure 2

- Does a clove of garlic have more flavor whole or chopped?
- Why is a ring of ice used in a punch bowl while cans of soda are normally put in crushed ice?

Did You Know That?

- The stems of cactus plants are thick and round to minimize surface area and store water. Most species of cactus do not have leaves since leaves would allow too much evaporation in the dry desert air.



Figure This!

Math Challenges for Families

Same birth month? **No Way!**



Figure This! In any group of six people, what is the probability that everyone was born in different months?

Hint: If there were only two people in a room, what is the probability they were born in different months? What if there were three people in the room?

The probability of an event is a measure of the chance that it will occur. Insurance companies use probability to set rates, weather forecasters use probability to predict weather patterns, and doctors use probability to determine treatments for disease.

Figure This!

Get Started:

Since there are 12 months in a year, a person's birth month has 12 different possibilities. For this problem, to make it simpler, assume the probability of being born in any given month is $1/12$. If there are two people in the room, what is the probability that the second person's birthday occurred in a different month from the first person's? If there are three people in a room, what is the probability that all three people have birthdays in different months?

Complete Solution:

With two people, the probability that the second person's birth month does not match the first person's is $11/12$.

A third person could be born in any of the other ten months and not match the first two. The probability that the third person's birth month will not match either of the other two is $10/12$. The probability that all three have different birth months is:

$$\frac{11}{12} \times \frac{10}{12} \approx 0.7638 \text{ or about } 76\%$$

Continuing this process, the probability that six people have different birth months is:

$$\frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12} \approx 0.2228 \text{ or about } 22\%$$

Thus, the probability that at least two of the six people do not have the same birth month is about 22%.

Try This:

- Choose a group of people, such as a classroom of students, a group of friends, or the members of a club. Count the number of people in the group and determine the probability that at least two of them were born in the same month.
- Check several classes, or groups of people, of the same size. In how many of the groups do two people share the same birthday? Use the results to estimate the probability that two people in a group that size share the same birthday.

Additional Challenges:

(Answers located in back of booklet)

1. About how many people does it take to have a 99% probability that at least two have the same birth month?
2. What is the probability that if you know the birthday of one person in a room, a second person has the same birthday?
3. What is the least number of people required to make sure that at least two have matching birthdays?

Things to Think About:

- With 30 strangers in a room, the probability of having at least two matching birthdays is about 71%.
- The probability that two people were born on the *same* day is different from the probability that two people were born on a *given* day.
- How many US presidents have had the same birthday? How many of them died on the same day?
- Think of an event that has a 100% probability of occurring.

Did You Know That?

- The probability that an event will occur and the probability that an event will not occur are called "complementary" probabilities. The sum of complementary probabilities is 1 or 100%.
- A probability of 0 means that the event cannot occur.
- The probability of any event is a percentage ranging from 0% to 100%.

Resources:

Books:

- Jacobs, H. *Mathematics, A Human Endeavor*. San Francisco, CA: W. H. Freeman and Co., 1970.

Websites:

- forum.swarthmore.edu/~isaac/problems/prob1.html



Figure This!
Math Challenges for Families

Who would you guess?

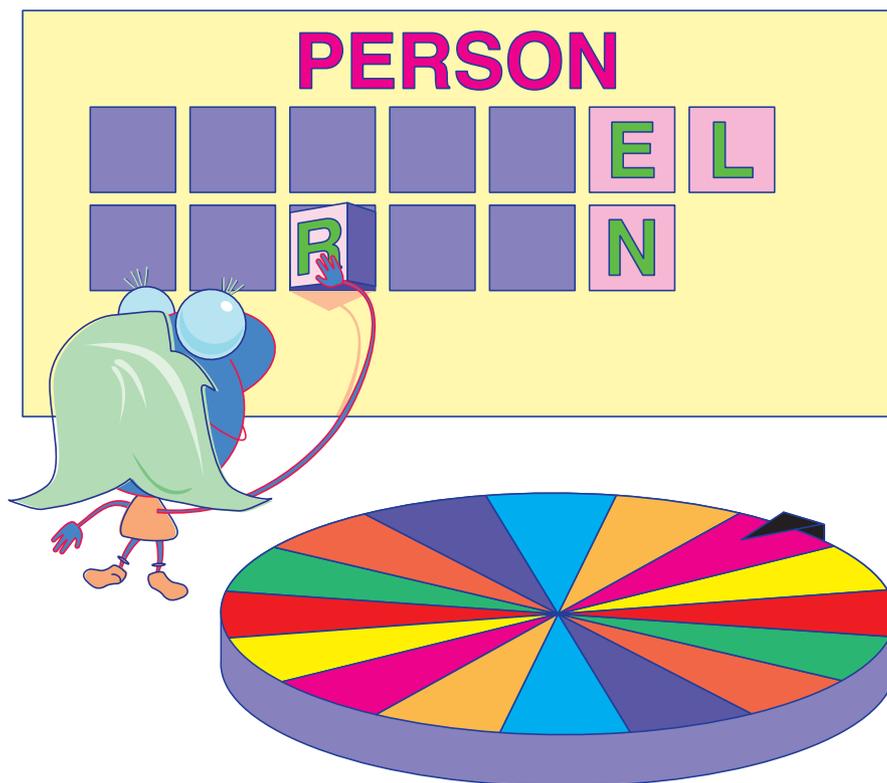


Figure This! In the game show, Wheel of Fortune™, contestants guess the letters of a word or phrase. The letters E, L, N, R, S, and T are given, if they occur in the puzzle. What three consonants and one vowel would you guess next?

Hint: Which letters are used most often?

Structure of language is important in breaking codes and in voice recognition. Spelling based on frequency of letter usage is fundamental in working with codes.

The most commonly used consonants after those listed are D, H and C. The next most common vowels are I and A. You might choose your set of letters from these.

Answer:

Figure This!

Get Started:

Make a guess; think of letter frequencies and combinations.

Complete Solution:

- The frequency of each letter can vary, depending on the sample. In English, E and T occur the most frequently. The table below shows the number of times each letter occurred in one sample of 1000 letters. In this sample, the letters D and H appear most frequently after those already given, and one of these might be a good choice.

A	B	C	D	E	F	G	H	I	J	K	L	M
73	9	30	44	130	28	16	35	74	2	3	35	25
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
78	74	27	3	77	63	93	27	13	16	5	19	1

Source: Sinkov, A. *Elementary Cryptanalysis*. Washington, DC: Mathematical Association of America, 1968.

Choose H, D, C and either I or O. If I is chosen, then the letters you see are:

I	C	H	E	L
	R	D		N

A possible answer is Michael Jordan.

- In a sample of approximately one million words of modern British English, Martin Wynne found that the following letters occur most frequently in the order given: E, T, A, O, I, N, S, H, R, D, L, and U. With this sample, H or D would still be a logical choice.

Try This:

- Choose a sample of 200–300 words from a magazine or newspaper. Work with a partner to count the number of times each letter is used. Make a list of the letters in the English alphabet. Working with a partner, read your words letter by letter and record the number of times each appears.
- Write a short paragraph without using the letters A and N.
- In the game of Scrabble™, players use letter tiles to form words and earn points. The following table shows the number of tiles available in Scrabble™ for each letter in the alphabet. The total number of non-blank tiles is 98; 2 blank tiles can be used as any letter.

A	B	C	D	E	F	G	H	I	J	K	L	M
9	2	2	4	12	2	3	2	9	1	1	4	2
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
6	8	2	1	6	4	6	4	2	2	1	2	1

Is the frequency of each letter in Scrabble™ about the same as its frequency in the sample given in the Complete Solution? What are the biggest differences you notice? How would you construct a “better” set of Scrabble™ tiles? How would you assign point values to each letter that represents letter frequencies?

- Do an Internet search using the following words: “codes and ciphers.”
- Play Hangman with a partner. In this game, you take turns guessing letters in each other’s words and try to avoid having a hangman consisting of a circle and six segments drawn. A part of the hangman is drawn with each mistake.



Additional Challenges:

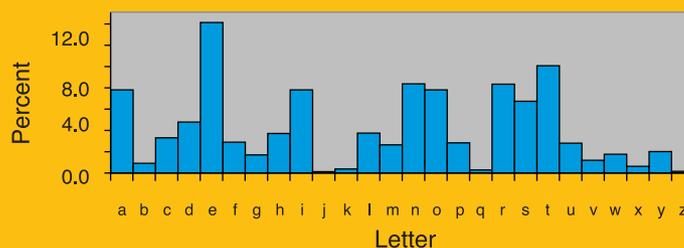
(Answers located in back of booklet)

- In the two examples below, each letter in the alphabet has been replaced by another letter, according to a code. When decoded, each string of letters reveals a famous quotation. What are the quotations?
 - “WKHVH DUH WKH WLPHV WKDW WUB PHQ’V VRXOV” (by Thomas Paine in “The Crisis”, 1776).
 - “VI VKKGZ V YVT FZZKN OCZ YJXOJM VRVT” (by Benjamin Franklin).
- What is unusual about each of the following sentences?
 - “In my opinion, I am saying an awfully abnormal group of words – a bunch of words which is unusually odd, uncommon, and surprising.”
 - “The quick brown fox jumped over the lazy dogs.”

Things to Think About:

- Would you expect the frequency of each letter in other languages to be the same as that in English?
- Would you expect the frequency of each letter in a book by Dr. Seuss to be the same as that in a newspaper?
- The graph shows the frequency of each letter as a percentage. Does the graph help you understand the frequency distribution of letters better than the table in the Complete Solution?

Letter Frequency



Did You Know That?

- Two classic stories in which deciphering a code plays an important role are *The Gold Bug*, by Edgar Allen Poe and *The Adventure of the Dancing Men*, by Arthur Conan Doyle.
- Computer programs available on the Internet can analyze the letter frequencies in any text selection.
- The study of codes and ciphers is called *cryptography*.
- The Enigma Machine, a rotor based cipher machine, was a device for coding and decoding. Originally intended to be used for commercial cryptography, it was adopted by the German Army and Navy prior to World War II for sensitive communications.
- The Navajo Indians' native language was used as a code during World War II, and the code was never broken.
- One of the most popular puzzles of *Games Magazine* is DSZQUPHSBNT! which involves breaking secret codes in which every letter is represented by another letter. Looking at letter frequencies is a productive way to begin to solve the puzzle.

Resources:

Books:

- Doyle, A. C. *The Complete Sherlock Holmes*. New York, NY: Doubleday, 1953.
- Poe, E. A. *Tales of Edgar Allan Poe*. York, PA: Random House. 1944.
- Sinkov, A. *Elementary Cryptanalysis*. Washington, DC: Mathematical Association of America, 1968.

Magazine:

- *Games*. Ambler, PA: Games Publications, Inc.

Websites:

For letter frequencies:

- www.vindustries.com/letterfreq

Enigma machine:

- wordzap.com/enigma

For breaking codes:

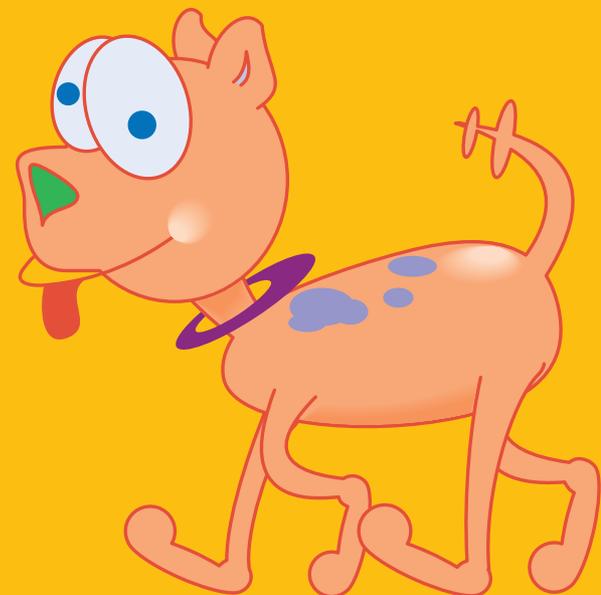
- www.teachingtools.com/GoFigurePages/Password.htm

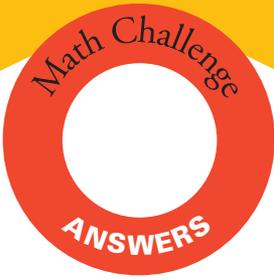
For cryptography:

- www.arachnaut.org/archive/freq.html
- www.kith.org/logos/words/upper2/ZZeus.html

Notes:

Tangent





Looking for answers?

Here are the answers for the
Additional Challenges section
of each Challenge.

Figure This!

Answers to Additional Challenges:

Challenge 49:

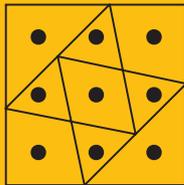
- There would be 10 rows of dots in the pattern 1, 3, 5, 7, 9, 9, 7, 5, 3, 1.
- The next pattern has 10 dots as shown.



- It is a general pattern to give the number of dots in the n th figure. It gives the numbers 1, 3, 6, 10, ...
- One possible solution is shown in the diagram below.



- One possible solution is shown in the diagram below.



Challenge 50:

- $H = 5 + 7d$, for $d \geq 1$
- Large dogs age faster after age 2; small dogs age faster initially.
 - As with a large dog, a small dog will have a human age of 40 after 5 years.
- When the dogs are 10 years old.
- The rate of change with each passing year is 5 not 7. It is constant after year 2, not year 1. The comparable formula is $H = 15 + 5d$, for $d \geq 2$.
- 5.

Challenge 51:

- 10.
- 1,000,000,000.
- Without the start bar, this portion of the bar code appears as follows:



Challenge 52:

- About 425 mice.
- About 20 trout.
- Student B will report the largest estimate; student C will report the smallest.

Challenge 53:

- The number of years between doublings has been decreasing.
- No.
- Approximately 688,004.

Challenge 54:

- Divide the area and perimeter of the cake's square top into five equal portions.
- Infinitely many with one cut, as long as the cut passes through the center of the rectangle.
- One way is to find the midpoint (M) of the base, \overline{BC} . A line from A to M will work, since the height is the same for each new triangle, and the bases are both one-half the original length.

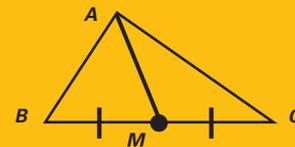
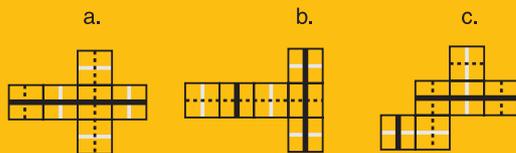


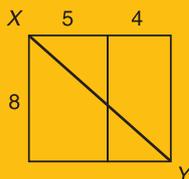
Figure This!

Challenge 55:

- 40 inches.
-



- About 12 inches. (One way to solve this problem is to form the rectangle below.)



- One possible pattern is shown here.



Challenge 56:

- At 20 gallons per week, about 1040 gallons. A small backyard pool contains about 8000 gallons. You could not swim in it.
- About 21,011,744,000 gallons.
- The price per liter is more expensive.

Challenge 57:

- It tessellates the plane so that there is no waste except at the end of paper rolls.
- Yes.
- A parallelogram.

Challenge 58:

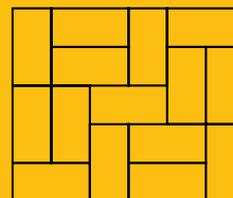
- The umbrella costs \$15; the beach ball costs \$5.
- The oval is worth 5, the triangle 12, and the rectangle 9.
- The number of apples is 130, and the number of bananas is 40.

Challenge 59:

- A completed table is shown below.

Room Size	6x3	6x6	6x9	6x12	6x15	6x18	6x21	6x24
No. of Ways	1	2	3	5	8	13	21	34

- Each number after the first two is the sum of the previous two numbers.
- 2.
- For example: The dimensions of the smallest possible rectangle are 5 by 6, where 1 unit represents the width of a rectangle. One arrangement is shown below.



Challenge 60:

- Because $7 \cdot 11 \cdot 13 = 1001$, dividing by these three numbers is the same as dividing by 1001. Any six-digit number in the form $abcabc$ divided by 1001 is abc . Another way of looking at this is that writing a three-digit number next to itself is like multiplying the original by 1001.
- If the ones digit is 9 in a two-digit difference, the result is 99. Otherwise, the sum of the first and third digits is 9; the middle digit is always 9.
- The net result of the steps is that you have added 1 to the original number.

