Math Challenge #73

A SHARD OR TWO

Figure This! Math Challenges for Families

How BIG was the plate?

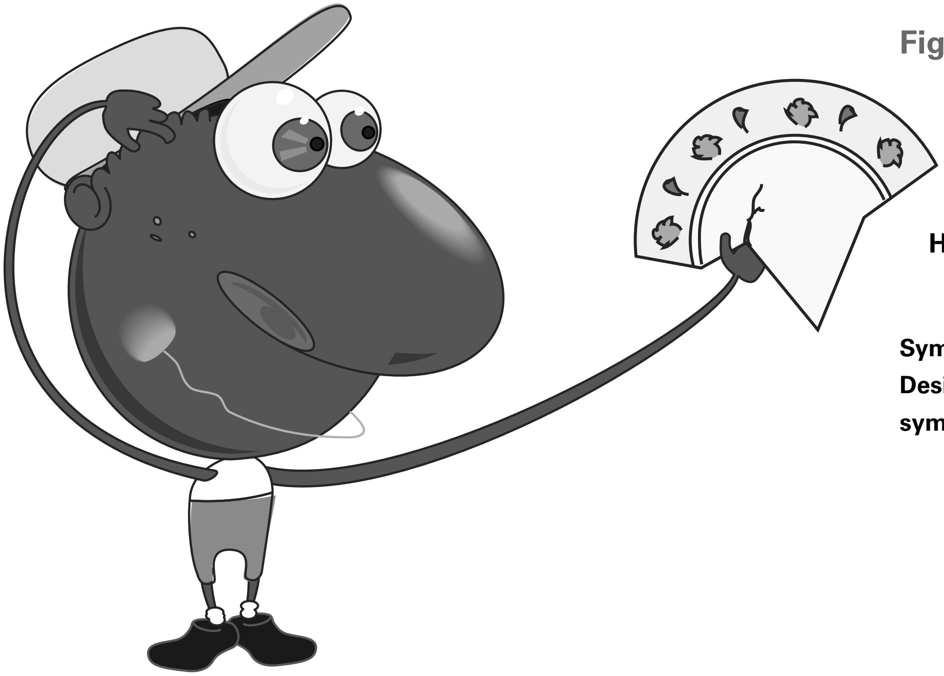


Figure This! While digging in an ancient city, an archeologist found this piece of a round plate. How can Ratio estimate the size of the original plate?

Hint: How can you fold a circle to find its center?

Symmetry is a property of some mathematical shapes. Designers, architects, bakers, and decorators all use symmetry in their work.

Answer:

Trace the rim of the piece of a plate on a sheet of paper. Fold the tracing of the rim onto itself to form a crease in the paper. Repeat this process using a different part of the arc. The point where the two creases intersect is the center of a circle. This circle is the same size as the original plate. (You can use a compass to complete the circle.)

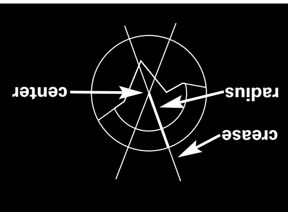


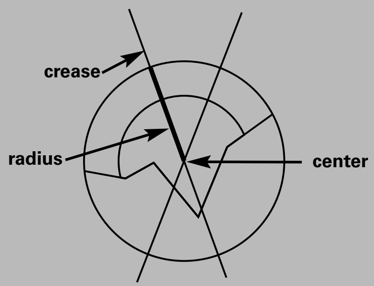
Figure This!

Get Started:

Trace the bottom of a glass to make a circle. Fold a circle of paper in half. Unfold it, then fold it in half again along a different line. What do you observe about the two creases?

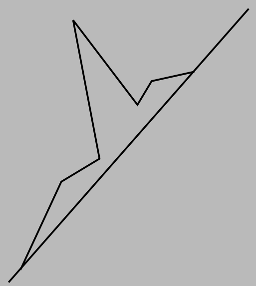
Complete Solution:

Trace the piece of a plate on a sheet of paper. Fold the tracing of the edge of the plate onto itself to form a crease in the paper. Repeat this process using a different part of the arc. The folds are along lines of symmetry for the circle that describes the shape of the original plate. These lines of symmetry contain diameters of the circle and meet at the center. The plate's radius is the distance from the center to the rim. Its diameter is twice the radius.



Try This:

- Sketch the figure below.



Fold the paper along the line, so that your drawing is on the outside, then trace the figure on the opposite side of the line. When you unfold the paper, the new image forms a figure with a line of symmetry.

- Look in the yellow pages of your local phone book for company logos that have lines of symmetry.
- Find a crossword puzzle and determine by folding if it has a line of symmetry.
- Draw a triangle. Through the midpoint of each side of the triangle, draw a line that forms 90˚ angles with the side (the perpendicular bisectors). These lines intersect at the center of a circle that contains each vertex of the triangle.

Additional Challenges:
(Answers located in back of booklet)

1. How many lines of symmetry does a rectangle have?
2. How could you find the center of a STOP sign?
3. What kinds of triangles have a line of symmetry?

Things to Think About:

- Does every figure have a center?
- What is a plane of symmetry?
- Why do museum curators restore ancient objects?
- How do paleontologists create a model dinosaur from fragments of fossilized bone?

Did You Know That?

- Many traditional Native American designs display symmetry.
- Scott Kim has developed a process of writing that involves "turn" symmetry (See the challenge "Upside Down.")
- Broken pieces or fragments of pottery are called potsherds, or shards.

Resources:
Books:

- Billstein, R., S. Libeskind, and J. Lott. *A Problem Solving Approach to Mathematics for Elementary Teachers.* Reading, MA: Addison Wesley Longman Publishing Co., 2000.
- Kim, Scott. *Inversions: A Catalog of Calligraphic Cartwheels.* Petersborough, NH: BYTE Books, 1981.
- Kim, Scott. *Alphabet Symmetry* (poster). Plains, NY: Cuisenaire/Dale Seymour Publications.

Website:

- **www.scottkim.com**

Math Challenge #74

THE RACE

Figure This!

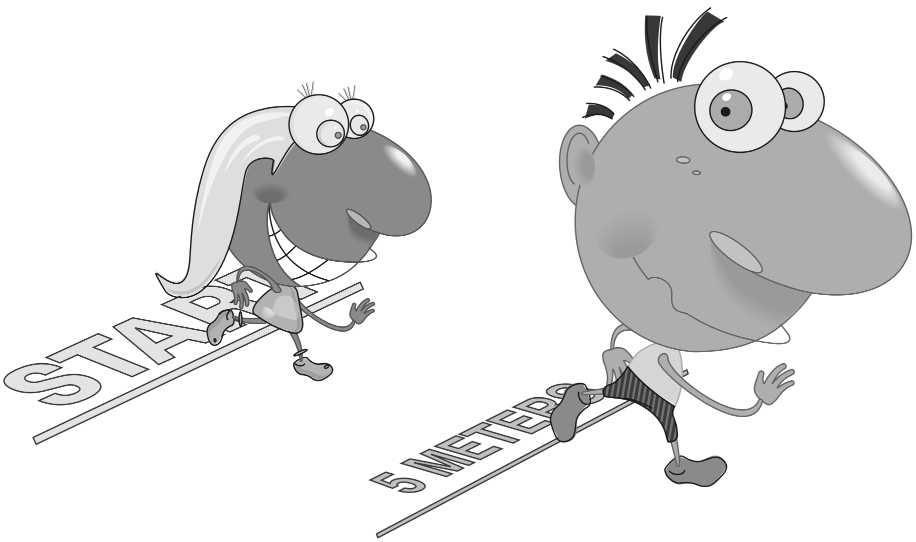
Math Challenges for Families

Who runs fffffaster?

Figure This! **When Polygon and Exponent ran a 50-meter race, Polygon crossed the finish line while Exponent was at the 45-meter mark. The two friends decide to race again. This time, Exponent starts 5 meters ahead of Polygon, who is at the starting line. If each runs at the same speed as in the previous race, who will win?**

Hint: Compare the distances traveled.

Mathematical problem solving requires logical reasoning. Lawyers, philosophers, electricians, and carpenters all use logical reasoning in their daily work.



Answer:

The race will end in a tie.

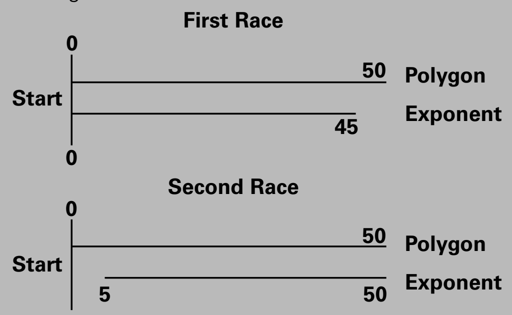
Figure This!

Get Started:

Draw a segment to represent each runner's position when the race is won. Draw another set of segments to represent the starting positions of the second race. What do you notice?

Complete Solution:

In the first race, Exponent ran 45 meters in the same time it took Polygon to run 50 meters. In the second race, the finish line is 45 meters from Exponent's starting position and 50 meters from Polygon's. Therefore, if each runs at the same speed as in the previous race, the race will end in a tie. See the diagram.



Try This:

- Run a race with a friend. After the race, decide on a fair handicap (the distance one of you gets to start ahead of the other faster person) for the winner, then race again. Who wins the second race?
- Look up the winning times for runners, Marion Jones (US) and Cathy Freeman (Australia), in the 2000 Olympics. How do these times compare with the world records for the distances they ran?

Additional Challenges:
(Answers located in back of booklet)

1. Polygon and Exponent decide to race a third time. In this race, Polygon starts 5 meters behind Exponent, who is at the starting line. If each runs at the same speed as before, who will win this time?
2. Imagine that Polygon and Exponent are running laps on a 50-meter oval. If each runs at the same speed as in previous races, after how many laps will Polygon be one whole lap ahead of Exponent?
3. If Exponent and Polygon start at opposite ends of the 50-meter track and run towards each other, where will they meet?

Things to Think About:

- Does the length of a race affect a runner's winning strategy?
- Why is chess called a game of logic?
- How are handicaps figured in golf? In what other sports are players allowed handicaps?

Did You Know That?

- The Greek mathematician Archimedes (ca. 287–212 B.C.) is sometimes considered the father of logic.
- Bertrand Russell (1872–1970), the English mathematician and philosopher, once said, "Mathematics is logic and logic is mathematics."
- Carpenters sometimes use a rope with knots spaced at 3, 7, and 12 units to make a right angle in a right triangle with sides 3, 4, and 5.
- There is an Association for Symbolic Logic.
- Former elementary teacher George Boole (1815–1864) developed an algebra of logic.

Resources:
Books:

- Gardner, M. *Goth: Paradoxes to Puzzle and Delight.* San Francisco, CA: W. H. Freeman and Co., 1982.
- House, P. (ed.). *Providing Opportunities for the Mathematically Gifted K-12* Reston, VA: National Council of Teachers of Mathematics, 1987.

Notes:

Math Challenge #75

GLIDE RATIO

FigureThis! Math Challenges for Families

How far can you FLY a paper airplane?

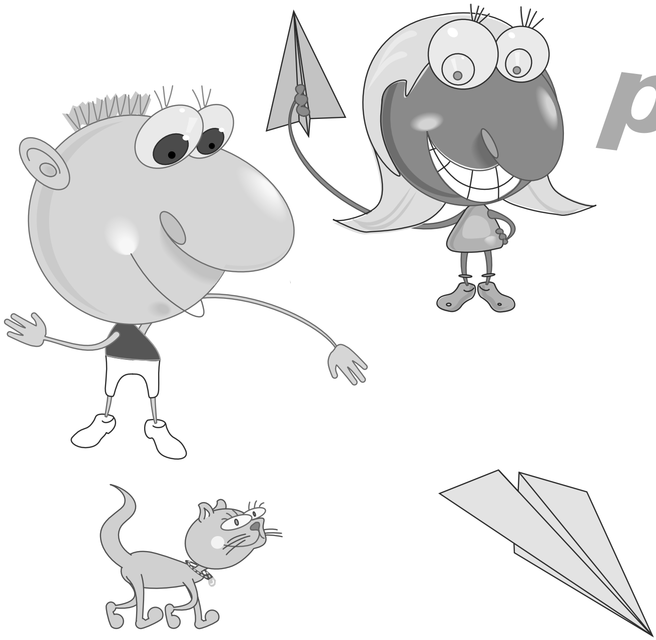


Figure This! **If a glider has a glide ratio of 1/10, it goes down 1 foot for every 10 feet it travels horizontally. Suppose two gliders start at the same height, one with a glide ratio of 0.3 and one with a glide ratio of 2/7. If they glide until they hit the ground, which one will have glided the farther horizontal distance?**

Hint: How can you compare the two glide ratios?

Comparing quantities expressed in different forms is critical in measurements, and in equations and rules. Mechanics repairing machines, computer programmers looking for efficient algorithms, and chemists working in laboratories have to find ways to compare quantities in different forms.

Answer:

The glider with the glide ratio of 2/7 will fly farther.

Figure This!

Getting Started:

How far does each glider go by the time it has dropped 1 foot? Draw a picture of the path of each glider.

Complete solution:

There are several ways to approach this challenge.

- One involves comparing how far the gliders descend after flying the same horizontal distance. To do this, write both ratios as fractions and find a common base (or denominator) for each one. For example, you could change both fractions to sevenths.

  A: Since $\mathbf{0.3 = \frac{0.3}{1}}$, we have $\mathbf{0.3 = \frac{7 \cdot 0.3}{7 \cdot 1} = \frac{2.1}{7}}$

  B: $\mathbf{\frac{2}{7}}$

  One glider (A) descends 2.1 feet for every 7 feet of horizontal travel, while the other (B) falls only 2 feet over the same horizontal distance. The glider that falls more over the same horizontal distance will hit the ground. Therefore, glider B will fly farther.

- Another way to analyze this situation is to determine how far each glider has gone horizontally when both have dropped 1 foot. To do this, rewrite each glide ratio as its equivalent fraction with a numerator of 1.

  A: $\mathbf{0.3 = \frac{0.3 \div 0.3}{1 \div 0.3} \approx \frac{1}{3.33}}$

  and

  B: $\mathbf{\frac{2}{7} = \frac{2 \div 2}{7 \div 2} = \frac{1}{3.5}}$

  This means that, after a 1-foot vertical drop, glider A travels about 3.3 feet, while B travels 3.5 feet. Therefore, glider B will fly farther.

Try This:

- Make two or more paper airplanes. Conduct a series of flight tests to estimate their glide ratios.
- Look up gliders and glide ratios on the web to find a discussion of glide ratios.

Additional Challenges:
(Answers located in back of booklet)

1. Three gliders have glide ratios of 0.21, 7/42, and 14%, respectively. Which one will travel farthest?
2. Three gliders have glide ratios expressed as 0.4, 1/2.5 and 0.04. Do any of these have a glide ratio of 2/5?
3. A glider with a glide ratio of 2/9 has just cleared a line of trees 20 feet tall. About 20 feet farther along its flight path, there is a fence 6 feet tall. Will the glider make it over the fence?
4. Polygon launches a glider with a glide ratio of 12% from the top of a 42-foot cliff. There is no wind. How far from the base of the cliff will the glider land? What is the actual distance traveled by the glider?

Things to Think About:

- What design elements might affect the glide ratio of a glider?
- Do gliders with greater glide ratios fly farther?
- Can a glider have a glide ratio of 0?
- How would you describe the glide ratio of a glider that drops straight down?

Did You Know That?

- The glide ratio can be determined by computing the tangent of the angle defined by the glider's path and the horizontal.
- An angle of depression is the angle from the horizontal looking down while the angle of elevation is the angle from the horizontal looking up.
- Some jets, such as the Harrier, can take off or land vertically.

Resources:
Books:

- De Lange, J. *Flying through Mathematics: Trigonometry, Vectors and Flying*. Scotts Valley, CA: Wings for Learning, 1991.

Notes:

Math Challenge #76 VISUALIZING

Figure This! Math Challenges for Families

Is seeing believing?

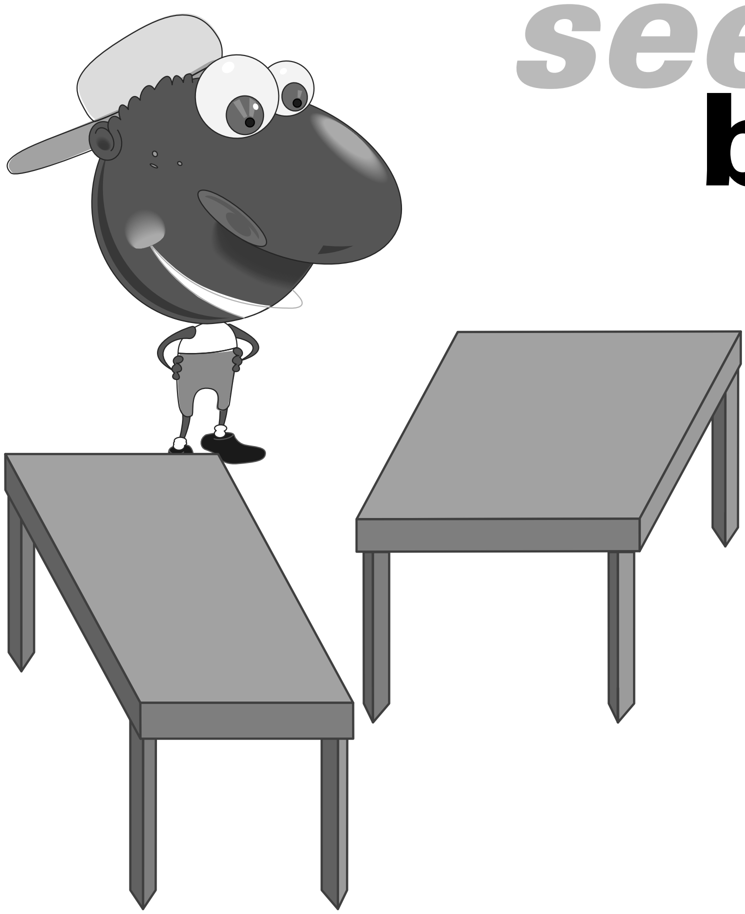


Figure This! Which table appears to have the larger top?

Hint: What do you have to know to compare the sizes?

Visualization is the ability to "see" an image in two- or three-dimensions. Visualization is important for architects, visual artists, and designers.

Answer:

The tops have the same area.

Figure This!

Getting Started:

Trace each top and compare them.

Complete solution:

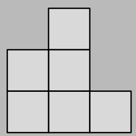
- If you trace one top, it will exactly fit on the other. The objects may appear different because people visualize two- and three-dimensional objects differently.
- Another strategy is to measure and compute the actual area of each table top. Doing this will show that the tops have the same area.

Try this:

- Walk around a chair or sofa in your home and notice how what you can see changes as you move.
- Using a sheet of grid paper, draw some letters of the alphabet so that they appear to be three-dimensional.
- The German artist Albrecht Dürer (1471–1528) created an engraving called "St. Jerome in his Cell." Look in a book or on the web to find a reproduction of that engraving. In the engraving, look for lines that intersect in the picture but would be parallel in real life.

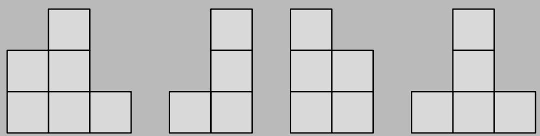
Additional Challenges:
(Answers located in back of booklet)

1. The diagram below shows the view of a building from the right-hand side.



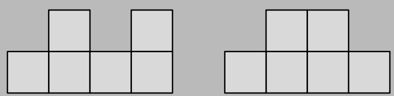
Right

Given this fact, are there any of the views below that could not represent the front of the building?



a **b** **c** **d**

2. What is the maximum number of blocks that can be used to create the views of the structure whose front and right side views are pictured?

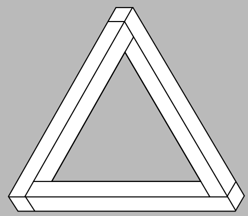


Front **Right**

3. Draw a cube. Then increase one dimension of the cube to draw a box that has three times the volume of the cube.

Things to Think About:

- A mirage is an imagined visualization in the desert.
- Many children's toys, such as Shape-O™, are designed to help children learn about visualization.
- Stick a pencil or straw into a glass of water. Does the object appear broken or bent? Why?
- Why does the moon look bigger when it is nearer the horizon, and smaller when it is higher in the sky?
- Could the triangular shape below be built of wood?



Did You Know That?

- The image that forms on your retina is flat, yet you see a world of shape, color, depth, and motion.
- Nature uses camouflage illusions that allows both animals and insects to match their surroundings so they are undetectable.
- The Poggendorf illusion, discovered in 1860, is one of the most famous distortion illusions known and uses colored lines passing through a rectangle.
- Mirage comes from the French word *mirer,* to look at; or *se mirer,* to be reflected. A mirage may include the appearance of lakes in deserts or on hot asphalt roads, the images of ships and icebergs (frequently seen as if inverted and suspended in the atmosphere of the Polar Regions), or "looming" as witnessed in mists or fogs.

Resources:
Books:

- Anno, Mitsumasa, *The Unique World of Mitsumasa Anno: Selected Illustrations* 1969-1977, Tokyo: Kodansha Ltd., 1977.
- Bergamini, D. *Life Science Library: Mathematics.* New York: Time Inc., 1963.
- Ernst, Bruno, *Adventures with Impossible Figures.* Norfolk, VA: Tarquin Publications, 1986.
- Ernst, Bruno, *The Magic Mirror of M. C. Escher.* Norfolk, VA: Tarquin Publications, 1985.
- McKim, R. *Experiences in Visual Thinking.* Monterey, CA: Brooks/Cole Publishing Co., 1972.

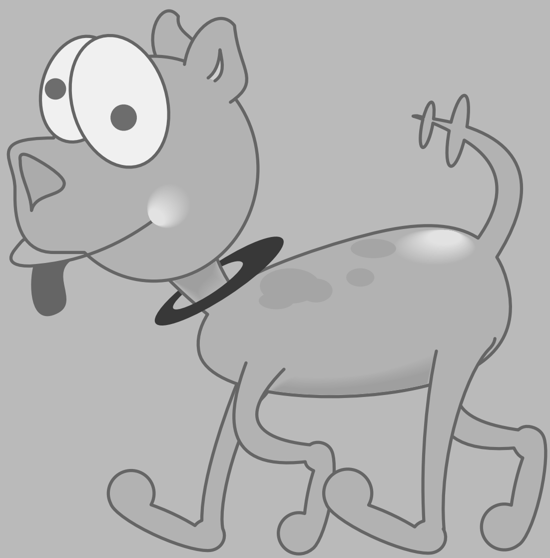
- Steinhaus, H. *Mathematical Snapshots.* New York: Oxford University Press, 1950.
- Turner, Harry. *Triad: Optical Illusions and How to Design Them.* New York: Dover Publications, 1978

Websites:

- **www.crs4.it/Ars/arshtml/arstoc.html**
- **www.saumag.edu/art/studio/chalkboard/lp-intro.html**
- **www.sandlotscience.com/Camoflage/Camouflage_frm.htm**

Notes:

Tangent



Math Challenge ANSWERS

FigureThis! Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

Challenge 73

1. If the rectangle also is a square, then it has four lines of symmetry. If not, then it has two, neither of which is a diagonal.
2. Find two lines of symmetry. They will intersect in the center.
3. A triangle with at least two sides of the same length (an isosceles triangle).

Challenge 74

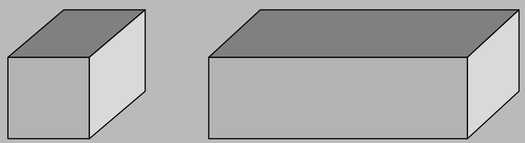
1. Polygon will win this race. Although they will be tied at the 45-meter mark, Polygon will run the last 5 meters faster than Exponent.
2. At their previous speeds, Polygon will run 500 meters (10 laps) in the same time that Exponent runs 450 meters (9 laps). Therefore, after 10 laps, Polygon will be 1 lap ahead of Exponent.
3. They will meet about 26.3 meters from Polygon's starting point (or 23.7 meters from Exponent's starting point).

Challenge 75

1. The one with the glide ratio of 14%.
2. The ratios 0.4 and 1/2.5 are both the same as 2/5.
3. Yes.
4. The glider will land at a horizontal distance of about 350 feet from the base of the cliff. Its actual flight path is about 353 feet long.

Challenge 76

1. No
2. 20 maximum
3. Answers will vary. One possible solution is shown here:



Challenge 77

1. Answers may vary. For example, box-office receipts of $9000 in week 5 and $6600 in week 6 will yield a profit of $1020.
2. About $8333 for week 5 and about $7142 for week 6.
3. $0.60.
4. About $6615.

Challenge 78

1. 20%.
2. It helps. The average of the six scores is 9.7. After dropping the high and low scores, the average is 9.725.
3. About 81%.

Challenge 79

1. To create an image that is 70% of the way to the smile, you must identify the number that is 70% (or 0.7) of the way from each number on the smile to its corresponding number on the frown. For example, the number 4 on the frown corresponds to 2 on the smile. The difference of 4 and 2 is 2. Find 70% of this difference, 0.7(2) = 1.4. Since 4 – 1.4 = 2.6, the corresponding square on the image should be 3 (rounding to the nearest whole number).

3	1	3
3	3	3

3	1	3
3	3	3

2. 110.
3. It will be 180 miles north and 120 miles east of Miami.

Challenge 80

1. 220.
2. 1 (The man with the wives was going the other way.)
3. 300.
4. Yes.