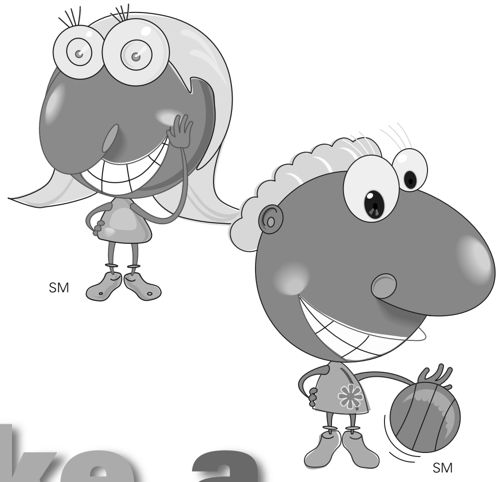
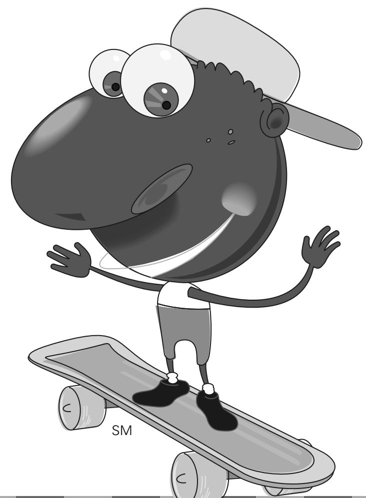
Figure This! Math Challenges for Families



Take a Challenge!

Set III: Challenges 33 - 48

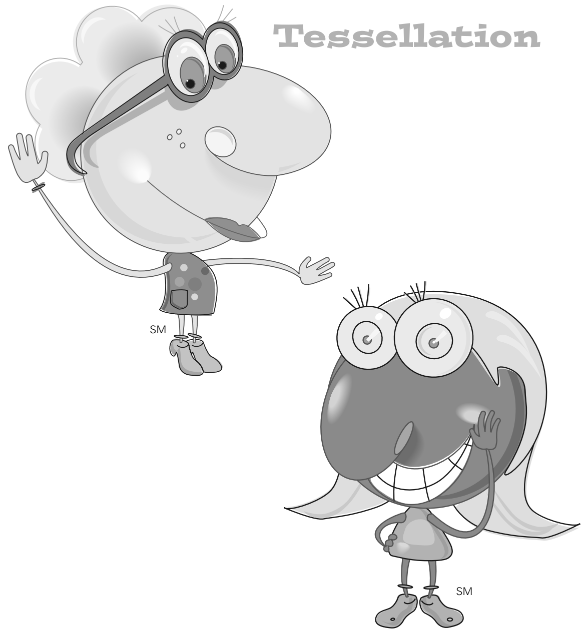


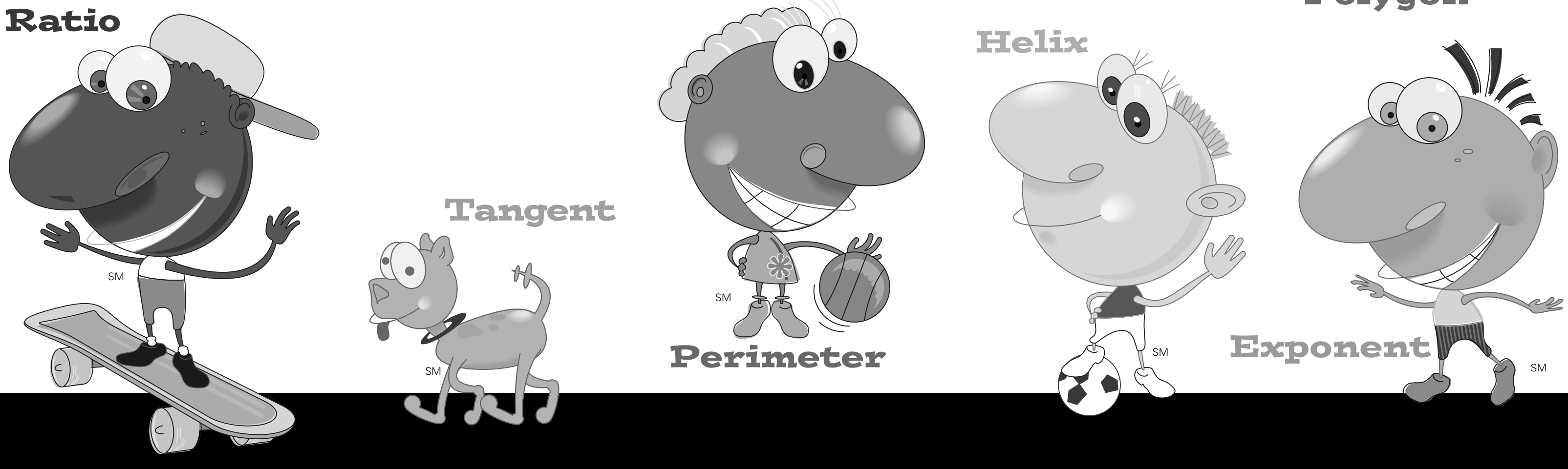
FigureThis!
Math Challenges for Families


Axis

Thank you for your interest in the **FigureThis! Math Challenges for Families.** Enclosed please find Challenges 33 – 48. For information about other challenges, go to www.figurethis.org.

The **Figure This!** Challenges are family-friendly mathematics that demonstrate what middle-school students should be learning and emphasize the importance of high-quality math education for all students. This campaign was developed by the National Action Council for Minorities in Engineering, the National Council of Teachers of Mathematics, and Widmeyer Communications, through a grant from The National Science Foundation and the US Department of Education.


Tessellation
Polygon


Ratio
Tangent
Perimeter
Helix
Exponent

We encourage you to visit our website at **www.figurethis.org** where you can find these and other challenges, along with additional information, math resources, and tips for parents.

Figure This!

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Math Challenge #33 ACCESS RAMP

Figure This! Math Challenges for Families

How STEEP can a ramp be?

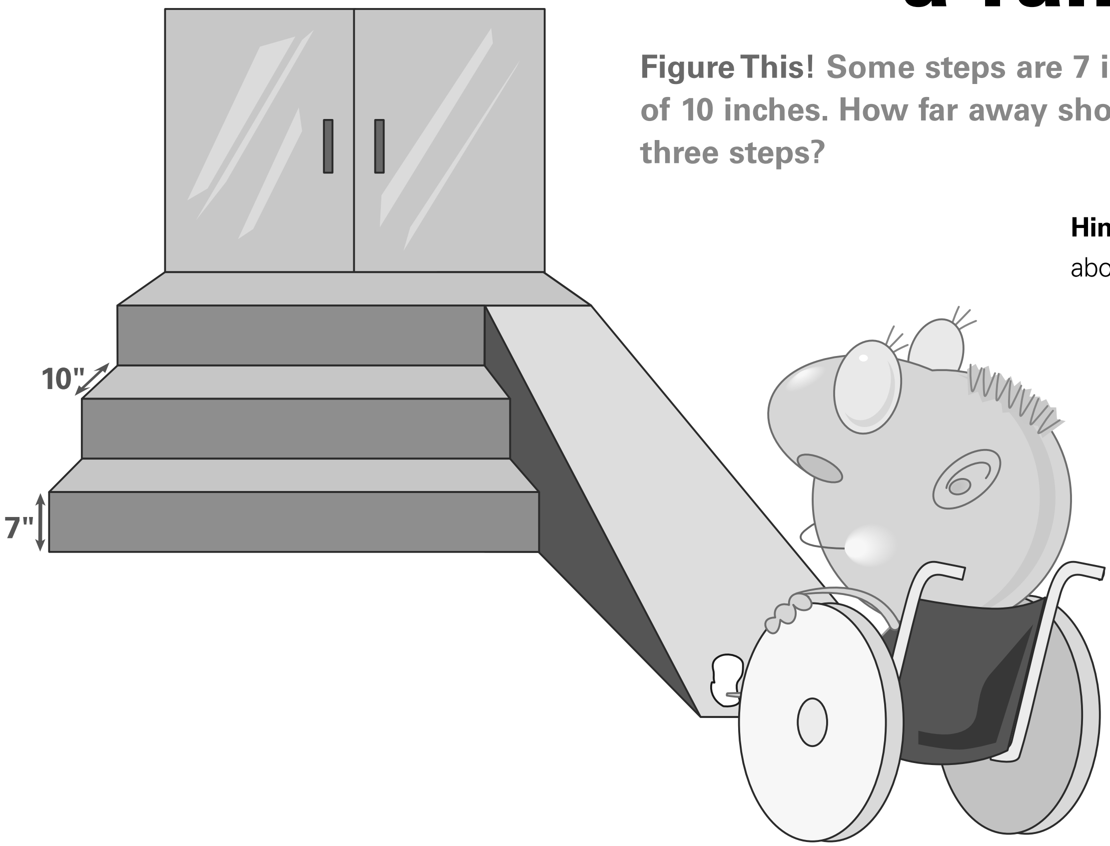


Figure This! Some steps are 7 inches high and have a width of 10 inches. How far away should the ramp start to go up three steps?

Hint: Access ramps usually go up about 1 inch for every 12 inches.

Slope is a measure of the steepness of an incline. Slope is used by civil engineers, builders, surveyors, and landscapers in constructing roads through mountains, stairs in houses, and drainage ditches.

Answer:
About 252 inches or 21 feet.

Figure This!

Get Started:

Sketch a diagram of the steps and find their total height. Then use the information given in the hint.

Complete Solution:

Since each step is 7 inches high, the three steps rise a total of 21 inches. The ramp should rise 1 inch for every 12 inches of horizontal distance. This means that the ramp should begin 21x12 = 252 inches away.

Try This:

- Place a small object, such as a penny, on a sheet of cardboard. Lift one end of the cardboard. How high must you raise the cardboard before the penny begins to slide? If the sheet of cardboard were longer, could you lift the end higher without making the penny slide?
- Measure the height and tread width of one step in your home, school, or business. Find the slope of the stairs containing the step (the ratio of the height to the tread width).
- Is there a skateboard ramp near your home? If so, estimate the slope of the ramp.
- Estimate a distance of 21 feet. Then measure a distance of 21 feet. How accurate was your estimate?

Additional Challenges:

(Answers located in back of booklet)

1. If the building in the challenge had four steps, how would you change the ramp?
2. As a glider travels through the air, it descends 1 foot for every 10 feet of horizontal ground distance it covers. If it descended 3 feet, how far did it actually travel?
3. The steepest parts of intermediate ski hills rise about 4 feet for every 10 feet horizontally, or have slopes of about 4 to 10. Is a hill with this slope steeper than a road with a 6% grade? A 6% grade means that a road ascends 6 feet for every 100 feet of horizontal distance traveled.

Things to Think About:

- Why are there official specifications for the slopes of roads, ramps, and staircases?
- Why do skiers and snowboarders use the switchback (zigzag) technique when descending some slopes?
- What is the typical height of a street curb? If the curb has an access ramp, what is its slope?
- Is the slope of the initial climb of a roller coaster more or less than the slope of the first descent?
- Could a road have a grade (slope) of 100%?

Did You Know That?

- According to US federal building codes, the maximum height for an access ramp is 30 inches. To reach entrances that exceed this limit, lifts or an elevator may be installed.
- Most cars cannot climb hills that have a slope of 30°(or rise about 1 foot for every 1.7 feet horizontally).
- Most local building codes describe a maximum stair rise of 8-1/4 inches and a minimum tread width of 9 inches.
- More than nine out of ten avalanches occur on slopes ranging from 25° to 45°.
- The ratio for slope is independent of units.
- Most black diamond ski slopes are around 30° to 35°, while a 40° slope would be considered the low end of extreme mountaineering. Parts of Tuckerman's Ravine on Mt. Washington have slopes of about 50°.
- Slopes are rates of change, a fundamental idea of calculus.
- Another way to express a slope uses the trigonometry notion of tangent.

Resources:

Books:

- Kleiman, G., et al. *Mathscape: Seeing and Thinking Mathematically. Roads and Ramps: Slopes, Angles, and Ratios*. Alsip, IL: Creative Publications, 1996.
- *Stairs: The Best of Fine Homebuilding*. Newtown, CT: Taunton Press, 1995.
- *The Guinness Book of World Records*, 1999. New York: Guinness Publishing Ltd., 1999.

Websites:

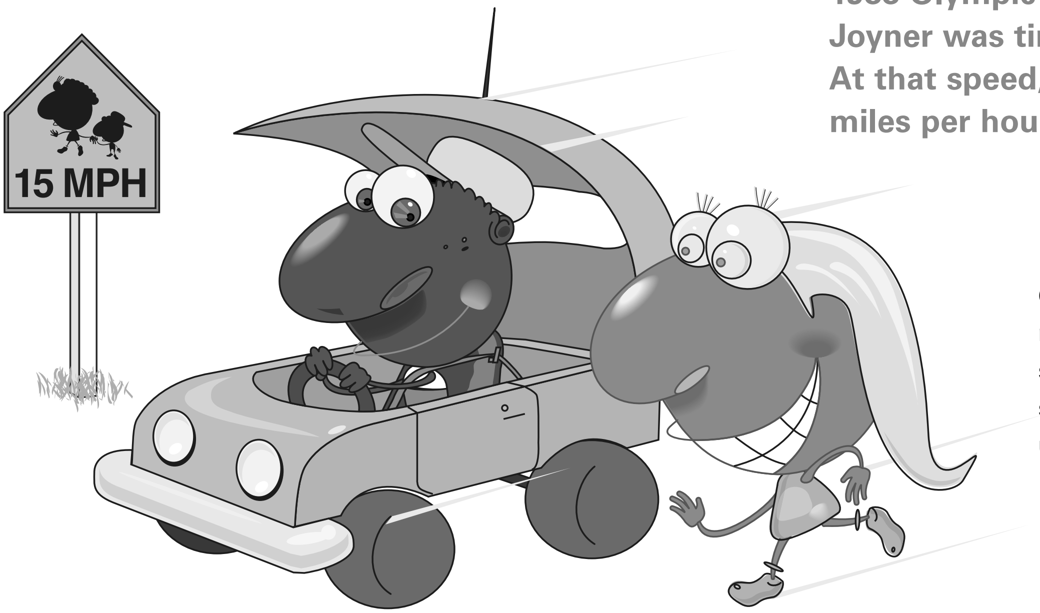
- **www.mroutdoors.com/columns/2000/0109out.html**
- **www.tuckerman.org/tuckerman/history.htm**
- **www.hometime.com/projects/howto.accrss/pc2aces2**

Math Challenge #34 FASTER?

Figure This! Math Challenges for Families

Can YOU run as ffffast as a car?

Figure This! During the 100 meter dash in the 1988 Olympic Games in Seoul, Florence Griffith-Joyner was timed at 0.91 seconds for 10 meters. At that speed, could she pass a car traveling 15 miles per hour in a school zone?



Hint: How many meters in a mile? How many seconds in an hour?

Conversion between units of measure is required from the kitchen to the construction site to the laboratory. Chefs, carpenters, scientists, and engineers all must convert units of measure in their work.

Answer:

Her speed would be about 24.6 miles per hour; she could pass the car.

Figure This!

Get Started:

There are 2.54 centimeters to an inch and 5280 feet to a mile. How many centimeters are in a meter? How many inches are in a foot; in a mile? What is her rate (distance divided by time)?

Complete Solution:

One method for converting between measures is called dimensional analysis. The conversions between measures are written as fractions so the common units cancel out.

$$\frac{10\ \cancel{m}}{0.91\ \cancel{sec}} \times \frac{100\ \cancel{cm}}{1\ \cancel{m}} \times \frac{1\ \cancel{in}}{2.54\ \cancel{cm}} \times \frac{1\ \cancel{ft}}{12\ \cancel{in}} \times \frac{1\ mi}{5280\ \cancel{ft}} \times \frac{60\ \cancel{sec}}{1\ \cancel{min}} \times \frac{60\ \cancel{min}}{1\ hr} \approx 24.6 \text{ miles/hour}$$

Her rate is about 24.6 miles per hour, and she could easily pass a car going at a rate of 15 miles per hour.

Try This:

- Many dictionaries contain conversion tables for measures. Find a conversion table and examine it. Are any of the conversions familiar to you?
- Look in a newspaper or website for currency exchange rates. How could you use the information you find to convert Spanish pesetas into Japanese yen?
- Compare the size of a liter and a quart.
- Have someone time how long it takes you to go 10 yards. What is your rate in miles per hour?

Additional Challenges:

(Answers located in back of booklet)

1. A waterbed mattress is 84 inches long, 60 inches wide, and 8 inches deep. There are 231 cu. in. in a gallon. How many gallons of water does it take to fill the mattress?
2. According to *Natural History* magazine, a cheetah is the fastest animal in the world with a speed of 6,160 feet per minute. How many miles per hour is that?
3. If it is 20˚ Celsius outside, would you need a jacket?

Things to Think About:

- Which measuring system is used in international track and field competitions?
- Roger Bannister, a British physician, broke the four-minute mile in 1954. Will someone break the three-minute mile? Is there a limit to the amount of time required to run a mile? If so, when do you think it will be reached?
- Why are speeds on the 100-meter or 200-meter dashes reported in meters/second instead of kilometers per hour?
- During World War II, American soldiers referred to kilometers as "kiddie miles." Where do you think this name came from?
- Preying animals, such as the cheetah, lion, and hyena, run faster than most other animals. Why?

Did You Know That?

- Since 1984, running events in the Olympics have been timed in hundredths of seconds because of electronic timing devices.
- The United States is the only developed country that does not use the metric system for its principal units of measurement.
- Almost all scientific measurements are made using metric units. The metric system is based on the decimal system (units of 10) and follows a consistent naming scheme using prefixes.
- The US National Aeronautics and Space Administration (NASA) spent $125 million on a spacecraft that flew 416 million miles over 9 1/2 months before crashing on Mars. The spacecraft crashed due to a contractor's error in converting pounds of force into another unit of force called newtons. One newton is the amount of force required to accelerate 1 kilogram of mass 1 meter per second each second.
- Many cars and trucks require metric tools for maintenance.
- One square centimeter (1 cm^2) is about the size of your little fingernail.
- The mass of 1 cm^3 of water at standard temperature and pressure is 1 gram.
- The US Conventional System of Measurement is a modified version of the British Imperial System, which is no longer in use.
- As of January 1, 2000, it is a criminal offense in Great Britain to sell most packaged and loose products using imperial measures (inches, pounds, and so on). One exception is precious metals.
- One of the few British Imperial units of measure that remains in world-wide use is the barrel, primarily for oil.

Resources:

Books:

- *The Guinness Book of Records 1999*. New York: Guinness Publishing Ltd., 1999.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Math Challenge #35 FIRE HYDRANT

Figure This! Math Challenges for Families

What shape is at the very top of a fire hydrant?

Figure This! The water control valve on the cover of a fire hydrant has five sides of equal length and five angles of equal measure. Many common household wrenches will not turn these valves. Why not?

Hint: Think about an ordinary household wrench. Most wrenches have two parallel sides; that is, the sides are everywhere the same distance apart.

The geometric shapes of many objects relate directly to their usefulness. For example, round tires produce a smooth ride, and airplane wings are designed to provide lift.

Answer:
Most household wrenches will not work on the valves of a fire hydrant because there are no parallel sides on the five-sided (pentagonal) value.

Figure This!

Get Started:

Draw a square. Are any sides parallel? Would a common household wrench open it? Does every shape with four sides the same length have parallel sides? How about a shape with five sides the same length and five angles of the same size?

Complete Solution:

All of the angles of a valve on top of a fire hydrant are the same size. The five sides have equal length. (The valve top is a regular pentagon.) No five-sided figure with these characteristics can have parallel sides. This means that an ordinary household wrench will not fit. This design makes opening hydrants difficult without the special wrenches carried by firefighters.

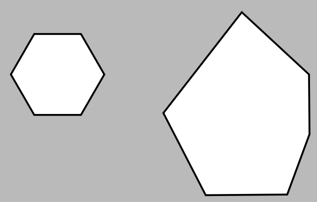
Try This:

- Tie a knot in a long, thin strip of paper. Carefully flatten the knot so the resulting figure has equal length sides. Cut off the excess paper. What shape did you make?
- Look out a window and describe all the sets of parallel lines you see.

Additional Challenges:

(Answers located in back of booklet)

1. What shapes of bolts can be turned with a common household wrench?
2. Would it be easier to use a wrench if the sides of a six-sided bolt are all the same length?



3. If each non-adjacent vertex of a pentagon is connected, diagonals are formed. The diagonals of any regular pentagon form a five-pointed star. How many triangles are there in the star?

Things to Think About:

- If two lines are parallel to a third line, are all three lines parallel?
- Some shapes have sides the same length but angles of different sizes.
- Some shapes have angles of the same size and sides of different lengths.
- A square is a rhombus, but a rhombus doesn't have to be a square.
- Why are pipe wrenches different from ordinary household wrenches?
- Unauthorized opening of fire hydrants frequently causes water pipe damage underground, and the resulting pressure can cause damage to home hot water heaters.

Did You Know That?

- The term "fire plug" dates from the early 1800s, when water mains were made of wood. When responding to an alarm, firefighters had to chop into the main waterline to connect their hoses. When they finished fighting the fire, they would seal the main with a "fire plug."
- The names *pentagon* and *hexagon* describe their respective numbers of angles (or sides). Early Greek mathematicians studied these shapes and gave them their names. In Greek, *penta*- means 5, *hexa*- means 6, *hepta*- means 7, *octa*- means 8, *nona*- means 9 and *deca*- means 10. Also, *gon* comes from a word meaning angle.
- An Allen wrench has a hexagonal cross-section.
- A regular polygon has sides the same length and interior angles the same size.
- Each of the diagonals of a regular pentagon is parallel to one of the sides.

Resources:

Books:

- Jacobs, H. *Mathematics: A Human Endeavor.* San Francisco: W.H. Freeman and Co., 1970.

Websites:

- **users.intermediatn.net/hillspainting/wrenches.htm**

Math Challenge #36 MAJORITY VOTE

Figure This! Math Challenges for Families

What percentage does it take to win a vote?

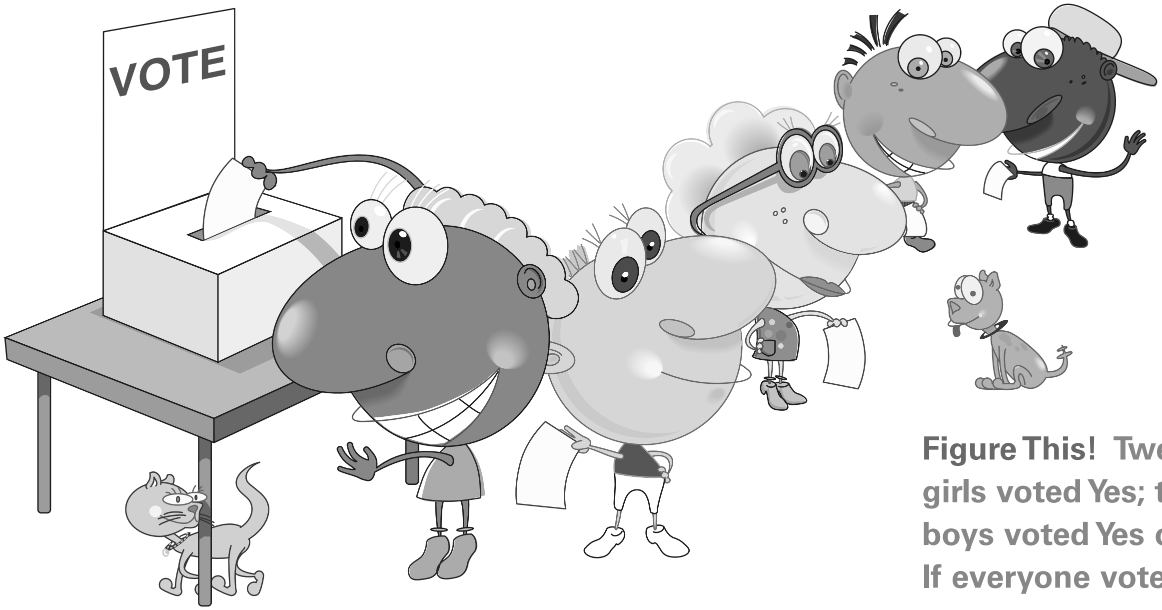


Figure This! Twenty-two percent of the girls voted Yes; thirty percent of the boys voted Yes on the same motion. If everyone voted, did the motion pass?

Hint: Suppose 40 boys and 50 girls voted. How many of each voted Yes?

Understanding percentages is necessary for people to make sense of information in the media, for businesses to summarize work data, for politicians to interpret the results of polls, and for manufacturers to make decisions about marketing.

Answer:

Assume winning means a simple majority or over half of the votes. If it takes at least half voting Yes to pass a motion, the motion did not pass.

Figure This!

Getting Started:

Suppose the number of boys and the number of girls were the same? Suppose there were many more boys than girls? Many more girls than boys?

Complete Solution:

There are different ways to solve the problem.

- Look at different numbers of boys and girls. Think about each of the following:

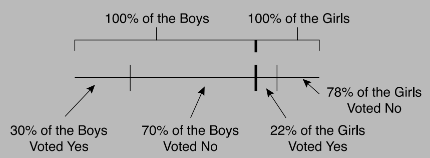
  (1) A class of 50 boys and 50 girls

  (2) A class of 0 boys and 100 girls

  (3) A class of 100 boys and 0 girls

  In neither of the extreme cases nor in any other case will the motion pass.

- Think about a line diagram showing the percentages where the numbers of boys and girls are not the same. Above the line, you see that 100% of the boys are depicted to the left of the bold line with 100% of the girls to the right. Below the line are the voting percentages of Yes and No for both boys and girls.



  To determine if the Yes vote wins, then the segments representing the Yes votes together have to be over half the length of the entire segment. The segments representing Yes will never add to half of the total length so the motion cannot pass.

- Think about the percentage who voted No: 78% of the girls voted No; 70% of the boys voted No. There is no way for over half of the total to vote Yes.

Try This:

- Think of a survey question that can be answered either "Yes" or "No." Ask a group of people your survey question and record the results for males and females. Find the percentage of males and the percentage of females who responded Yes. Then calculate the total percentage of people who said Yes. Does adding the percentages of Yes votes by males and by females result in the total percentage of people who said Yes?
- Suppose a motion passes with a 52% vote. Find some possible ways to break this into percentages for males and percentages for females who voted Yes. Is it possible that only males supported the motion? Only females?

Additional Challenges:
(Answers located in back of booklet)

1. There are 400 boys and 420 girls in a school. If 18% of the boys are left-handed and 10% of the girls are left handed, what percentage of students in school are left-handed?
2. A survey on soda preference had the following results: 25% liked Pepsi™, 30% liked Coca-Cola™, and 10% liked both. How many liked neither soda?
3. A newspaper survey reported that 42% of voters in Western County favored the Democratic candidate, and 25% of the voters in Central County favored this candidate. The newspaper reported a 17% difference in the number of voters favoring the Democratic candidate. Is this conclusion true?
4. Ninety-nine percent of the people in the United States have a television; 40% have 3 or more televisions. Is it valid to conclude that 59% of the people do not have more than 2 televisions?

Things to Think About:

- You cannot add percentages unless they were calculated using the same population and are measuring mutually exclusive events.
- When can you add, subtract, multiply, or divide percentages?
- Washington DC's population grew –14.5% from April 1, 1990 to July 1, 1999.
- Is it ever possible to have a percentage over 100?

Did You Know That?

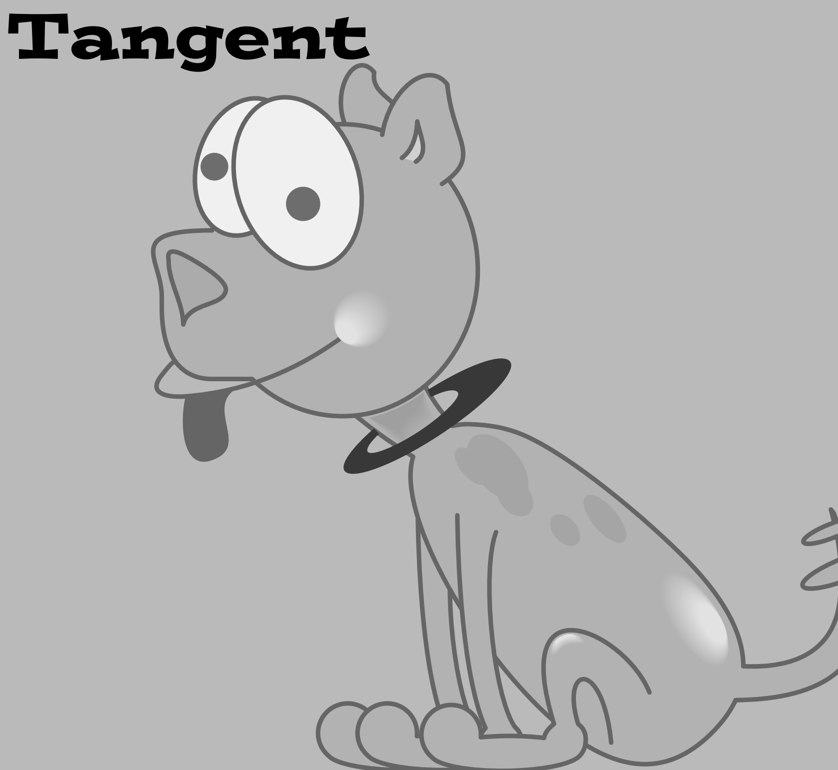
- Under US election rules, it is possible for presidential candidates to win with less than 50% of the popular vote.
- Percentages are often used to compare quantities from populations of different sizes.
- A rate of 10 per 1000 is the same as 1%.
- A 50% discount is the same as half price.
- There are many different methods of voting. For example, a simple majority (or plurality) method of voting has as winner the choice receiving the most votes. This method of voting can be unfair if there are more than two choices.

Resources:

Books:

- Malkevitch, J. *The Mathematical Theory of Voting*. Lexington, MA: COMAP, Inc., 1985.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Notes:



Math Challenge #37

MAP COLORING

Figure This! Math Challenges for Families

How many colors of states are on a map of the US ???

Figure This! Mapmakers use different colors on states that share a border. What is the least number of colors needed to color a map of the states west of the Mississippi River in this way?

Hint: Try to fill in the map with as few colors as possible. Then try to show why using fewer colors would not work.

Assigning different colors to objects or decisions is a useful technique for analyzing complex situations. Similar methods can be applied to scheduling meetings, routing air traffic, and designing computer circuit boards.

Answer:
Four colors.

Figure This!

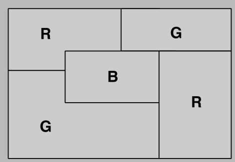
Get Started:

You can color a state or write a symbol for the name of a color in a state. Start in one corner of the map. Then begin coloring neighboring states, adding another color only when necessary.

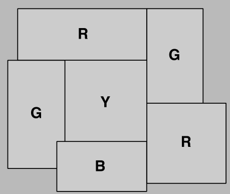
Complete Solution:

The least number of colors needed for this map is four. One possible solution uses the colors red (R), green (G), yellow (Y), and blue (B).

It is not possible to use fewer than four colors. If a state is surrounded by an even number of states, alternate two colors as you move around the states. The surrounded state will require a third color. See the diagram.



If a state is surrounded by an odd number of states, the surrounding states require three colors, and the surrounded state requires the fourth color.



Try This:

- Get a map of your state showing all counties. Color the map so that neighboring counties have different colors. How many colors did you need?
- Find a map of the US where states are colored. Do neighboring states get different colors? How many colors does the map use?
- Make a map that requires only three colors and that is different from the one pictured.
- Find maps of the US in 1790, when there were only the 13 original colonies. How many colors does that map need? How about a map of the US in 1810, or 1850?

Additional Challenges:

(Answers located in back of booklet)

1. Draw a map that has at least seven regions requiring only three colors.
2. Imagine that you are planning an ideal city. The city has five neighborhoods, and highways connect each neighborhood with every other neighborhood. To save driving time, none of these highways must intersect. What is the fewest number of overpasses you will need to build? How would your answer change if the city had six neighborhoods?

Things to Think About:

- On some surfaces, a map can be drawn requiring more than four colors.
- How many colors would you need to color the “four corners” region of the United States (Arizona, New Mexico, Utah, and Colorado)?
- In the western United States, Utah and Nevada are surrounded by an odd number of states. Are there any eastern states that have this feature?

Did You Know That?

- In 1852 British student Frederick Guthrie asked whether any map drawn on a piece of paper can be colored with no more than four colors. Guthrie’s question became the “Four-Color Problem.”
- A. B. Kempe, a lawyer, published a proof of the Four-Color Problem in 1879, but P. J. Heawood found an error in Kempe’s proof in 1890. The problem remained unsolved until 1976 when mathematicians Kenneth Appel and Wolfgang Haken of the University of Illinois gave a proof based on more than 1000 hours of computer calculations.
- In 1890, Heawood determined that it requires at most seven colors to color any map on a doughnut.
- The number of colors needed for a map drawn on other than flat surfaces was determined in 1968, eight years before the Four-Color Problem was solved.

Resources:

Books:

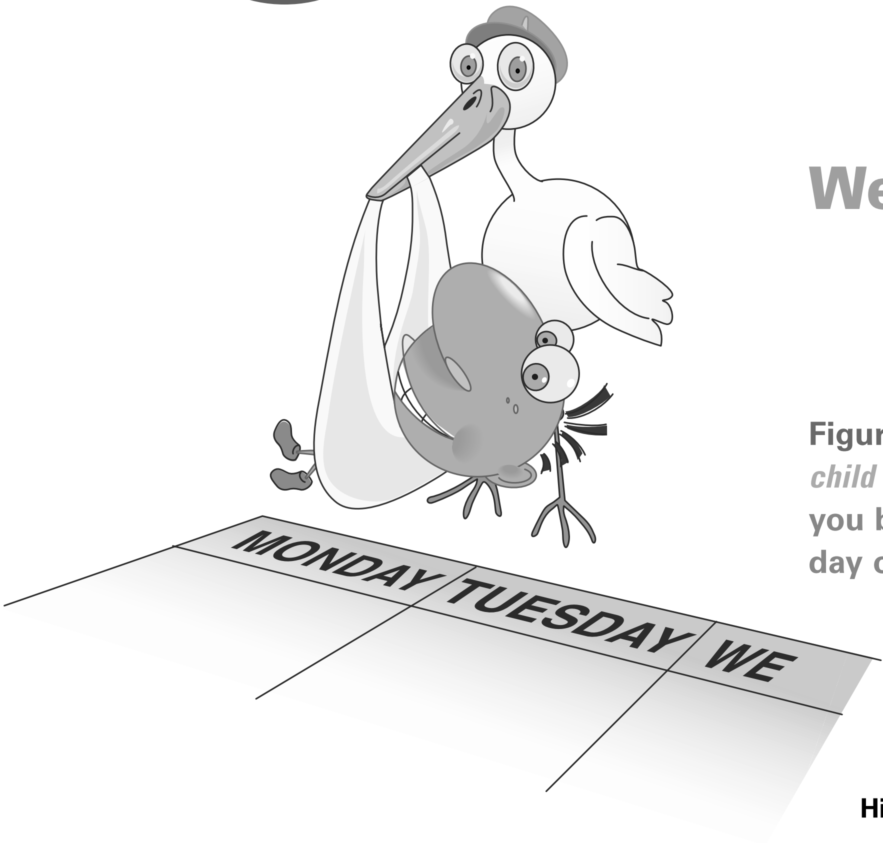
- Bergamini, D. *Mathematics, Life Science Library*. New York: Time Incorporated, 1963.
- Francis, R. *The Mathematician’s Coloring Book, HIMAP Module 13.* Arlington, MA: COMAP, Inc., 1989.
- Montana Council of Teachers of Mathematics. *Integrated Mathematics: A Modeling Approach Using Technology*, Level 4, Vol. 1. Needham Heights, MA: Simon & Schuster Custom Publishing, 1997.

Websites:

- **www.mathispower.org**
- **www.math.utah.edu/~alfeld/talks/S13/4CMP**
- **www.math.ucalgary.ca/~laf/colorful/4colors**

Math Challenge #38 MONDAY'S CHILD

Figure This! Math Challenges for Families



Were you born on a Monday?

Figure This! ***Monday's child is fair of face; Tuesday's child is full of grace.*** **On what day of the week were you born? Can you devise a method to find the day of the week for any date?**

Hint: January 1, 2000 was a Saturday; January 1, 1999 was a Friday. (But don't forget leap years. The year 2000 is a leap year.)

An algorithm is a step-by-step process for completing a task. Algorithms are used by those who follow routines or recipes in their work: computer scientists, factory workers, statisticians, and chefs, among others.

Answer:

Your answer depends on your date of birth. For example, a student born on December 31, 1986, was born on Wednesday.

Figure This!

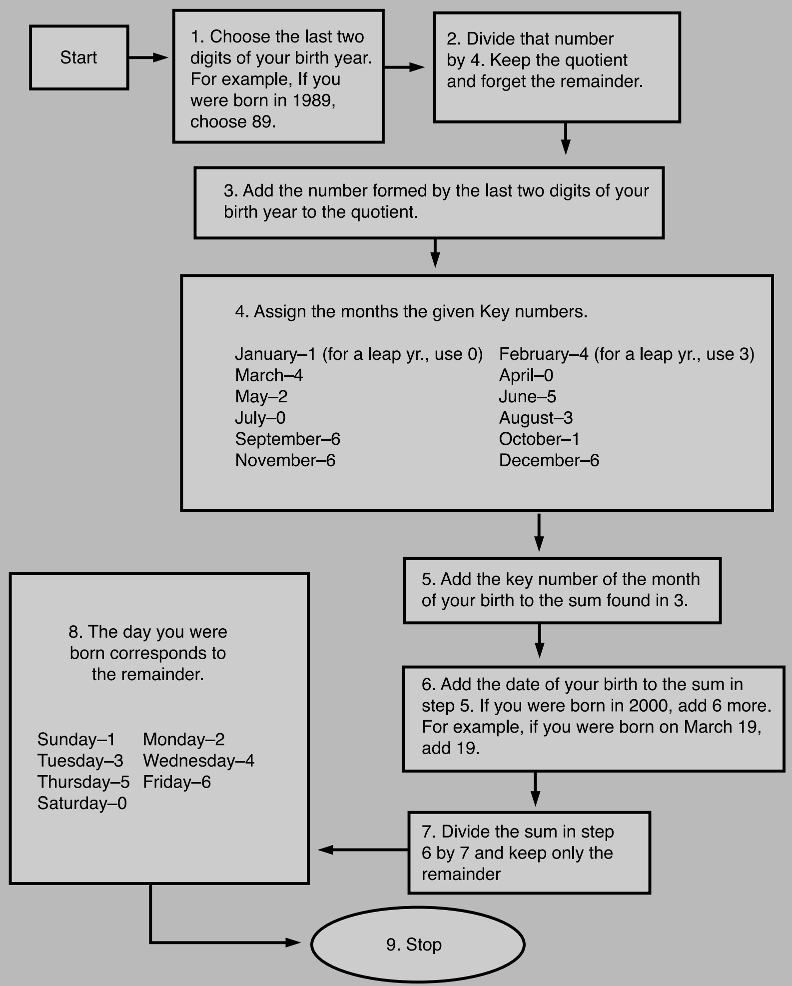
Get Started:

List the number of years between this year and the year of your birth. Then count the number of leap years that are included.

Complete Solution:

There are many ways to approach this problem.

- A general approach to this problem involves creating a step-by-step procedure for determining the day of the week for a past date. One way to describe such a procedure is with a flowchart. The flowchart given here finds the day of the week for any date from 1900 to 2000 (*The Farmer's Almanac*, p. 113).



- Another way to find the day of the week of your birth follows: Consider the number of days in a year. Non-leap years have 365 days. Since 365 ÷ 7 gives a remainder of 1, then any particular date moves ahead one day of the week in the following year. Going back a year, each date moves back one day of the week. Since leap years have 366 days, and 366 ÷ 7 gives a remainder of 2, any date moves ahead two days of the week in the year following a leap year.

  This means that when going back in time, each date moves back two days of the week for each leap year. (Note: If the current year is a leap year, then only dates after February 29 will have moved two days ahead.)

  Suppose that you were born on December 31, 1986. From the hint, you know that January 1, 2000 was a Saturday. That means that December 31, 1999 was a Friday. The year 1986 was 13 years before 1999. The years 1988, 1992, and 1996 were all leap years. That means that the day of the week moves backward 13 + 3, or 16 days. Since 16 ÷ 7 gives a remainder of 2, you should count back two days from Friday to get Wednesday.

Try This:

- Use one or both of the methods described in the complete solution to find the day of the week on which another member of your family was born.
- To use the flowchart for birth years from 2000 to 2099, subtract one from the sum in step seven. Find the day of the week for your 100th birthday.
- Create a flowchart for some task or chore that you do at home. See if a family member can follow your flowchart.

Additional Challenges:

(Answers located in back of booklet)

1. On July 4, 1976, the United States of America celebrated its bicentennial. What day of the week was it?
2. Below are two algorithms for finding the cost with tax of an item. At original cost c dollars and a tax rate of t per dollar,

$$c + ct$$
$$c \cdot (1 + t)$$

do the two expressions represent the same amount?

Things to Think About:

- Why might it be important to know the day of the week for certain dates?
- Compare the two methods given in the complete solution to find the day of the week. How do you think the key numbers in the flowchart are determined?
- A formula is an algorithm.
- The process of doing long division is an algorithm.

Did You Know That?

- One of the first recorded mathematical algorithms was developed by Eratosthenes (about 230 BC) for finding prime numbers.
- Our present calendar is the Gregorian calendar, named for Pope Gregory XIII in 1582. Great Britain adopted the Gregorian calendar in 1752 and forced its use in the American colonies. In 1923, Greece also adopted the Gregorian calendar. Since then, it has become the standard calendar for nearly all the countries in the world.
- Although Thanksgiving day is usually observed on the fourth Thursday in November, its actual date in any year must be decided by proclamation of the sitting president of the United States.
- The first man to walk on the moon, Neil Armstrong, landed there on a Sunday, July 20, 1969.
- When developing software programs, computer scientists often use flowcharts to organize their work.
- To change the flowchart to handle years before 1900, add 2 to the sum in step 6. For years from 1753 to 1800, add 4 to the sum in step 6.

Resources:

Books:

- Glenn, W., and D. Johnson. *Fun with Mathematics*. Sacramento, CA: California State Department of Education, 1960.
- Rand McNally & Company. *The Real Mother Goose*. Eau Claire, WI: E. M. Hale and Co., 1953.
- *The Old Farmers Almanac*. Dublin, NH: Yankee Publishing Inc., 1999.

Websites:

- **www.geceventures.com/year2000/about_calendars.htm**

Notes:

Axis

Math Challenge #39

PATIOS

Figure This!
Math Challenges for Families

Does bigger perimeter mean bigger area?

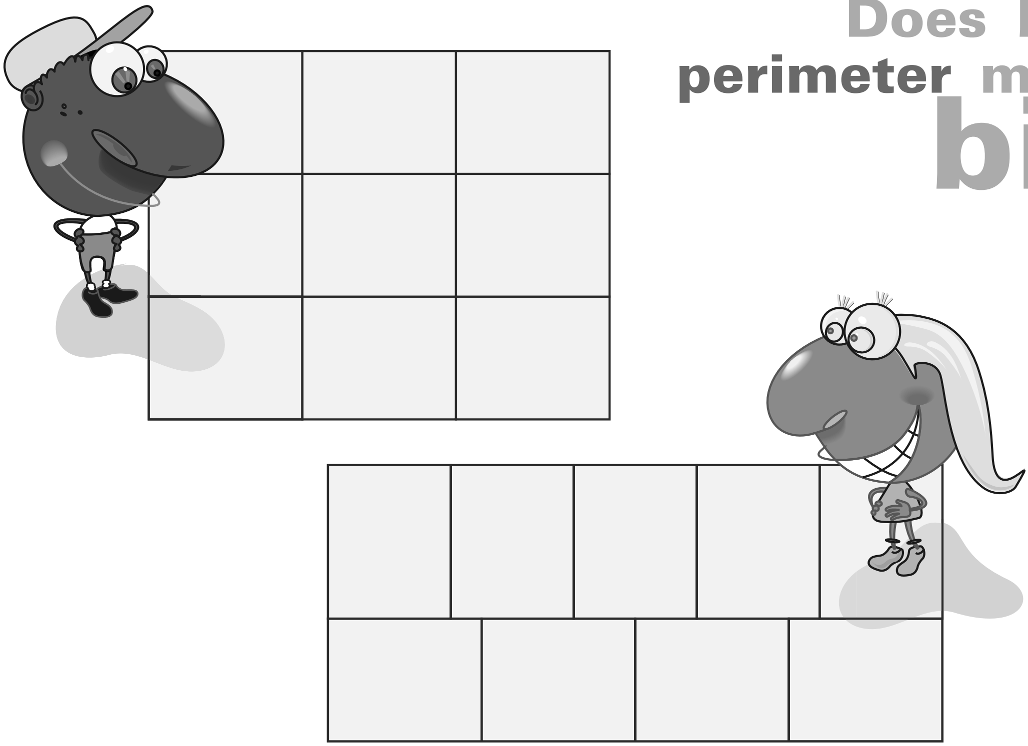


Figure This! Helix and Polygon both used the same number of identical concrete pieces to make their patios. The area of each patio is the same: 180 square meters. What are the dimensions of a single piece of concrete?

Hint: Notice how the pieces fit together on Polygon's patio. What is different about the way the pieces fit on Helix's patio?

Area is an important mathematical concept. Architects, real estate agents, artists, and surveyors all use area in their work.

Answer:
A tile is 4 m by 5 m.

Figure This!

Get Started:

Find the area of one of the concrete pieces. How many of the short sides equals a long side of Polygon's patio?

Complete Solution:

The area of each patio is 180 square meters (m2), and each is made from nine identical rectangular concrete pieces. This means that the area of one piece is 180 ÷ 9, or 20 m2. Since the area of the rectangle equals length (L) times width (W), you know that L • W = 20. From the way in which the pieces are arranged in Polygon's patio, you can see that four lengths is the same as five widths. In other words, the ratio of length to width is 5 to 4. As it happens, 5 • 4 = 20. This means that the length is 5 m, while the width is 4 m.

- Another way to look at this is as follows. You know from Polygon's patio that four lengths equals five widths. Since 4L = 5W, then:

  L = 5/4 • W

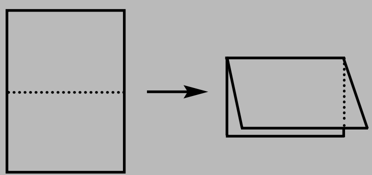
  You also know the area of the piece: L • W = 20.
  Substituting for L,

  (5/4 • W) • W = 20
  5/4 W^2 = 20
  W^2 = 4/5 • 20
  W^2 = 16
  W = 4 (Widths cannot be negative.)

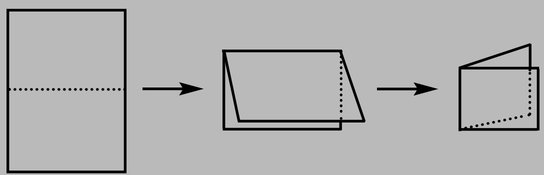
  Substituting 4 for W, you can then determine that L = 5.
  So a single concrete piece has dimensions 4 m by 5 m.

Try This:

- Take two 8 1/2 in. by 11 in. sheets of paper. Fold one of them horizontally as shown.



  Continue folding this sheet horizontally four more times. Take the other sheet of paper and make the first fold exactly as you started above. Then make the second fold vertically as shown.



  Continue folding this piece of paper three more times alternating horizontally and vertically. Compare the areas of the rectangles folded with the two different sheets.

- Estimate the perimeter and area of the top of your bed. Then measure your bed and calculate the perimeter and area. How close were your estimates?
- Draw two rectangles with the same area but different perimeters.
- Tie the ends of a piece of string together to form a circle. Use the string to form various rectangles. The perimeter doesn't change. Does the area?

Additional Challenges:

(Answers located in back of booklet)

1. The distance around a figure is its perimeter. What are the perimeters of the patios in the challenge?
2. The patio in the diagram is made of identical tiles and has an area of 180 m^2. What is its perimeter?

3. If given only Helix's patio in the challenge, you would not get a unique answer. If you were given only Polygon's patio, you would find a unique answer. Explain why.
4. Use squares with a side length of 1 unit to create different shapes that have a perimeter of 8 units. Which of the shapes has the greatest area?

Things to Think About:

- There are rectangles with the same perimeter but different areas.
- The patterns in many wood or tile floors are made with rectangles. Why do you think this is so?
- As the perimeter of a rectangle increases, the area may either increase or decrease.

Did You Know That?

- Guess, check, and revise is sometimes the most efficient way to solve a problem.
- For a rectangle with a given perimeter, the shape with maximum area is a square.

- All squares are rectangles but not all rectangles are squares.
- The least common multiple of two whole numbers *a* and *b* can be found by making a square from rectangles with dimensions of *a* x *b*.
- A baseball diamond is a square 90 feet on a side.

Resources:

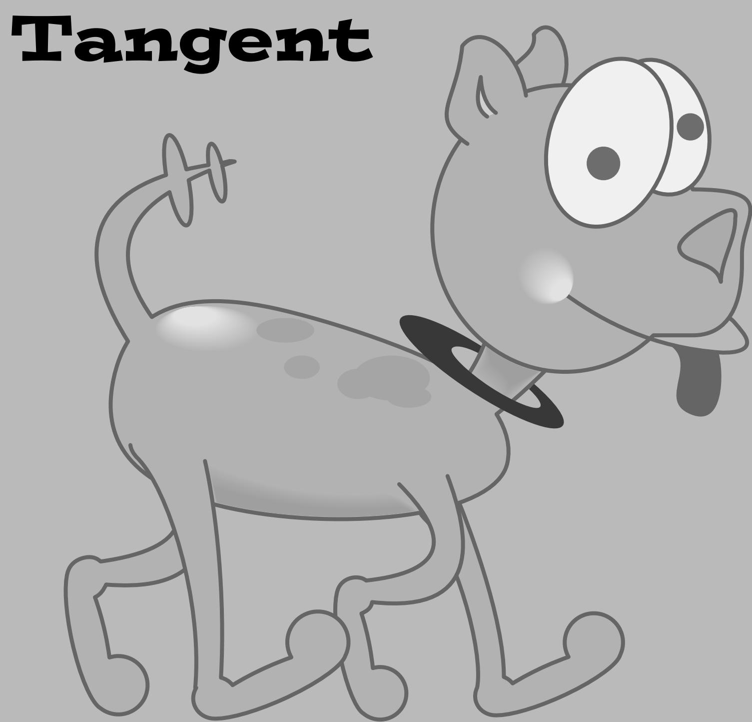
Books:

- Conrad, S., and D. Flegler. *Math Contests for High School, Volume 1*. New York: Math League Press, 1992.
- Mumme, J. *"Investigating Perimeter and Area."* In *Teaching with Student Math Notes*, Volume 2 (ed. E. Maletsky). Reston, VA: National Council of Teachers of Mathematics, 1993.

Websites:

- **web.singnet.com.sg/~raynerko/index.htm**
- **www.forum.swarthmore.edu/dr.math/problems/cducote2.16.96**

Notes:



Math Challenge #40
PUTT PUTT

Figure This!
Math Challenges for Families

Can you make a hole-in-one?

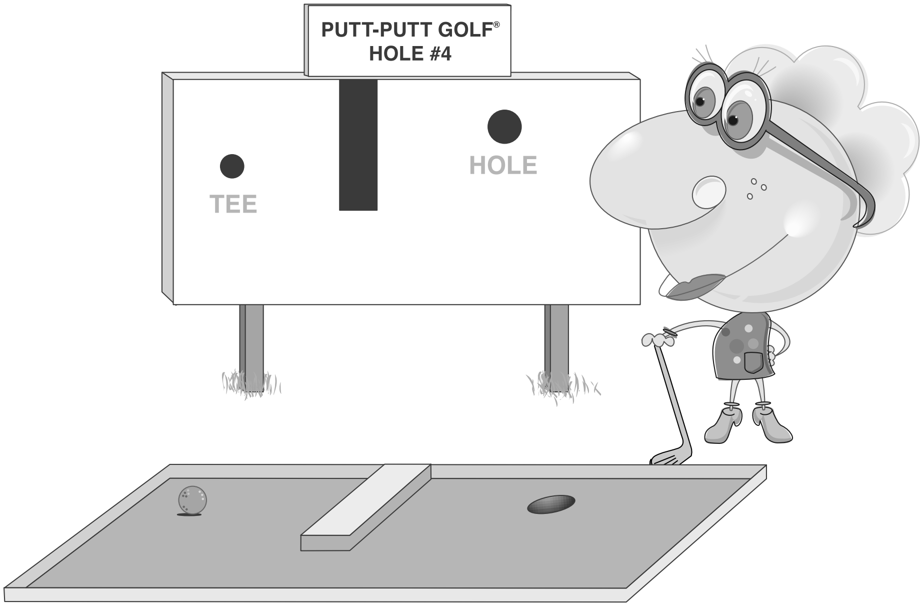


Figure This! **How would Tessellation hit a golf ball from the tee to make a hole-in-one?**

Hint: The angle at which the ball bounces off the wall will have the same measure as the angle at which it hit the wall.

A golf ball bounces off the side of a miniature course or a pool ball bounces off the edge of a pool table with a hard cushion, the same way light bounces "off" a mirror. Opticians who make lenses, biologists using microscopes, and astronomers using telescopes, as well as makers of satellite dishes and periscopes, are concerned with this principle in their work.

Answer:
The lines in the drawing show a path of a hole-in-one.
The marked angles, 1 and 2, have equal size. You should aim for point P.

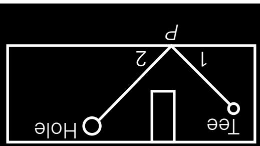
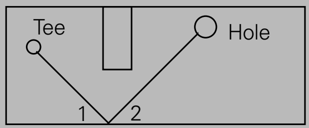


Figure This!

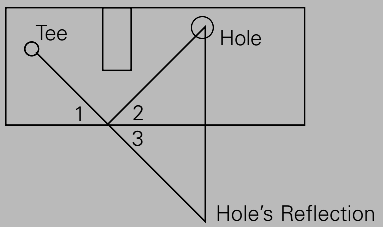
Get Started:

How can you decide where to aim? Since a direct path from the tee to the hole does not exist, you must bounce the ball off the wall. Draw lines from the tee to the edge and the hole to the edge to find a point that makes the bounce angles, 1 and 2, equal.



Complete Solution:

One way to locate the proper place to aim is to think of the sidewall as a mirror. If you looked into this mirror from the tee, the hole's reflection would appear to be in the location shown in the diagram.

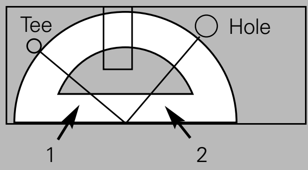


As mentioned in the hint, the angle at which the ball leaves the wall will be the same as the angle at which it hit the wall. If you aim the ball at the point where the line from the tee to the hole's image intersects the wall, then it will bounce into the hole.

This can be proven mathematically as follows: Because the two triangles on either side of the wall in the diagram are mirror images, they are exactly the same size and shape (congruent). This means that angle 2 is the same size as angle 3. Angle 1 and angle 3 are also the same size, because they are vertical angles. (Vertical angles are angles formed by two intersecting lines.) Angle 1 is the same size as angle 2 because both are the same size as angle 3. This means that the path to the hole shown on the diagram is the same path the ball will take as it bounces off the wall.

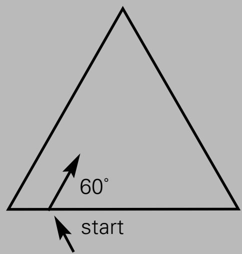
Try This:

- Design a hole for a miniature golf course; then play a game with a friend to show how the ball can bounce off the sides and into the hole. (Try this on a computer.)
- Measure the distance from the ball to the hole along the path the ball travels in the challenge. Suppose the ball hits the boundary anywhere else on the way to the hole. Measure this other path and compare it to the first. What do you find?
- Trace the diagram in the challenge on a sheet of paper. Insert thumb-tacks at the tee and the hole. Stretch a rubber band around the thumb-tacks. Place a protractor on the diagram and pull the rubber band to the center of the protractor as shown.



Slide the protractor until the angles marked 1 and 2 are the same size. Now measure the length of the stretched rubber band. If the angles are not the same size, how do you think the length of the rubber band will change?

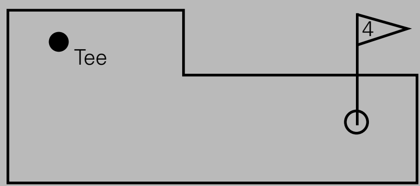
- The diagram below shows a beam of light entering a triangular box with mirrored walls. Trace the path of the light as it bounces off the walls. No matter where you start, what happens to the path?



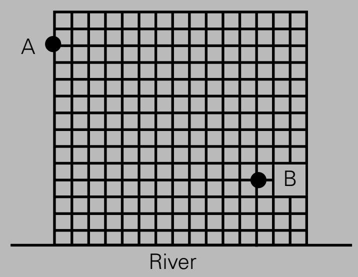
Additional Challenges:

(Answers located in back of booklet)

1. In the Complete Solution to the Challenge, the hole was reflected. How would the solution change if the tee were reflected?
2. Show how to make a hole-in-one on this miniature golf course by bouncing the ball off two walls.



3. Amesburg (A) is 12 miles from the Straight River, while Belleville (B) is 4 miles from the river. The county wants to build a water pumping station on the river to serve both towns. To reduce costs, engineers would like to locate the station so that the sum of the distances from the pumping station to the towns is as small as possible. Where should they build the station? (This happens when the "bounce" angles are equal.)



Things to Think About:

- Is there any way to place a hole on a miniature golf course so that no matter where you aim the ball from the tee, you can make a hole-in-one every time?
- Is there any way to place a hole on a miniature golf course so that no matter where you aim the ball, you can never make a hole-in-one?

Did You Know That?

- Although the properties of reflected light were known to other ancient philosophers and scientists, it was Heron of Alexandria (about AD 75) who developed the related mathematics.
- The longest recorded hole-in-one was 447 yards, made by Robert Mitera in Omaha, Nebraska in 1965.
- Kenneth Schreiber, a legally blind golfer, shot a hole-in-one in 1997 in Bayonet Point, Florida.
- The method of reflecting is sometimes used to find distances that can not be measured directly.

Resources:

Books:

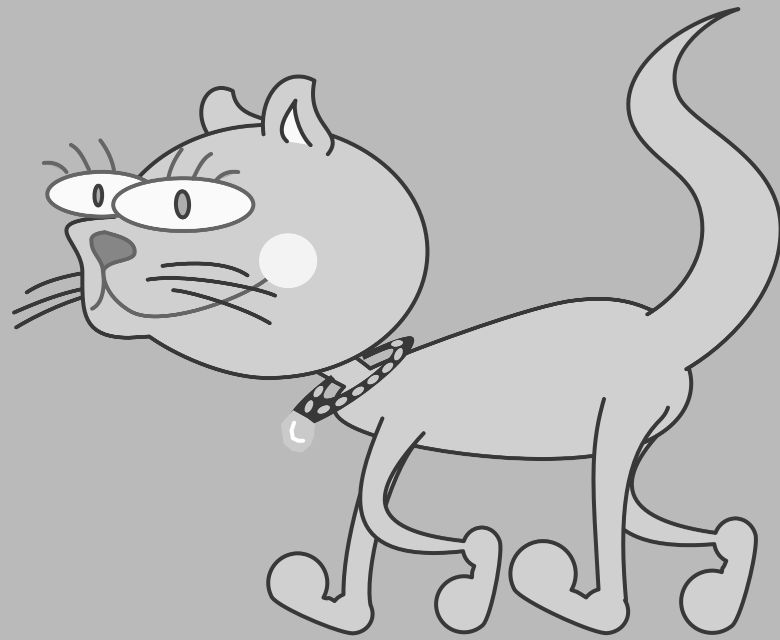
- Jacobs, H. *Mathematics*: *A Human Endeavo*r. San Francisco: W. H. Freeman Co., 1970.
- *The Guinness Book of Records 1999*. New York: Guinness Publishing Ltd., 1999.

Websites:

- **www.tenet.edu/teks/math/clarifying/cageoc.html**
- **www.illuminations.nctm.org**

Notes:

Axis



Math Challenge #41 RADAR BILLS

Figure This! Math Challenges for Families

Do you have a radar bill in your pocket?

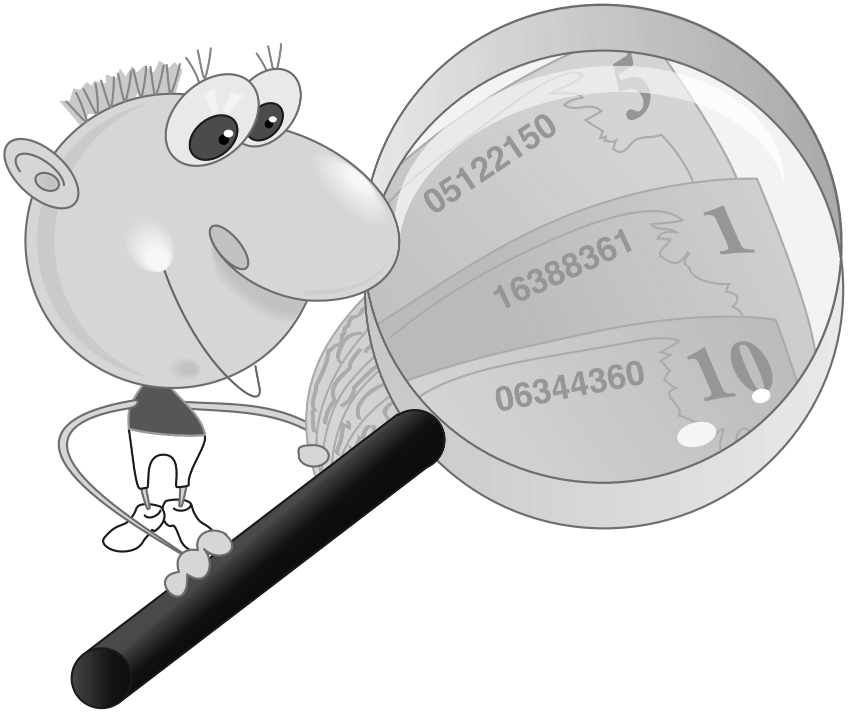


Figure This! **Paper money, such as dollar bills, with serial numbers that read the same backwards as forwards are sometimes called "radar bills." How common are radar bills?**

Hint: Serial numbers on US bills have eight digits. How many different serial numbers are possible?

Symmetry and repeating patterns are crucial to the study of mathematics. Artists, scientists, and designers all use these properties in their work.

Answer:
In the 100,000,000 eight-digit serial numbers from 00000000 to 99999999, there are 10,000 that read the same forwards and backwards. Considering this fact, you might expect 1 in every 10,000 bank notes to be a radar bill.

Figure This!

Get Started:

Begin with a similar, but simpler problem. Suppose there were only two digits in a serial number. In this case, a radar bill would have to have a serial number with two identical digits: 00, 11, 22, 33, 44, and so on. There are a total of 10 of these serial numbers out of the 100 possible 2-digit numbers. If there were four digits, the serial numbers would range from 0000 to 9999. Think of the four digits as two groups of two. If the last two digits were 01, the number of the radar bill would be 1001. How many radar bills would there be altogether for four-digit serial numbers?

Complete Solution:

Since there are eight digits, the serial numbers can range from 00000000 to 99999999. In other words, there are 100,000,000 possible serial numbers. Think of the eight digits as two groups of four. The last four digits of a radar bill serial number determine the entire number. For example, if the last four digits are 0001, then the entire number must be 10000001. Since the four digits that determine the radar bill serial number range from 0000 to 9999, there are 10,000 possible radar bill numbers. Since there are 10,000 possible radar bill numbers out of 100,000,000 serial numbers, you might expect 1 of every 10,000 bills to be a radar bill.

Another way to think about this is to look at the following pattern:

Number of Digits in Bill	Number of Radar Bills
2	10
4	100
6	1000
8	10,000

Following this pattern, there could be 10,000 radar bills that have eight digits.

Try This:

- Look at some dollar bills to see if you can find a radar bill.
- Words or phrases that read the same forwards and backwards are called palindromes. Think of at least five words besides radar that are palindromes.

Additional Challenges:

(Answers located in back of booklet)

1. Suppose that the serial numbers on a bank note contained nine digits. How many radar bills would be possible?
2. In the United States, all telephone numbers in a given area code have seven digits. If there were no restrictions on the digits, how many possible "radar" telephone numbers would there be?
3. Is a serial number more likely to begin with a 0 or a 1?

Things to Think About:

- Does knowing any four digits of a radar serial number let you determine the entire number?
- What is the significance of the capital letters at the beginning and end of each serial number on a US bill?
- Why does a star follow some serial numbers on a US bill?

Did You Know That?

- US currency is printed in sheets of 32 bills organized in an 8 x 4 arrangement. The last two digits of the serial numbers are the same for all 32 bills on the sheet. One hundred of these sheets are stacked, cut to size by a guillotine, then bundled.
- The newer versions of US currency have two letters in front of the serial number and one behind it.
- Some collectors specialize in collecting radar notes, notes with many 7's in the serial number, or notes with the same serial numbers from different Federal Reserve Banks.
- In the year 1999, the US government began issuing paper money that was less likely to be counterfeited. The bigger picture is one of the reasons. There is a hologram behind the picture.

Resources:
Books:

- Blocksma, M. *Reading by the Numbers: A Survival Guide to the Measurements, Numbers, and Sizes Encountered in Everyday Life.* New York: Viking Penguin, 1989.

Websites:

- **www.strawberries.com/home.html**
- **www.ping.be/~ping6758/index.shtml**
- **www.bep.treas.gov/allfacts.htm**
- **www.jakesmp.com/Specials_part_013.htm**
- **www.drbanks.com/currency/glossary.html**

Math Challenge #42 ROSE BOWL

Figure This! Math Challenges for Families

Can a football team score 11 points in a game?

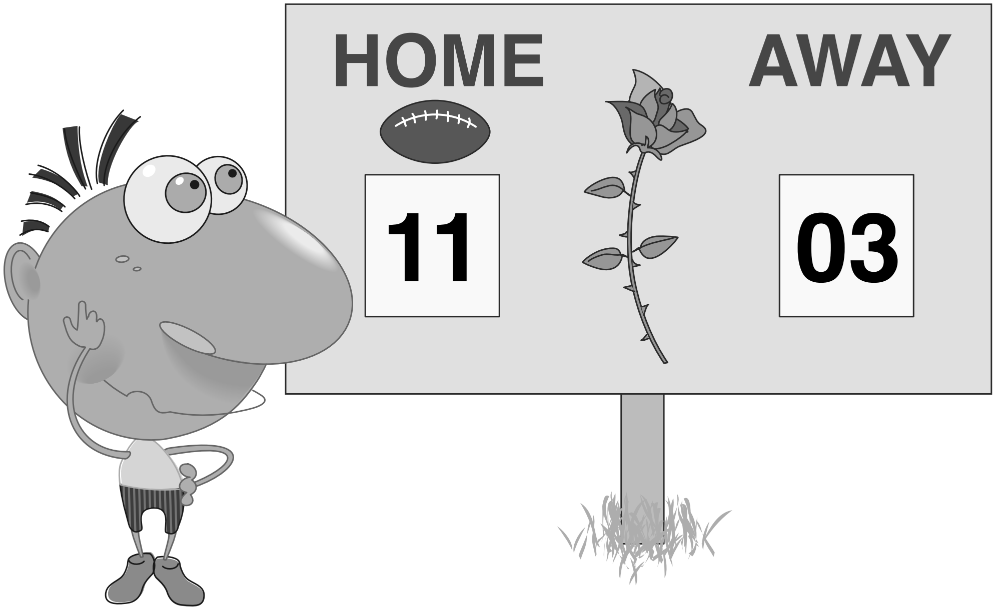


Figure This! In the history of college football's Rose Bowl, no team's final score has ever been 11 points. How many different ways are there for a team to score 11 points?

Hint: In American football, a team may score points in the following ways:

8 points (touchdown and 2-point conversion)

7 points (touchdown and 1-point conversion)

6 points (touchdown and no conversion)

3 points (field goal)

2 points (safety)

Making a list or a table is a method of organization used in problem solving and in prioritizing work. People in business and industry use this strategy in their jobs as well as to simplify daily chores.

Answer:
There are 5 different ways to score 11 points.

Figure This!

Get Started:

What would have to be scored with a touchdown and a 2-point conversion (8 points) to produce 11 points? A table can help you make sure that every possibility has been considered, and that no case has been counted more than once.

Possible Scores

	8 pts	7 pts	6 pts	3 pts	2 pts	Total Points
Possible Combinations of Points	1	0	0	1	0	8 +3 = 11

Complete Solution:

The table below shows the five different ways to score exactly 11 points.

Possible Scores

	8 pts	7 pts	6 pts	3 pts	2 pts	Total Points
Ways	1	0	0	1	0	8 + 3 = 11
Score	0	1	0	0	2	7 + 2 x 2 = 11
Could	0	0	1	1	1	6 + 3 + 2 = 11
Be	0	0	0	3	1	3 x 3 + 2 = 11
Made	0	0	0	1	4	3 + 4 x 2 = 11

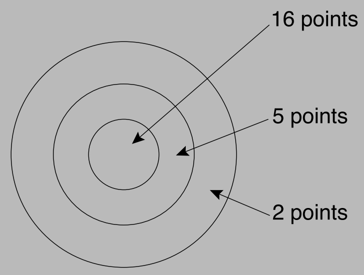
Try This:

- Search newspapers, almanacs, magazines, or websites for the final scores of football games. How often did you find a final score of 11 points?

Additional Challenges:

(Answers located in back of booklet)

1. In the nearly 100-year history of the Rose Bowl, there were six occasions when a team's final score was 10 points. In how many ways can 10 points be scored?
2. Are there any point totals that cannot be made in American football?
3. If you land four darts in this dart board, what scores are possible?



4. Six is called a "perfect number" because the sum of all its factors is twice itself, or 12. What is the next perfect number?

Things to think about:

- What total number of points, other than 11, might be rare in a football game?
- Which is more common, a field goal or a touchdown?
- What final score do you think has occurred most often?

Did You Know That?

- The first Rose Bowl was played in 1902. The final score was Michigan 49 and Stanford 0.
- The most points ever scored in the Rose Bowl was 49 by Michigan in 1902 and again in 1948.
- In 18 Rose Bowl games, one of the teams had a final score of 0. Seventeen of these shutouts occurred prior to 1954.

Resources:

Books:

- *The World Almanac and Book of Facts 2000.* Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.Rosebowl.org**
- **www.ncaa.org**

Math Challenge #43 SEEING STARS

Figure This! Math Challenges for Families

Can you **draw** a picture of the stars on an American flag?

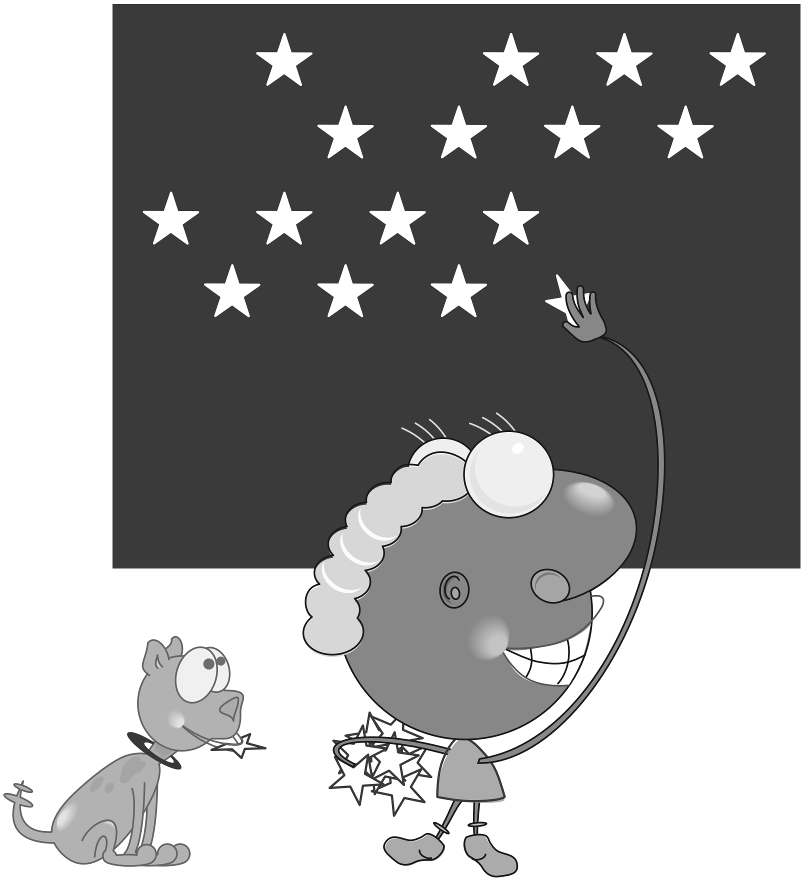


Figure This! **The American flag has 50 stars, one for each state. The rows are of two different lengths. Each row has one more star or one fewer star than the row next to it. Use these clues to figure out how the stars are arranged.**

Hint: Select a number of stars for one row; then use the information given to test some possible patterns.

Mathematics has been defined as the study of patterns. Biologists, geologists, architects, designers, and computer scientists all use patterns in their work.

Answer:
Using the clues, there are six possible arrangements. The actual US flag has 9 rows of stars: 4 rows of 5 stars, and 5 rows of 6 stars.

Figure This!

Get Started:

By studying the clues, you know that if there are 2 stars in the first row, one possibility is that there are 3 stars in the next row, 2 stars in the following row, and so on. That pattern could look like this:

2 + 3 + 2 + 3 + …

Using this pattern, is it possible to reach a sum of 50 stars?
How about 3 + 4 + 3 + 4 + … ?

Complete Solution:

- One way to approach this problem is to test all the possible patterns. Knowing that the numbers of stars in alternating rows differ by 1, determine which patterns allow an arrangement of 50 stars. For rows of 2 and 3 stars, you could have:

  2 + 3 + 2 + 3 +…+ 2 + 3 = 50.

  Ten rows of 2 stars and 10 rows of three stars equal 50 stars.

  With rows of 3 stars and 4 stars, you have

  3 + 4 + 3 + 4 +…+ 3 + 4 = 49

  The closest that you can get to 50 is 49, and 49 plus another row of either 3 or 4 stars will not make 50 stars. So rows of 3 and 4 will not work. Continuing to test possibilities in this way, you should find that there are six solutions that satisfy the clues given in the challenge. The actual American flag has 4 rows of 5 stars and 5 rows of 6 stars.

No. of Stars in Row	No. of Rows	No. of Stars in Next Row	No. of Rows	Total No. of Stars
1	16	2	17	1x16 + 2x17 = 50
2	10	3	10	2x10 + 3x10 = 50
4	5	5	6	4x5 + 5x6 = 50
5	4	6	5	5x4 + 6x5 = 50
12	2	13	2	12x2 +13x2 = 50
16	1	17	2	16x1 + 17x2 = 50

- Another way to think about this problem is to consider the sums of the pairs of numbers in which one number is 1 more than the other. Using 2 and 3, for example, the sum is 5. Since 10 sets of 5 make 50, there could be 10 rows of 2 stars and 10 rows of 3 stars. Using 3 and 4, the sum is 7. Seven sets of 7 is 49, and neither 3 nor 4 can be added to 49 to get 50. Therefore rows of 3 and 4 will not work. This process can be continued to find the rest of the possible solutions.

Try This:

- How would you arrange the stars if the United States included 51 states?
- Look up flags of different countries in a dictionary, an atlas, or on the Internet. What patterns can you find?
- Draw each of the possible flags from the challenge. Which do you like the most?

Additional Challenges:

(Answers located in back of booklet)

1. How could the 50 stars be arranged in 5 rows so that every row had one more star than the one before it?
2. What is the least positive number such that when you divide by 2, the remainder is 1; when you divide by 3, the remainder is 2; when you divide by 4, the remainder is 3; and when you divide by 5, the remainder is 4?
3. A band director found that if the band members lined up two at a time, three at a time, four, five, or even six at a time, there was always one person left over. However, if they lined up seven at a time, no one was left over. If there were fewer than 500 students in the band, how big was the band?

Things to Think About:

- Are there some basic patterns in flags that occur over and over again?
- Who decides what each new flag should look like?
- The original US flag had 13 stars arranged in a circle, along with 13 stripes. In 1795, when Kentucky and Vermont were added to the original 13 US states, the flag featured 15 stars and 15 stripes. When more states were added, however, the designers returned to 13 stripes. Why do you think this happened?

Did You Know That?

- While making the pattern for the first American flag, Betsy Ross was reportedly able to create a 5-pointed star from a single sheet of paper with one cut.
- June 14 is Flag Day, commemorating the adoption of the US flag by the Continental Congress in 1777.
- The first US flag had 13 stars, while the second had 15 stars. Since states sometimes entered the union in groups, no US flags had 14, 16, 17, 18, 19, 22, 39, 40, 41, 42, or 47 stars.
- New US flags can be introduced only on the Fourth of July.
- The "star-spangled banner" described in the US national anthem is the flag with 15 stars and 15 stripes.
- The state flag for Hawaii is the only state flag that includes the flag of a foreign country.
- Not all state flags are rectangular.

Resources:

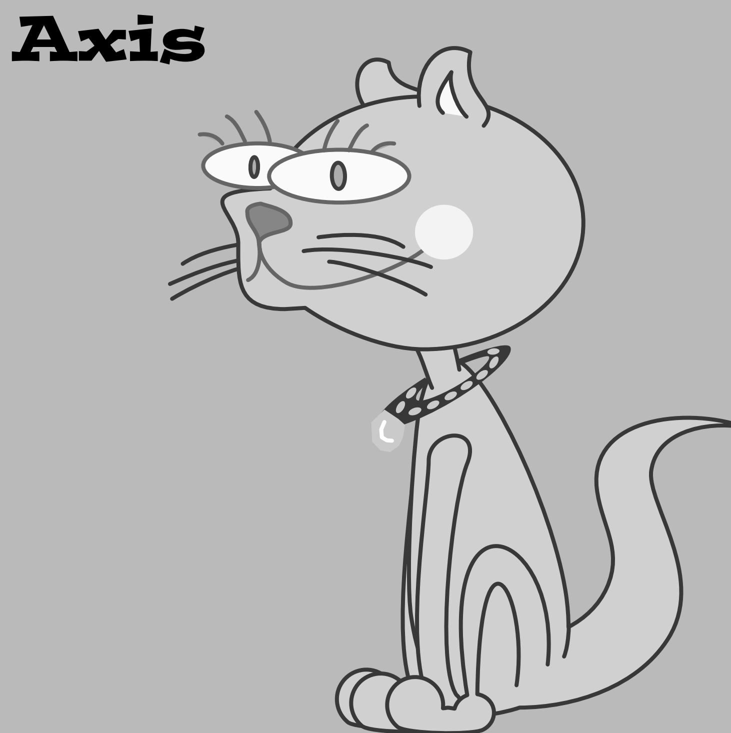
Books:

- "New Stars for Old Glory." *National Geographic*, July, 1959.
- Olson, A. *Mathematics Through Paper Folding*. Reston, VA: National Council of Teachers of Mathematics, 1987.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.crwflags.com/fotw/flags/us-1777.html**
- **www.ushistory.org/betsy/flagtale.html**
- **www.nationalgeographic.com/ngm/archive/index.html**

Notes:



Math Challenge #44

TABLE FOR 19

Figure This! Math Challenges for Families

How much room do you need at a table?

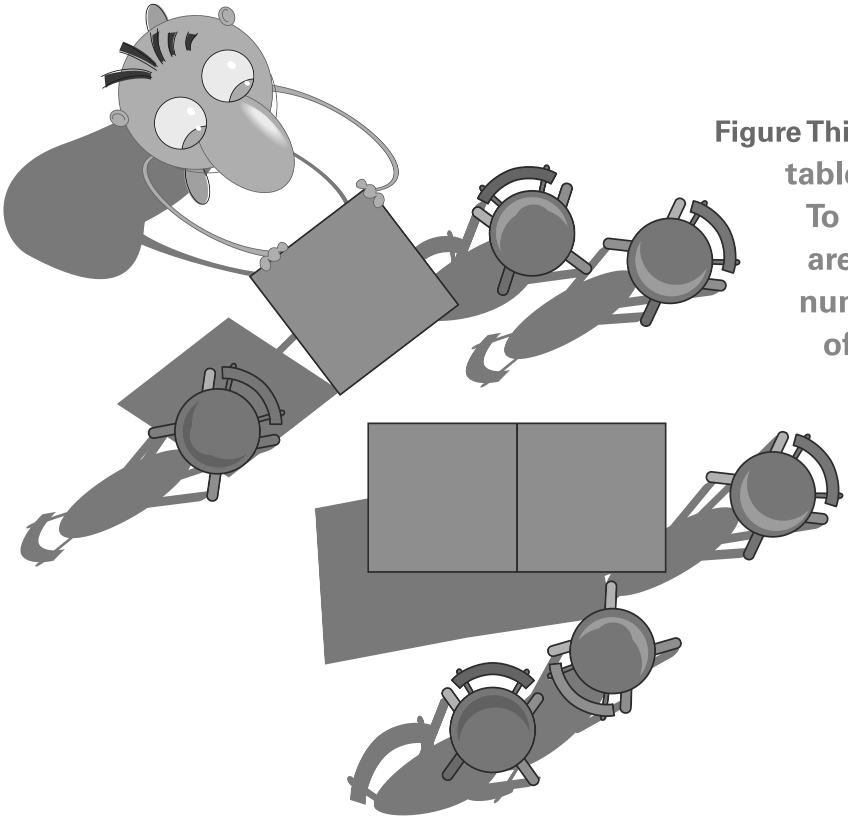


Figure This! **Polygon's Restaurant has square tables that seat one person on each side. To seat larger parties, two or more tables are pushed together. What is the least number of tables needed to seat a party of 19 people who want to sit together?**

Hint: How many people could sit at two tables pushed together? How many could sit at three tables pushed together?

Finding patterns and arranging geometric shapes are used by architects, landscapers, quiltmakers, and carpet layers in their work.

Answer:
9 tables.

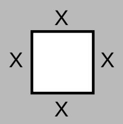
Figure This!

Get Started:

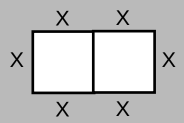
Use squares of paper (or square crackers) to represent the tables and model ways to seat people. Start with one square and see how many people can be seated. If you join two squares together, what happens to the number of seats?

Complete Solution:

- There are several ways to solve this problem. Using the hint, one table can seat four people.



Adding another table takes away one place and adds three places for a net gain of two seats.



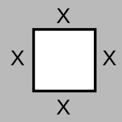
Reasoning in this way, when an additional table is added, one seat is lost and three are gained for a net gain of two. Continuing for three, four, and so on, at least nine tables are required to seat 19 people. There are many different possible arrangements of the tables.

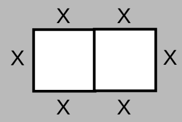
- Another way is to make a chart and look for patterns.

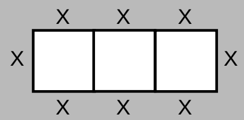
Number of Tables	Number of People Seated
1	4
2	6
3	8
⋮	⋮

The pattern indicates an increase of two seats each time. Continuing the pattern, nine tables will seat 20 people but eight will only seat 18. Thus, nine tables are required for 19 people.

- Thinking geometrically leads to a general rule for the seating pattern. For every table arrangement, you can always seat one "at each end" with as many people on each side as there are tables. Examples follow:







With the two people at the ends and twice the number of people as there are tables seated at the sides, a general rule for *n* tables would allow seating a maximum of $2 + 2n$ people.

For 19 people,

$$2n + 2 \geq 19$$

$$2n \geq 17$$

$$n \geq 8.5$$

This means that at least nine tables must be used.

Additional Challenges:

(Answers located in back of booklet)

1. What is the maximum number of people who can be seated at seven tables put together?
2. If every seat is filled, what is the least number of people that can be seated in an arrangement of nine tables?
3. Think about the picture created by adding tables. Find a different version of the general rule found in the Complete Solution of the Challenge.

Things to Think About:

- If you were a waiter, would it be easier to seat a large group at one big table or two smaller tables? How about serving them?
- Why do some restaurants use round tables?
- How is the computer game Tetris™ related to this challenge?

Did You Know That?

- Arrangements of squares are called polynominos. Dominos are polynominos with two squares.
- According to *The Guinness Book of Records*, the greatest number of people simultaneously participating in a toast was 78,276 on February 27, 1998 in the United States.
- Fred Magel of Chicago, IL dined out 46,000 times in 50 years while rating the quality of restaurants.

Resources:

Books:

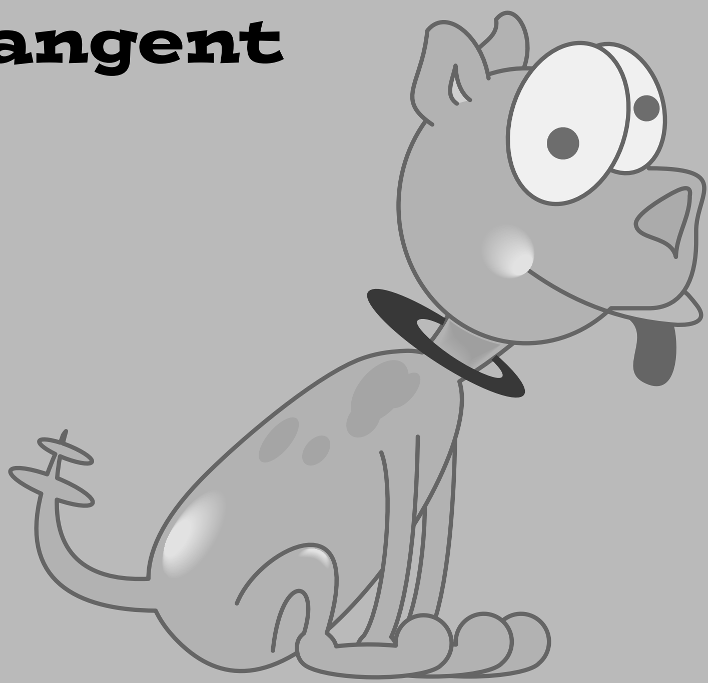
- Burns, M. *Spaghetti and Meatballs for All!* New York: Scholastic, Inc. 1998.
- *The Guinness Book of Records, 2000*. New York: Guinness Publishing, Ltd., 1999.

Websites:

gallery.uunet.be/luxil/2dtetris.htm

Notes:

Tangent



Math Challenge

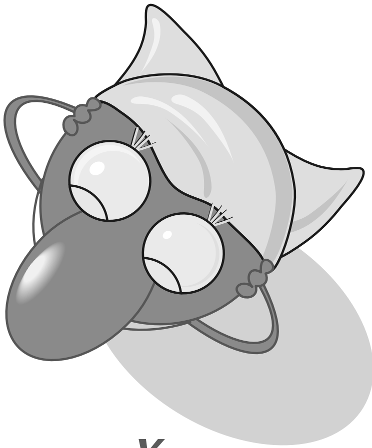
#45

THREE SQUARES

Figure This!

Math Challenges for Families

When do **two** squares make a **new** square?



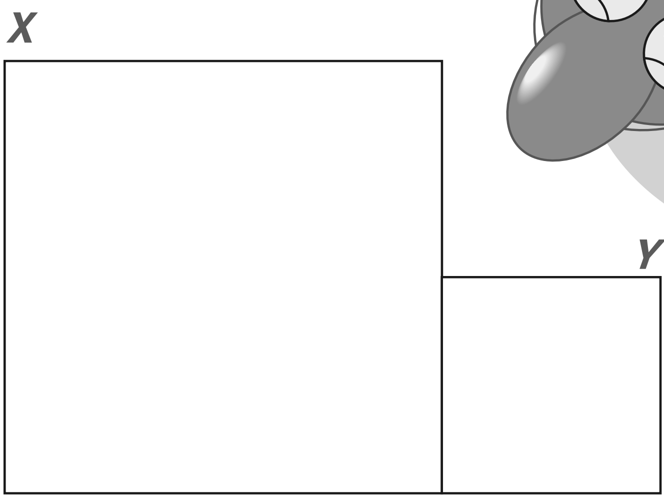


Figure This! Can you make a new square from two squares?

Hint: Cut two squares from a sheet of paper and tape them together as in the diagram. Find a point *Z* along the bottom of the two squares so that angle *XZY* is a right angle. Then use a pair of scissors.

The Pythagorean Theorem states that for any right triangle, $a^2 + b^2 = c^2$, where c is the length of the longest side, and a and b are the lengths of the other two sides. This relationship is often used to find the distance between two points and is fundamental in construction, engineering, and the sciences.

Answer:

Draw segments *XZ* and *YZ* as in the picture. The three pieces can be formed into a square as shown. Any two squares can be used to make a third square in this manner.

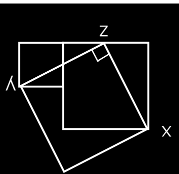


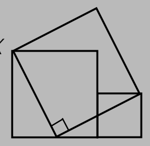
Figure This!

Get Started:

To locate point Z as described in the hint, place another sheet of paper on top of the two squares. Place one corner of the sheet on the bottom of the two squares. Slide the paper until one edge touches point X and an adjacent edge touches point Y. Cut the triangle pieces off and try to form a square by re-arranging all pieces.

Complete Solution:

The Hint and the Answer suggest using a piece of paper to find point Z and construct a square. (The edges of the paper will lie on two sides of the new square. Carpenters would use a carpenter's square for finding this point.)



A different way to find point Z is to mark off the length of a side of the small square along the bottom of the larger square starting at the left. This length locates point Z. Because the total length of the bottom is the sum of the lengths of a side of each square, Z also separates the bottom into two lengths that are the lengths of the sides of the squares. Draw segments XZ and YZ. The two right triangles are the same size and shape because each has a right angle and the two smaller sides are each a length of the original squares. Because the triangles are the same size and shape, XZ and YZ are the same length. They become sides of the new square. Using the angles of the triangles along the base, angle XZY can be shown to be a right angle.

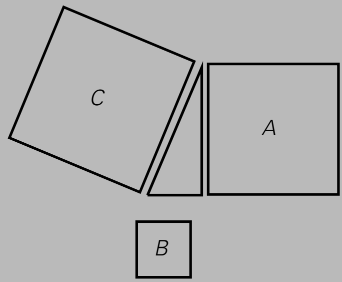
Try This:

- Make a square with 3-inch sides and another square with 4-inch sides. Use these two squares to make a new square. How long is each side of the new square?
- Show that segment XY in the Challenge is the diameter of a circle that passes through point Z.

Additional Challenges:

(Answers located in back of booklet)

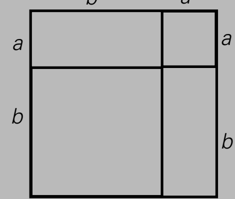
1. A Pythagorean triple is a set of three counting numbers a, b, and c so that $a^2 + b^2 = c^2$. For example, the numbers 3, 4, and 5 make a Pythagorean triple because $3^2 + 4^2 = 5^2$, or $9 + 16 = 25$. Do the numbers 6, 8, and 10 make a Pythagorean triple?
2. Find a different Pythagorean triple that contains 5.
3. Find a Pythagorean triple in which 7 is the smallest number and the larger numbers differ by 1.
4. Can all of the numbers in a Pythagorean triple be odd?
5. This diagram shows a right triangle and three squares, A, B, and C. The sides of each square have the same length as one of the sides of the triangle.



Make a larger square using square C and four copies of the right triangle. Make another large square from squares A and B and four copies of that same right triangle. Use the new square to show that the sum of the areas of squares A and B equals the area of square C.

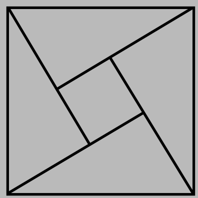
Things to Think About:

- Ancient Egyptians were able to create right angles for building and surveying using a rope with 12 equally-spaced knots to form a right triangle.
- Any triangle can be cut and reformed into a rectangle.
- The two smaller numbers of a Pythagorean triple cannot both be odd.
- Any multiple of a Pythagorean triple is also a Pythagorean triple.
- The Pythagorean Theorem holds for any numbers a, b, and c that form the sides of a right triangle.
- If two right triangles have corresponding sides the same length, the triangles have to be the same size and shape (congruent).
- This diagram shows that $(a + b)^2 = a^2 + 2ab + b^2$



Did You Know That?

- Pythagoras was a Greek mathematician (about 580–500 BC) who formed a secret society based partly on mathematical discoveries. The motto of his followers is said to have been "All is number."
- What is called the Pythagorean Theorem was known to the Egyptians as early as 2000 BC and to the Babylonians in 1700 BC.
- The Hindu mathematician Bhaskara (about 1114-1185) proved the Pythagorean Theorem simply by drawing this picture and saying "Behold!"



- Lewis Carroll, author of *Alice in Wonderland*, and US President James Garfield wrote proofs of the Pythagorean Theorem.
- French mathematician Pierre Fermat (1601–1665) built on the Pythagorean Theorem by proving that no cube is the sum of two cubes and no fourth power is the sum of two fourth powers. He believed that the same was true for all whole-number powers greater than 2. He wrote in the margin of a notebook that he had a proof of his belief, but the margin was too small for the proof. Only in 1995, more than 300 years later, did British-born mathematician Andrew Wiles find the key that proved Fermat correct.
- Choose any two different positive counting numbers a and b with a the bigger of the two. If $x = 2ab$, $y = a^2 - b^2$ and $z = a^2 + b^2$, then x, y, and z form a Pythagorean triple.

Resources:
Books:

- Bell, E. *The Last Problem*. Washington, DC: Mathematical Association of America, 1990.
- Loomis, E. *The Pythagorean Proposition*. Reston, VA: The National Council of Teachers of Mathematics, 1940.

Websites:

- **cut-the-knot.com/pythagoras/index.html**
- **www.geom.umn.edu/-demo5337**

Notes:

Axis

Math Challenge #46 TIME ZONES

FigureThis! Math Challenges for Families

It's afternoon in San Francisco, what time is it in Cairo, Egypt?

Standard Time Zones of the World

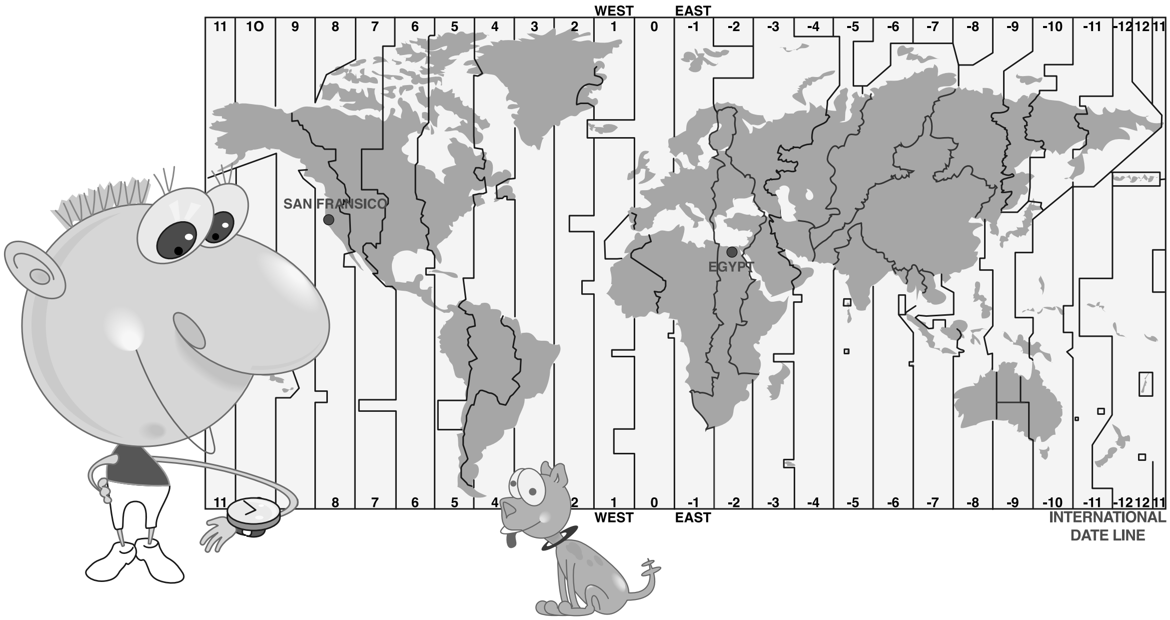


Figure This! After Helix gets home from school in San Francisco, can he call his grandfather in Cairo, Egypt, and expect to find him awake?

Hint: Think about the number of time zones separating San Francisco and Cairo, Egypt.

Determining the time in different regions of the world involves the use of positive and negative values. A similar approach is sometimes used to describe profit and loss, or to describe changes in climate.

Answer:
There is a 10-hour time difference between San Francisco and Cairo, Egypt. If Helix called at 3:00 PM from California, it would be 1:00 AM the next day in Egypt. His grandfather might be asleep.

Figure This!

Get Started:

Find San Francisco, California and Cairo, Egypt on a world map with time zones marked. How many time zones separate the two cities? When school ends in San Francisco, what time is it in Cairo?

Complete Solution:

- Global time is calculated from an imaginary line, called the Prime Meridian that extends from the North Pole to the South Pole through Greenwich, England. Each time zone to the east of Greenwich is numbered with a negative (-) number; while each zone to the west is numbered with a positive (+) number. Cairo is at –2, which indicates it is two time zones east of Greenwich. San Francisco is at +8, which indicates it is 8 time zones west of Greenwich. This means that a total of 10 time zones separates the two cities. Therefore, 3:00 PM in San Francisco is 1:00 AM the next morning in Cairo.
- Another way to do this challenge follows: San Francisco is at +8 on the map. Cairo is at –2. Using mathematical operations, $+8 - (-2) = 10$. When it is 3:00 PM in San Francisco, it is 10 hours later in Cairo, or 1:00 AM the next morning.

Try This:

- Using a marking pen, draw the points on a basketball or a beachball that represent the North Pole, the South Pole, San Francisco, and Cairo. Locate Greenwich, England and draw a line from the North Pole to the South Pole through Greenwich. To represent each time zone, draw more lines from the North Pole to the South Pole at intervals of 15˚, until you reach Greenwich again. Count the number of time zones between San Francisco and Cairo.
- Look in a telephone directory for a chart of time zones. Use the chart to determine what the current time is in each time zone of the United States.

Additional Challenges:

(Answers located in back of booklet)

1. What would be a reasonable time to call Cairo from San Francisco?
2. The International Date Line is located halfway around the world from Greenwich, England, at about the 180th meridian. When crossing from the west, the date is advanced one day. When crossing from the east, the date is set back one day. If it is noon on January 1 in San Francisco, what is the date and time in Singapore?
3. The New York Stock Exchange is open from 9:30 AM to 4:00 PM Eastern Standard Time. What are the hours in the Pacific Time Zone when the New York Stock Exchange is open?
4. Are there places in the world where the difference in time zones is 20 hours or more?

Things to Think About:

- What does the earth's rotation have to do with time zones?
- Why is there an International Date Line?
- Why were the time zones established by using 15˚ angles?
- Why is there Daylight Saving Time?
- It is possible to leave one city and arrive in another city "before" you left the original city.

Did You Know That?

- The United States covers six time zones: Eastern, Central, Mountain, Pacific, Alaska, and Hawaii-Aleutian. US territories cover four more time zones. Russia covers 11 time zones.
- In order to accommodate local geography, the boundaries indicating actual time zones are not straight lines.
- Some US states, such as Indiana, are in two time zones.
- Two US territories, Guam and Wake Island, are on the other side of the International Date Line from North America.
- In the US, Daylight Saving Time, which first originated during World War I, begins at 2:00 AM on the first Sunday in April and ends at 2:00 AM on the last Sunday in October.
- Hawaii and Arizona do not observe Daylight Saving Time.
- A 24-hour clock is used by the military and most scientists. In the 24-hour time system, the hours are numbered from 0 to 23, and there are no AM and PM designations. With this time system, 0 o'clock is possible.
- A typical time zone differs from its neighboring zones by one hour, but some time zones differ by a fraction of an hour. For example, the time on the Canadian island of St. John's differs from that in the rest of Newfoundland by 30 minutes. The time in Nepal differs from neighboring India by 15 minutes.

Resources:

Books:

- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.askjeeves.com.**
- **www.cstv.to.cnr.it/toi/uk/timezone.html**
- **www.lib.utexas.edu/Libs/PCL/Map_collection/world_maps/World_Time_ref802649_1999.pdf**
- **time.greenwich2000.com**

Math Challenge
#47
VCR

Figure This!
Math Challenges for Families

Do you always get 6 hours of recording on a 6-hour tape?

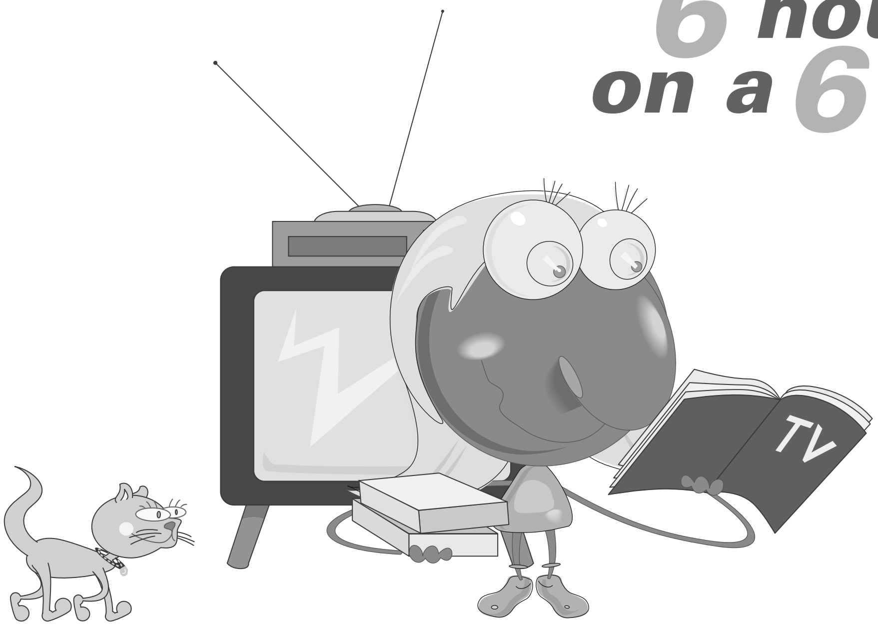


Figure This! Suppose the setting SP (standard play) on a VCR allows 2 hours of recording with an ordinary 120-minute tape. Changing the setting to EP (extended play) allows 6 hours of recording. After taping a 30-minute show on SP, the VCR is reset to EP. How many more 30-minute shows can be recorded on this tape?

Hint: What fraction of the tape was used to record the first show?

Proportional reasoning involves simple fractions, ratios, and probabilities. It is used by people comparing grocery prices, economists describing population densities, architects making scale models, and chemists determining the relative strengths of solutions.

Answer:
Nine more shows.

Figure This!

Get Started:

How many half-hour shows could be recorded on the SP setting? What part of the tape was left after the first half-hour show was recorded?

Complete Solution:

- You can consider this problem in terms of minutes or in units of half-hour shows. Using only the SP setting, the tape can record 120 minutes of shows. If one 30 minute show has already been recorded, 1/4 of the tape has been used.

Type of Setting	Minutes Used	Total Minutes Available on Tape	Part of Tape Used
SP	30	120	30/120, or 1/4
EP	(1/4) • 360, or 90	360	90/360 or 1/4

With 1/4 of the tape filled, 90 minutes of EP has been used, so that there are 360 – 90 or 270 min left. At 30 minutes per show, nine additional shows can be taped.

- If you are using half-hour units, there are four half–hour shows on SP in two hours. One half–hour show used 1/4 of the tape. Three-fourths of the tape is left. With EP, you can do 12 half–hour shows, and then you have enough tape for 3/4 • 12 or 9 shows remaining.
- Yet another way to consider this problem is to use proportions as follows:

$$\frac{30 \text{ minutes}}{120 \text{ minutes available on SP}} = \frac{? \text{ minutes}}{360 \text{ minutes available on EP}}$$

Because 3 • 120 = 360 and 3 • 30 = 90, there are 90 minutes used on the EP setting, so that there are 360 – 90, or 270 minutes available which allows for 9 thirty-minute shows on EP.

Similarly 30 • 360 = 120 • (number of minutes) so the number of minutes is 90.

Try This:

- Select a favorite half-hour TV show. Set your VCR to EP, then record only the program itself, editing out all the commercials and stopping the VCR as soon as the show ends. How much time is used for commercials? How many programs could you get on a 6 hour tape without commercials and credits?
- Select two different channels. Time the news and the amount of time for commercials on the channels. How do the proportions of commercials and news differ?
- Think about a movie that you have seen both in a theater and on television. Do the movies in the different settings take the same amount of time? How do the times compare with the same movie on a rental tape?
- What proportion of Saturday morning TV shows are cartoons?

Additional Challenges:

(Answers located in back of booklet)

1. Some VCRs have a third setting, LP, which allows 4 hours of recording on an ordinary 120-minute tape. Suppose you taped one 30-minute show on SP, changed the setting to LP, then taped another 30-minute show. If you then change the setting to EP, how many more 30-minute shows would fit on the tape?

2. The table shows average TV watching times, in hours and minutes, for women, men, and teenagers in May 1998 based on a random sample of the population. Of the time that each group spends watching TV (on average), which group spends the largest proportion on Saturday morning?

Group	Mon-Fri 10:00 AM-4:30 PM	Mon-Fri 4:30 PM-7:30 PM	Mon-Sun 8:00 PM-11:00 PM	Sat 7:00 AM-1:00 PM	Mon-Fri 11:30 PM-1:00 AM
Women (18 and older)	5 hr. 41 min.	3 hr. 47 min.	9 hr. 1 min.	42 min.	1 hr. 32 min.
Men (18 and older)	3 hr. 19 min.	2 hr. 47 min.	8 hr. 11 min.	35 min.	1 hr. 26 min.
Teenagers (12-17)	1 hr. 59 min.	2 hr. 56 min.	5 hr. 51 min.	42 min.	50 min.

3. Use the chart in Number 2. Of the time that each group spends watching TV (on average), which group spends the largest proportion watching primetime (Monday - Sunday from 8 PM to 11 PM)?

Things to Think About:

- Proportions can be helpful for determining the heights of objects that cannot be easily measured. For example, the approximate height of a flagpole, tree, or building can be determined using shadows and a known measure, such as your height.
- In 1997, 98% of US households owned at least one television set. This represented about 98 million homes. Of these, 84% also owned a VCR.

Did You Know That?

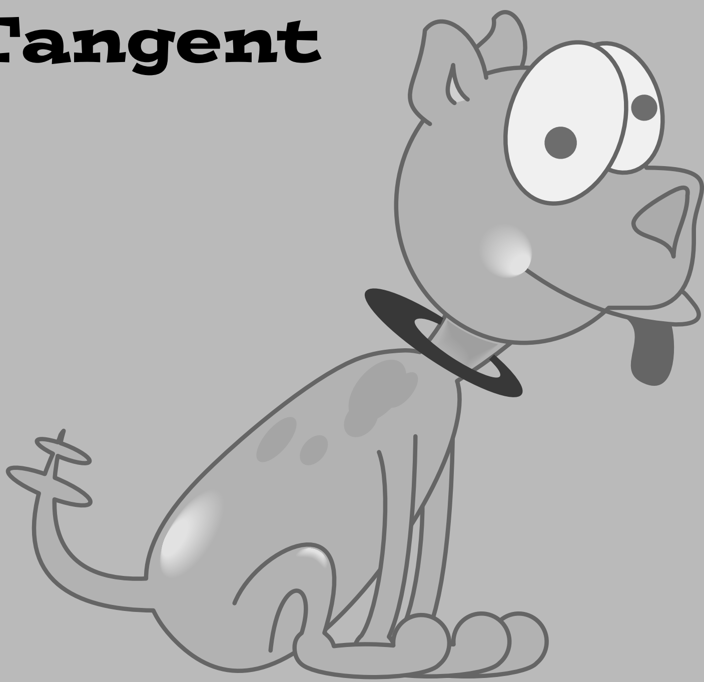
- Videocassettes are available in several different lengths, including one that can record up to 8 hours on EP.
- Videodiscs are less expensive to produce than cassettes, and offer superior sound, color, and picture quality.
- A camcorder combines a VCR and a video camera in one machine.

Resources:
Books:

- *Encyclopedia Americana, Vol. 28*, Grolier Inc., Danbury, Connecticut, 1998.
- *Collier's Encyclopedia, Vol. 23*, New York: Nuffield, Publications, 1997.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Notes:

Tangent



Math Challenge #48 WORKING HOURS

FigureThis! Math Challenges for Families

How much time do teens spend on the job?

Hours Worked by High School Seniors (per week)

Hours	Frequency
none	36%
1-5 hours	7%
6-10 hours	9%
11-15 hours	11%
16-20 hours	17%
21+ hours	21%

Source: National Center for Educational Statistics, National Assessment of Educational Progress. (Hours are rounded to the nearest whole number; percentages total more than 100 due to rounding.)

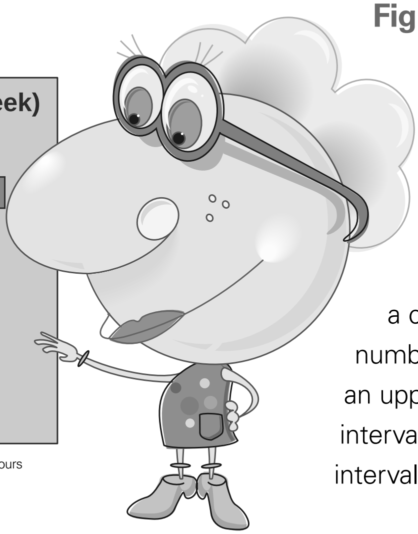


Figure This! What's the average number of hours high school seniors work per week?

Hint: Assume that there are 100 students in the survey data. The average (mean) is the total of all the hours a class of seniors worked divided by the total number of seniors. Statisticians usually assume an upper bound for each category that keeps the intervals the same width; in this case, the upper interval for the 21+ category could be 25.

An average is often used to summarize a set of numbers. Averages describe data on housing prices, wages, athletics, and academic performance.

Answer:

Between 9 and 11 hours per week depending on your approach to the problem.

Figure This!

Get Started:

If the data represent 100 students, how many students would fall in each category? How could you estimate the number of hours worked in each category? Making a chart may help you organize the information.

Complete Solution:

There are many ways to do this problem. The answer can only be approximated because of the way the data are reported.

- If you assume that the table shows data for 100 students, then the 7% who worked from one to five hours would correspond to seven students who worked one to five hours. To find an average (mean), you need to estimate the total number of hours worked. Consider the seven students who worked from one to five hours. Taken together, the least time they could have worked is seven hours. The greatest they could have worked is 35 hours. (The actual number is probably somewhere in between these values.) The first chart shows the least number of hours the 100 students could have worked, while the second chart shows the greatest number of hours they could have worked.

Least Number of Hours	Number of Students*	Least Number of Total Hours
0	36	0
1	7	7
6	9	54
11	11	121
16	17	272
21	21	441
Total	**100**	**895**

***Estimated**

Greatest Number of Hours	Number of Students*	Greatest Number of Total Hours
0	36	0
5	7	35
10	9	90
15	11	165
20	17	340
25*	21	525
Total	**100**	**1155**

***Estimated**

To find the lower value for the average number of hours worked per week, divide the total number of hours by the number of students: 895/100 = 8.95. The larger value for the average workweek can be found in the same way: 1155/100 = 11.55. Using this approach, an estimate for the average for a high school senior is between 8 1/2 hours and 12 1/2 hours.

- Another way to estimate the average uses an average for the hours worked in each category.

Smallest Number of Hours	Largest Number of Hours	Average Number of Hours	Number of Students*	Total Hours Worked
0	0	0	36	0
1	5	3	7	21
6	10	8	9	72
11	15	13	11	143
16	20	18	17	306
21	25*	23	21	483
		Total	**100**	**1025**

***Estimated**

To find a value for the average, divide the total number of hours by the number of students: 1025/100 = 10.25. This method results in an average of about 10 1/4 hours.

Try This:

- Survey some seniors in a high school near you to find how many hours per week they work. How do your results compare to the information in the challenge?
- Make up a set of numbers in which the median (or middle number) is the same as the mean or average.
- Make up a set of numbers in which the median is not the same as the mean or average.

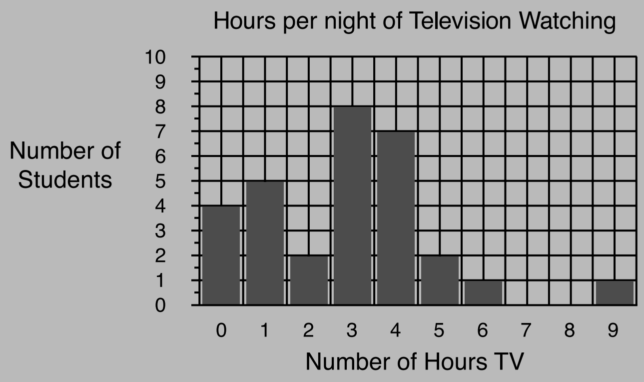
Additional Challenges:

(Answers located in back of booklet)

1. Would your estimate for the number of hours worked per week in the challenge change if you had used 200 students? 1000 students?
2. Which number should be omitted from the following set to result in an average of 20?

   12, 18, 8, 18, 21, 32, 25, 36, 25,10

3. This graph shows the results of a class survey of TV viewing habits. Use this data to find the average number of hours per night this class spent watching TV.



Things to Think About:

- Why is it important to use 0, representing those who said they did not work at all, in finding the average?
- Thirty-six percent of US high school seniors in the survey did not work at all, while 21% of them worked more than 21 hours per week.
- Is it possible for two very different sets of data to have the same mean?
- If there are one or more unusually large numbers with respect to the rest of the data in a data set, the mean may be much larger than seems typical. If there are one or more unusually small numbers in the set with respect to the rest of the data, the mean may be much smaller than seems typical.
- Why is it reasonable to use 25 as an upper bound for the 21+ category in the data of the challenge?

Did You Know That?

- In 1999, US teens watched television for an average of 12 1/4 hours per week (Nielson Media Research).
- The average earnings in 1998 for people 18 or older was $16,124 per year (without a high school diploma); $22,895 per year (with a high school diploma); and $40,478 per year (with a university degree). (US Census Bureau).
- On average, fast-food cooks make $6.29 an hour, service-station attendants make $7.34 an hour, machinists make $14.35 an hour, mail carriers make $16.39 an hour, and physical therapists make $27.49 an hour (US Bureau of Labor Statistics).
- In 1998, US production workers averaged 34.6 hours per week on the job. Their average hourly wage was $12.77; their average weekly earnings were $441.84 (US Bureau of Labor Statistics).
- In 1997, the US metropolitan area with the highest average annual salary was San Jose, California. Workers there earned an average of $48,702 per year (US Bureau of Labor Statistics).
- In 1998, men with a bachelor's degree earned an average of $50,272 per year, while women with a bachelor's degree earned $30,692 (US Census Bureau).
- On average, elephants in captivity live 40 years, guinea pigs 4 years, and opossums 1 year. Animals in the wild rarely live to their maximum potential life span.
- Whenever a set of data is summarized by an average or mean, a measure to indicate how the data are spread around the mean should be reported as well. One such measure is called the standard deviation. It describes a typical amount that the data values may differ from the mean.

Resources:

Books:

- *World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.
- *Current Population Reports.* Washington, DC: US Census Bureau, March 1998.

Websites:

National Center for Educational Statistics, National Assessment of Educational Progress:

- **nces.ed.gov/**

US Census Bureau:

- **www.census.gov/hhes/income/histine/p.16.html**

US Bureau of Labor Statistics:

- **stats.bls.gov/blshome.html**

Math Challenge
ANSWERS

Figure This!
Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

Figure This!

Answers to Additional Challenges:

Challenge 33:

1. The ramp would have to begin 7 feet farther away, or about 28 feet in all.

2. About 30.2 feet.

3. Yes.

Challenge 34:

1. About 175 gallons.

2. 70 miles per hour.

3. No, because the temperature is about 68˚ Fahrenheit.

Challenge 35:

1. Any shape with at least one pair of parallel sides, including squares and regular hexagons (the most common shape).

2. The bolts with the sides the same length would be easier, because you could use the wrench on any of three sets of parallel sides.

3. 10.

Challenge 36:

1. Approximately 14% are left-handed.

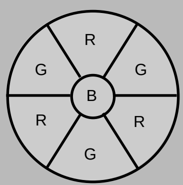
2. 55% liked neither soda.

3. Not necessarily.

4. No.

Challenge 37:

1. There are many possibilities. In the example shown here, the regions are colored red (R), blue (B), or green (G).



2. A city with five such neighborhoods will need at least one overpass. A city with six neighborhoods requires at least three overpasses.

Challenge 38:

1. Sunday.

2. Yes.

Challenge 39:

1. The perimeters are 58 m and 54 m.

2. The perimeter is 56 m.

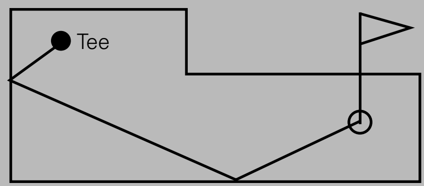
3. Since the arrangement of pieces in Helix's patio does not show a relationship between length and width, you cannot figure out the dimensions of a single piece. There are many pairs of numbers that multiply to make 20.

4. A square with a side length of 2 units.

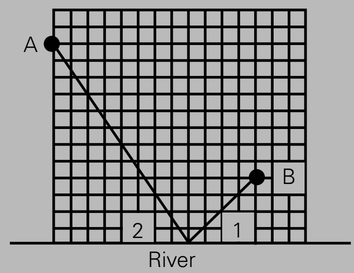
Challenge 40:

1. It wouldn't change.

2. One possible solution is shown here:



3. The sum of the distances is least when angles 1 and 2 in the diagram are equal. In this case, that distance is 20 miles.



Challenge 41:

1. 100,000.
2. 10,000.
3. Equally likely.

Challenge 42:

1. There are five different ways to score exactly 10 points, the same number of ways as for 11.
2. It is impossible to score 1 point. Any other number of points is possible.
3. 8, 11, 14, 17, 20, 22, 25, 28, 31, 36, 39, 42, 50, 53 or 64.
4. 28.

Challenge 43:

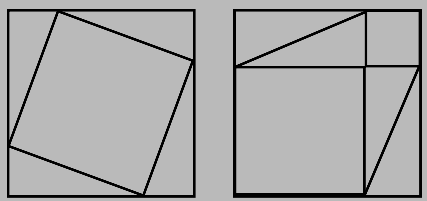
1. The stars could be in rows with 8, 9, 10, 11, and 12 stars.
2. 59
3. 301 students.

Challenge 44:

1. 16.
2. Arranging the tables in a 3 x 3 square leaves only 12 available seats.
3. Among the answers are $2 \bullet 3 + 2(n - 2)$ and $2(n + 1)$.

Challenge 45:

1. Yes, since $6^2 + 8^2 = 10^2$.
2. 5, 12, 13.
3. 7, 24, 25.
4. No.
5. The two large squares below have the same area.



Taking away the four right triangles from each large square shows that the remaining areas are equal.

Challenge 46:

1. Early in the morning or late at night. For example, 8:00 AM in San Francisco would be 6:00 PM in Cairo; 10:00 PM in San Francisco would be 8:00 AM in Cairo.
2. It is 4:00 AM on January 2.
3. 6:30 AM until 1:00 PM.
4. Yes. For example, the difference in time from Japan to Western Samoa is –20 hours.

Challenge 47:

1. 7 complete shows.
2. Teenagers (12 - 17).
3. Men (18 and older).

Challenge 48:

1. No. The estimated average would be the same in all three cases.
2. 25.
3. 2.87 hours per night.