Math Challenge #9

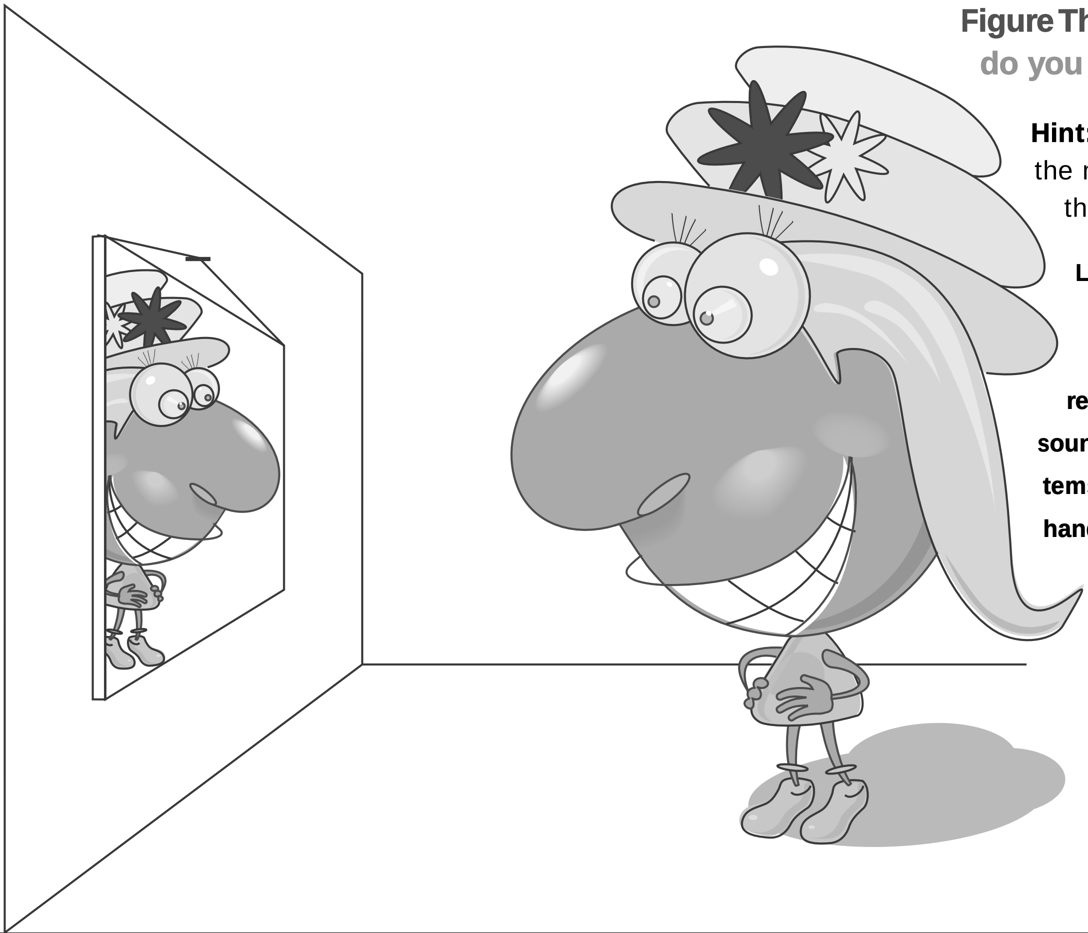
MIRROR, MIRROR...

Figure This!
Math Challenges for Families

Oh! mirror, mirror

"Mirror, mirror, what do I see? Does backing up show more of me?"

Figure This! How much of yourself do you see in a small mirror?



Hint: Begin by measuring the height of the mirror and the amount of yourself that you can see.

Looking in mirrors involves angles, reflections, lines of sight, and triangles. Understanding how these are related is important in the design of sound stages, theatres, and security systems. Such knowledge can also come in handy when playing billiards, racquetball, tennis and some video games.

Answer:
When you look in the mirror and then back up, you see exactly the same amount of yourself.

Figure This!

Get Started:

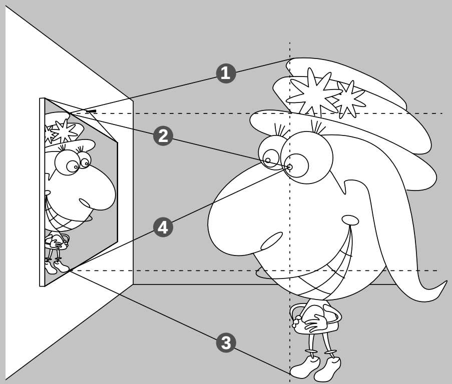
Start by standing about 2 feet from a wall. Have a friend hold a small mirror against the wall so that you can see only a portion of yourself; back up and see if you can see more of yourself.

Challenge:

Measure the height of the mirror. Measure how much of yourself you see in this mirror (the uppermost part to the lowermost part you see). Compare the measurements. Try other mirrors. Stand in different places. Draw a picture. Put the measurements in your picture. What do you notice?

Complete Solution:

The drawing shows that you see twice as much of yourself as the length of the mirror. Suppose that the mirror is hung flat against the wall so that the top of the mirror is halfway between your eye and the top of your hat. Lines 1 and 2 show the "line of sight" from your eye to the mirror and from the mirror to the top of your hat. Lines 3 and 4 show the "line of sight" from your eye to the mirror and from the mirror to your foot. The horizontal dotted lines show the heights of the mirror's top and bottom. The distance from the top of your hat to the dotted line indicating the top of the mirror is the same as the distance from this line to your eye. The distance from your eye to the dotted line indicating the bottom of the mirror is the same as that from this line to your foot. Thus, you can see twice the length of the mirror.



Try drawing the figure closer to the mirror or farther away. Although the angles change, the part of the body seen in the mirror is always twice the length of the mirror.

Try This:

- Use masking tape to tape one end of each of three long strings to the bottom of the mirror.

  Back up from the mirror and hold one string up to your eye. Have your friend hold a second string horizontal to the floor and tape it to your body where it hits. Standing straight up, look in the bottom of the mirror and have your friend tape the third string to the lowest point on yourself that you can see. (Leaning forward can affect the outcome so try to stand straight up.)

  Have your friend carefully measure the distance from your eye to the second string and from the second string to the third string. Are the measurements close to being the same? (They should be!)

  Back up and try it again. Did these measurements change?

Additional Challenges:

1. If a person can see his entire body in a 42 in. mirror, how tall is he?
2. If you had a 42-inch mirror, where would you place it on the wall to see your whole body?
3. If you want to be able to see yourself head-to-toe in a mirror, how big does the mirror have to be and where on the wall should you hang it?

Things to think about:

- Many fun houses and shopping malls have curved mirrors. What do you see when you look in such a mirror? What happens when you step back from this mirror?
- What type of mirrors make you look larger? smaller?
- Suppose a friend is directly behind you, standing still, when you look in a mirror. When you move back and forth, do you see more or less of your friend?
- If you stand in front of a counter looking at a mirror and back up, you see more of yourself. Why?

Did You Know That?

- According to the BBC World News, the Russian space project, Znamya, has been trying to place a large mirror in space in order to reflect the sunlight down on northern cities.
- The tallest human on record was 8 ft 11.1 in. tall. Robert Pershing Wadlow (1918–1940) would have needed a 54 inch mirror to see himself from head to toe.
- A reflection is an isometry, a geometric motion that preserves distance.

Resources:

Books:

- Desoe, Carol. *Activities for Reflect-It™ Hinged Mirror*. White Plains, NY: Cuisenaire Company of America, 1994.
- Walter, Marion. *The Mirror Puzzle Book*. Jersey City, NJ: Parkwest Publications, Inc., 1985.

Film:

- Harvard-Smithsonian Center for Astrophysics. *A Private Universe*. Washington, DC: Annenberg/CPB Math and Science Collection, 1987. [Materials include a 20-minute videocassette and *A Private Universe Teacher's Guide*.] **www.learner.org**

Website:

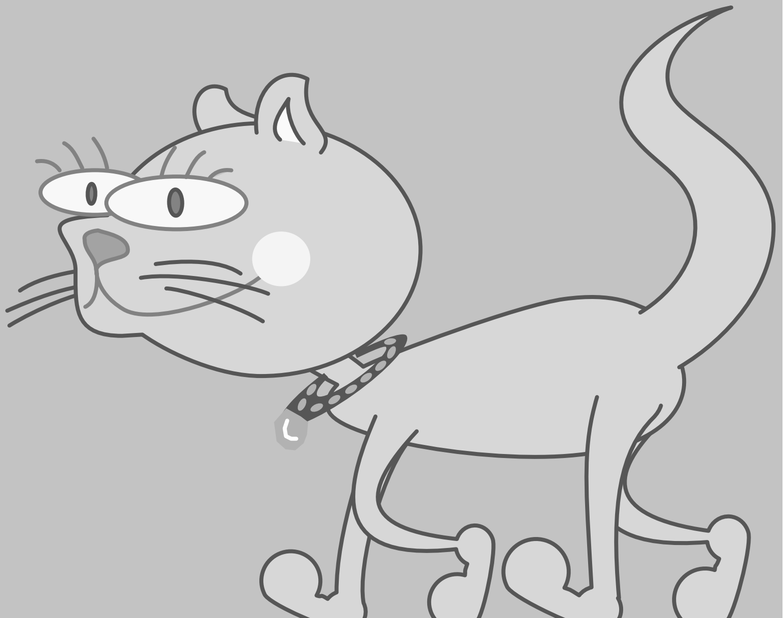
- Link to article on the Russian space mirror project, Znamya 2.5: **news.bbc.co.uk/hi/english/sci/tech/newsid_272000/272103.stm**

Answers to Additional Challenges:

(1.) What you see in the mirror is twice as long as the mirror. A person 84 inches tall or less can see all of himself in such a mirror.

(2.) This depends on your height. The top of the mirror should be at least 2 to 3 inches above your eye level so you can see the top of your head. Unless you are over 84 inches tall, you will then be able to see your feet.

(3.) (See the first Additional Challenge.) The length of the mirror depends on your height. You should hang it so that the bottom of the mirror is halfway between your feet and eyes.



Math Challenge #10 WHAT'S MY ANGLE?

Figure This! Math Challenges for Families

SPREAD out your fingers

and look at your hand. Do people with big hands have larger angles between their fingers?

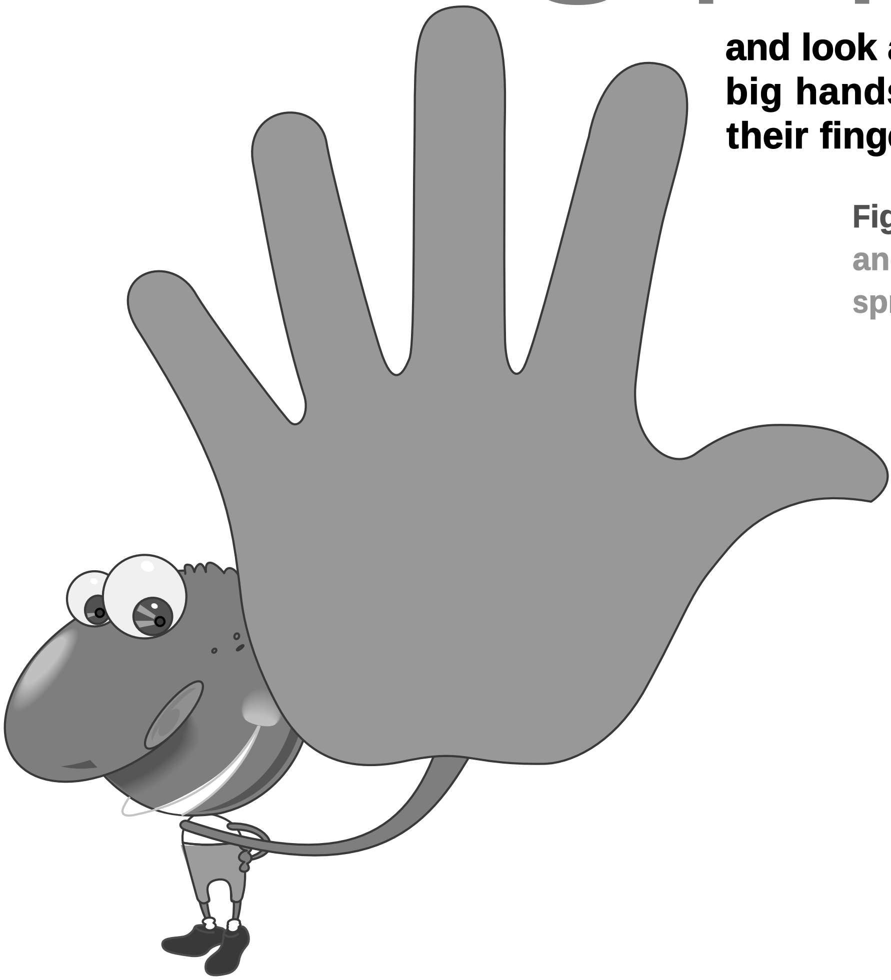


Figure This! Estimate the measures of the angles between your fingers when you spread out your hand.

Hint: When you hold your hand so that your thumb and index finger form an "L," the angle formed measures about 90°.

Angles are important geometric shapes. They are used in designing many things from airplanes to golf clubs.

Answer:

The size of your hand does not make any difference in the size of the angles.

The measures are approximately 90°, 45°, 20°, and 20°.

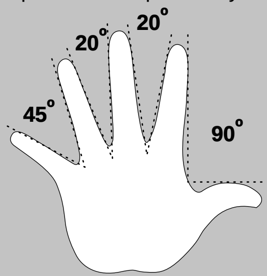
Figure This!

Get Started:

The angle measure between your thumb and index finger when your hand is shaped like an " L " is about 90°. How do the angle measures between your other fingers compare to this angle measure? Is the angle measure larger or smaller? Half as big? Draw the 90° angle between your thumb and index finger. Sketch an angle with half that measure. Use your sketch to estimate the measures of the other angles.

Complete Solution:

The largest angle, between the thumb and index finger, measures about 90°. (A 90° angle is a right angle.) The other angle measures vary somewhat depending on the person. One possibility is shown below:

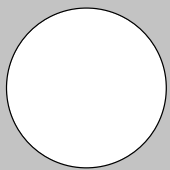


The lengths of the fingers do not affect the angle measures.

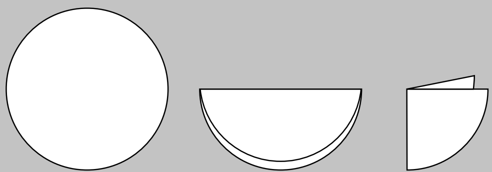
Try This:

- Measure an angle by making your own angle measurer or protractor using the following steps.

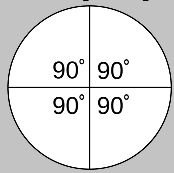
1. Cut out a circular piece of waxed or other lightweight paper. (You may want to trace the bottom of a large cup.)



2. Fold the circle in half, and then fold the result in half.



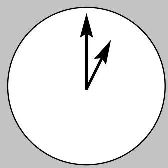
3. Unfold the paper and open the circle. You should see four right angles at the center of the circle. Each right angle has measure 90°.



4. Refold the circle on the same crease marks. Then fold the paper in half one more time. Unfold the paper to reveal eight angles, each of which measures 45°.
5. Refold the paper on the previous creases. Then fold in half one more time. How large is each new angle? Label each new angle with the appropriate measure.
6. To measure an angle with your protractor, place the center of the circle on the point of the angle and line up one of the creases with a side of the angle. Estimate the angle's measure by finding where the other side aligns on the protractor.

Additional Challenges:

1. Suppose a bicycle wheel turns around exactly once. This is a 360° revolution. How far would the bicycle wheel have moved on the ground?
2. On a compass, north has a heading of 0°. What is your heading when you are going east, south, or west?
3. What is the measure of the angle between the hands on a clock if one hand is on the number 12 and the other is on the number 1?



4. What do you think a negative angle might be?

Things to Think About:

- Why do you think Tiger Woods worries about the angle at which he hits a golf ball?
- What angles are important in designing a bicycle? Why?
- Many chairs recline or lean back. What kind of angles can you make in a reclining chair or in the driver's seat of a car?
- Angles are formed by tree branches and in the veins of a leaf. How big are these angles?
- What types of angles can you find on a cereal box?

Did You Know That?

- The legs and upper body of an astronaut floating in space form a natural angle of about 128°.
- People recovering from knee surgery track their progress by achieving flexibility to a certain angle measure.
- To measure angles between joints, orthopedic surgeons and physical therapists, use an instrument called a goniometer.
- Angle measure is related to the number system of the ancient Babylonians.
- Angles can be measured in radians and grads as well as degrees.

Resources:

Books:

- Desoe, Carol. *Activities for Reflect-It ™ Hinged Mirror.* White Plains, NY: Cuisenaire Company of America, 1994. **www.cuisenaire-dsp.com**
- Greenes, Carole, Linda Schulman-Dacey, and Rika Spungin. *Geometry and Measurement*. White Plains, NY: Dale Seymour Publications, 1999. **www.cuisenaire-dsp.com**
- Page, David A., Philip Wagreich, and Kathryn Chval. *Maneuvers with Angles*. White Plains, NY: Dale Seymour Publications 1993. **www.cuisenaire-dsp.com**

Answers to Additional Challenges:

(1.) Answer:
The answer depends on the size of the wheel. You can find this distance by conducting an experiment. Mark the tire at a point where it touches the ground and mark the point on the ground. Roll the bicycle ahead until the mark is once more on the ground and mark the distance it traveled. Measure the distance between the two marks on the ground. Another way to find the answer is to measure the diameter of the wheel, then use the fact that the distance around a circle is its circumference. Circumference of a circle = π • diameter where π is about 3.14.

(2.) Answer:
90°, 180°, and 270°, respectively.

(3.) Answer:
The angle has measure 30° because 12 angles of 30° each make 360°. Each of the other similarly formed angles also measure 30°.

(4.) Answer:
If positive angles are measured in one direction (for example, counterclockwise), negative angles would be measured in the opposite direction.

Math Challenge #11
PLAY BALL!

Figure This!
Math Challenges for Families

What's round, hard, and sold for $3 million?

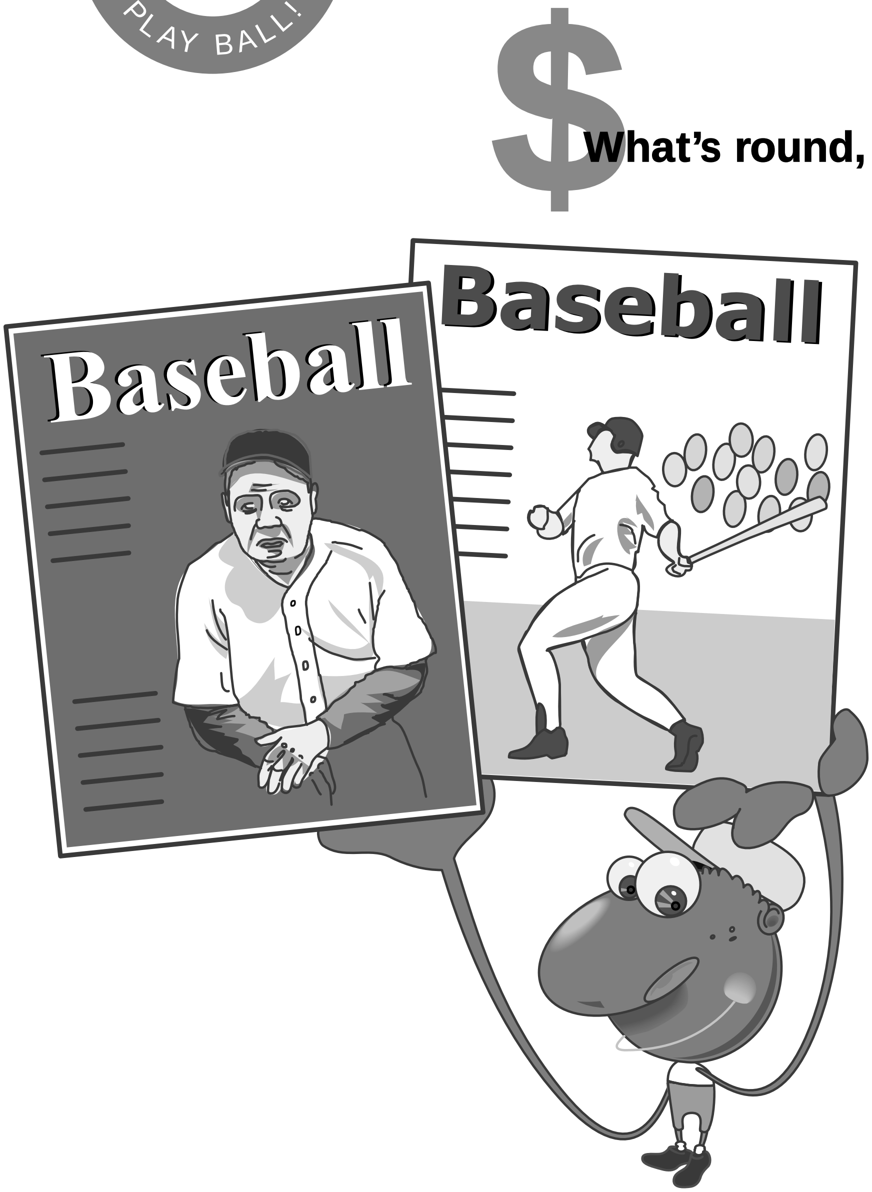


Figure This! Mark McGwire became baseball's home run king in 1998 with 70 home runs. His 70th home run ball sold for slightly over $3 million in 1999. Babe Ruth, an earlier home-run king, hit 60 in 1927. His home-run ball was donated to the Hall of Fame. Suppose that Ruth's ball was valued at $3000 in 1927 and, like many good investments, doubled its value every seven years. Would you rather have the value of Ruth's ball or McGwire's?

Hint: How many times would you need to double the value of Ruth's ball to reach the value of McGwire's?

Compound interest and rate of change over time affect many quantities. Bankers, stockbrokers, and population biologists have to understand this kind of change in their work.

Answer:
You decide. If the value of Ruth's ball started at $3,000 doubled every seven years since 1927, its value in 1997 would be approximately $3,072,000.

Figure This!

Get Started:

Assume that Ruth's ball was valued at $3000 in 1927. What was its value seven years later? Try making a table.

Complete Solution:

Suppose Ruth's ball had a value of $3000 in 1927. If the price doubled in seven years, the ball would be worth $6000 in 1934. In seven more years, its value would double again.

Year	Value
1927	3000
1934	$2 \cdot 3000 = 6000$
1941	$2 \cdot 2 \cdot 3000 = 2^2 \cdot 3000 = 12{,}000$
1948	$2 \cdot 2 \cdot 2 \cdot 3000 = 2^3 \cdot 3000 = 24{,}000$
1955	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3000 = 2^4 \cdot 3000 = 48{,}000$
	. . .
1997	$2^{10} \cdot 3000 = 3{,}072{,}000$

The year 1997 was 70 years after 1927, so there would be 10 sets of 7 years during that time. By 1997, Ruth's ball would have a value of $3,072,000. Since it would have a greater value than McGwire's in 1997, it would have a greater value in 1999.

Try This:

- Find out how much annual interest you could earn on a savings account at a local bank. If you invest money at this rate, how long would it take your money to double? Are there any conditions you would have to consider?
- As a family, talk about any loan that you may have for a college education, a house, a car, or an appliance. What was the original price? What is the total amount that you would spend by the time the loan is paid off completely?

Additional Challenges:

1. If you invest $10 at an annual interest rate of 7%, in about how many years will your money double?
2. If your investment earns 5% annual interest, how many years will it take to double your money?
3. What annual interest would you have to earn to double your money in seven years?
4. A new baseball costs about $6. How many baseballs could you buy with $3 million?
5. Suppose the number of water lilies in a pond doubles every day. If the pond was half covered on Monday, when was it only one-fourth covered with lilies? When would it be completely covered?
6. What would be the value of Babe Ruth's ball in 1999?

Things to Think About:

- A banker's general rule to find the number of years needed to double an investment is to divide 70 by the interest rate. Try this rule on the challenges.
- Banks have computers programmed to use fractional powers to calculate interest earned on savings accounts and interest owed on loans.

Did You Know That?

- The $3,000,000 price of Mark McGwire's baseball was 23 times that of any baseball previously sold and five to six times the highest price paid for any other sports artifact.
- A number, *e*, named for Leonhard Euler (1707-1783), is used in computing continuous interest.

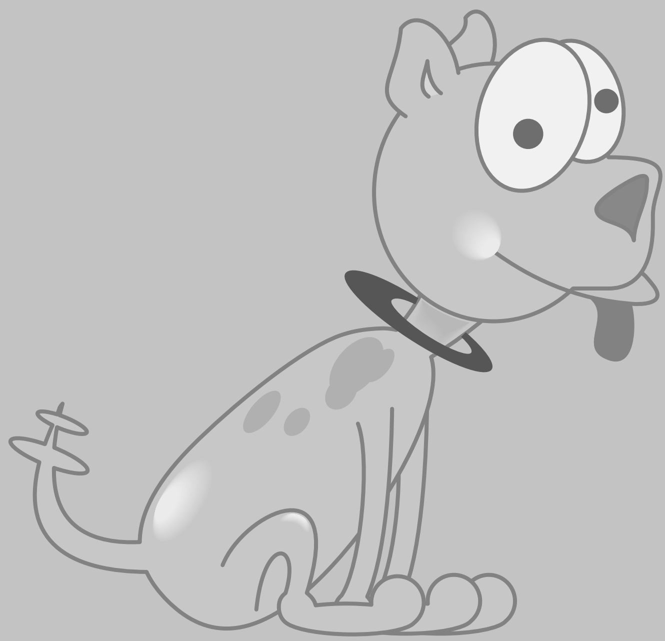
Resources:

Books:

- Sports Illustrated. *Home Run Heroes—Mark McGwire, Sammy Sosa and a Season for the Ages*. New York: Simon & Schuster, 1998.
- "Baseball." *The Encyclopedia Americana*. International Edition. Bethel, CT: Grollier, Inc. 1998.
- "Mark McGwire vs. Sammy Sosa: The 1998 Home Run Race." *The World Almanac and Book of Facts 1999*. Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.majorleaguebaseball.com**
- **www.baseballhalloffame.org**



Answers to Additional Challenges:

(1.)
Assuming that no withdrawals are made and that all interest is reinvested, the balance will reach $20 during the tenth year.

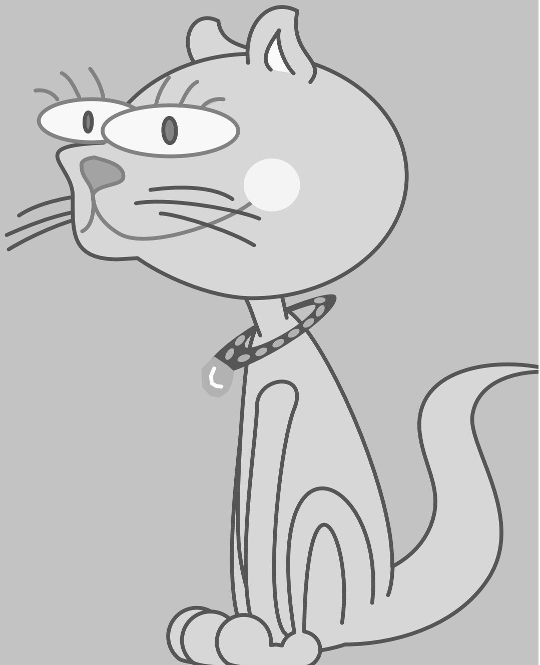
(2.)
The money will double during the 14th year.

(3.)
About 10.4%.

(4.)
500,000 balls.

(5.)
The pond would be one-quarter covered on Sunday. By Tuesday, it would be totally covered.

(6.)
The year 1997 was 70 years after 1927, so there are 10 sets of 7 years during that time. The value would double 10 times, giving $2^{10} \cdot 3000$, or $3,072,000. The year 1999 is 72 years after 1927, so there are 72/7 sets of 7 years. The value would be $2^{72/7} \cdot 3000$. Using a calculator, this is about $3,744,810.



Math Challenge #12 CHOCOLATE

Figure This!
Math Challenges for Families

Which of these chocolate-covered cookies would YOU eat?

mmm... chocolate

Figure This! Suppose you love chocolate. The top of each cookie is covered with the same thickness of chocolate. If you wanted to choose the cookie with more chocolate, which one would you pick?

Hint : Think about how to measure the area of the top of each cookie.

There are no simple ways to find the exact areas of irregular shapes, such as land masses or living cells. Estimating these areas can be important in land-use planning and medical research.

Answer:
The chocolate lover would choose the cookie with the rough edges.

Figure This!

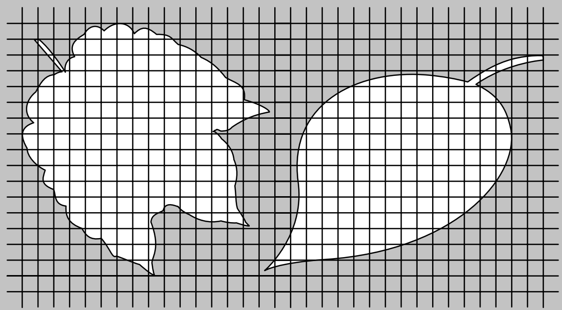
Get Started:

Trace the cookies on a sheet of paper and cut them out. How do you think their areas compare? Would graph paper help?

Complete Solution:

There are many ways to do this challenge.

- Trace the cookies on graph paper and count the number of squares each one covers. The smaller the squares, the better the estimate of the area.



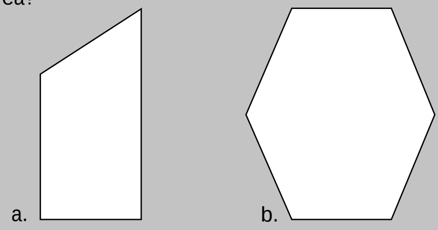
- Cut out the cookies, put one on top of the other and cut off the parts of one that are not covered by the other. Try to fill in the extra space with the parts you cut off. If you cannot cover all of the first cookie with the parts of the second, the first one is larger. If you have pieces of the second cookie left when the first is covered, the second one is larger.
- Cover each cookie with something small (cereal or rice) and then compare the two quantities.

Try This:

- Find the area of a nearby baseball field.
- Find an irregular shape in the room around you. Estimate its area.
- Draw an irregular shape on a piece of paper. Choose some point inside the shape and call that the “center.” Find the lengths from the center to different points on the edge of the shape. [If a segment goes outside the shape, add the lengths of the pieces that are inside the shape. Find the average of all the lengths. Let this average be the “radius” of the shape. Use the formula for the areas of a circle ($\pi \times$ radius $\times$ radius, or about $3.14 \cdot r \cdot r$). This should be a good estimate of the area.

Additional Challenges:

1. You can use a string to find the perimeter of each cookie. Some people might think that the cookie with the greater perimeter will have the greater area. Do you agree? Why or why not?
2. You can find the area of some figures by dividing them into rectangles, squares, or triangles. How could you divide the shapes below to find the area?



Things to Think About:

- Some people say that a coastline has infinite length. What could they mean by this?
- When people talk about buying so many yards of carpet, they are really talking about square yards; with yards of concrete or sand, they are really talking about cubic yards.

Did You Know That?

- Square measure is reasonable to find areas because square regions can cover a flat surface with no overlapping and no holes.
- Although you usually read only about the area covered by an oil slick, it also has volume.
- A planimeter is a tool that measures the area of irregular shapes by tracing the perimeter of the figure. A planimeter involves the concepts of polar coordinates.

Resources:

Books:

- Gravemeijer, K., M. A. Pligge, and B. Clarke. “Reallotment.” In *Mathematics in Context*. National Center for Research and Mathematical Sciences Education and Freudenthal Institute (eds.). Chicago: Encyclopaedia Britannica Educational Corporation, 1998.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and R. Phillips. *Connected Mathematics: Covering and Surrounding*. Palo Alto, CA: Dale Seymour Publications, 1996.

Answers to Additional Challenges:

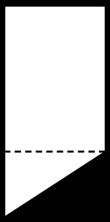
(1.)

For the same perimeter, some shapes have smaller areas, while others have greater areas.

(2.)

a. Divide the shape into a triangle and a rectangle.
b. Divide the shape into a rectangle and two triangles.

a.



b.