Math Challenge #61 STATUE OF LIBERTY

Figure This! Math Challenges for Families

IS THE STATUE OF LIBERTY'S NOSE TOO long??

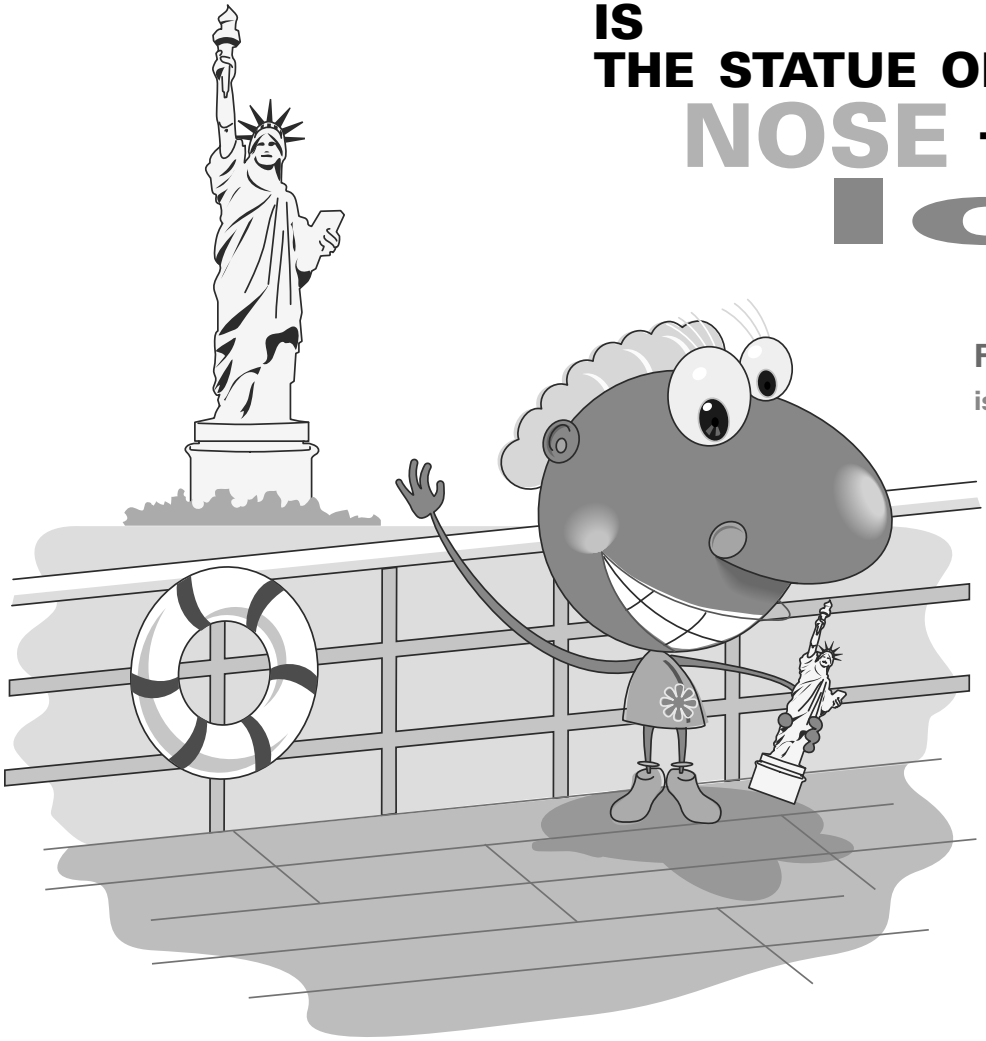


Figure This! The arm of the Statue of Liberty is 42 feet long. How long is her nose?

Hint: How long is your nose? How long is your arm?

Similarity and scaling underlie design and model building. Architects, clothing designers, toy makers, and civil engineers all use scaling in their work.

Answer:
The actual length of her nose is about 4 feet, 6 inches.

Figure This!

Get Started:

Measure the lengths of your nose and your arm in the same units. About how many times longer than your nose is your arm? What might this tell you about the dimensions of the Statue of Liberty?

Complete Solution:

- There are several ways to think about this problem. One way is to assume that approximately the same relationship holds between your dimensions and those of the Statue of Liberty. If your nose is 3" long and the length of your arm is 24" long, then the length of your nose is 3/24, or 1/8, of the length of your arm. If your nose is 1/8 of the length of your arm, then the nose of the Statue of Liberty might be 1/8 the length of her arm. In general, the length of her nose would be:

$$\frac{\text{Length of your nose}}{\text{Length of your arm}} \bullet \text{Length of Arm of Statue of Liberty}$$

- Another approach is to assume that the Statue of Liberty is similar in scale to an average person. In this case, the lengths of corresponding body parts should have the same ratio. (To estimate the lengths for an "average person," take a group of people, measure them, and find the average measures. Answers will vary depending on measurement errors and what was used as an average.) Using this method, the relationship would be:

$$\frac{\text{Length of nose of Statue of Liberty}}{\text{Length of typical nose}} = \frac{\text{Length of arm of Statue of Liberty}}{\text{Length of typical arm}}$$

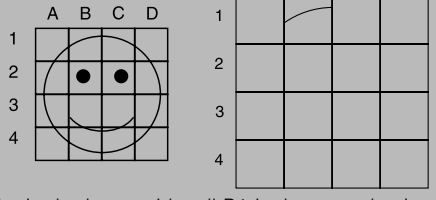
For the example above:

$$\frac{\text{Length of nose of Statue of Liberty}}{3''} = \frac{42'}{24''}$$

Length of nose of Statue of Liberty = 3" (42'/24"), or 5 1/4'

Try This:

- Measure the lengths of a friend's nose and arm. Find the ratio of these measurements. How does this ratio compare to yours?
- Measure the length, width, and height of a Hot Wheels™ car and a real car. Does the Hot Wheels™ appear to be a scale model?
- You can use similarity and scale to enlarge a picture. Select a simple cartoon. Draw a square around it, then use horizontal and vertical lines to divide the square into 16 smaller squares, as shown in the first diagram. Now draw a larger square, also divided into 16 smaller squares. In each cell in the larger square, draw the corresponding part of the figure in the cartoon.



For example, in the large grid, cell B1 is the reproduction of cell B1 in the small grid. Try this method to enlarge a cartoon.

Additional Challenges:
(Answers located in back of booklet)

1. An HO gauge model train is scaled 1/8 inch to a foot. If an HO caboose is 4 inches long, what is the length of the real caboose?
2. The average height of a Lilliputian (from Jonathan Swift's *Gulliver's Travels*) is slightly less than 6 inches. The height of a Lilliputian is about what fraction of your height?
3. When translating a textbook, 15 pages of English resulted in 17 pages of Spanish. Based on this information, about how many pages of English would be necessary for a 187 page Spanish book?

Things to Think About:

- If a T-shirt shrinks, is it a scale model of its original shape before it was washed?
- Are enlargements of pictures always scale models of the original?
- Architects use scale models to help determine weight loads and structural strength.
- Do different-sized boxes of the same breakfast cereal have the same length-to-width ratios?

Did You Know That?

- The movies *Honey I Shrunk the Kids* and *Honey We Shrunk Ourselves* involve scale models to make people look small.
- Some maps are scale models of actual areas.
- In 2000, the National Building Museum contained a scale model of the White House.
- Sometimes people are posed in oversized furniture to make them look small.
- The relationship:

$\frac{a}{b} = \frac{c}{d}$, where none of a, b, c, or d is 0,

also can be written in any of the following ways:

$$\frac{a}{c} = \frac{b}{d} \qquad \frac{d}{b} = \frac{c}{a} \qquad \frac{b}{a} = \frac{d}{c}$$

- The Statue of Liberty was originally called Liberty Enlightening the World. It was completed in Paris in 1884, then shipped to New York.
- If you double all the dimensions of a quart carton, the new container will hold 2 gallons (8 quarts).
- The index finger of the Statue of Liberty is 8 feet long.

Resources:

Books:

- Billstein, R., and J. Williamson. *Middle Grades Math Thematics*. Evanston, IL: McDougal Littell, 1998.
- Swift, J. *Gulliver's Travels*. Oxford: Oxford University, 1998.
- *The World Almanac 2000*. Mahweh, NJ: World Almanac Books, 1999.

Website:

- **world.std.com/~dpowsner/hoscale.html**
- **www.readingrailroad.org/rdgcab.htm**

Notes:

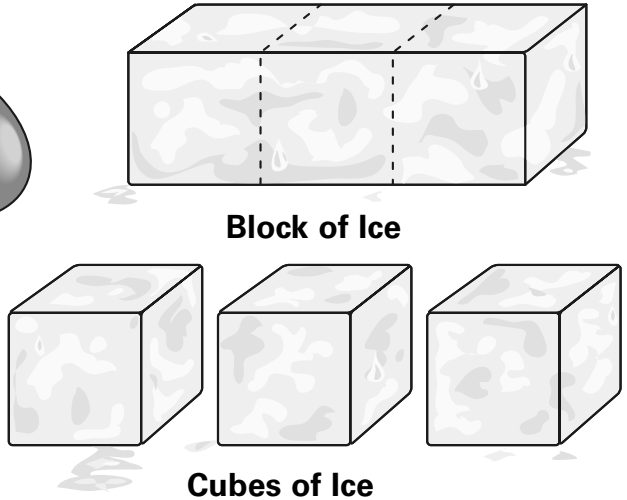
Tangent

Math Challenge #62 KEEPING COOL

Figure This! Math Challenges for Families

When should you buy block ice or crushed ice?

Figure This! Which typically melts faster, a single block of ice or the same block cut into three cubes?



Block of Ice

Cubes of Ice

Hint: Compare the exposed areas.

Surface area is a critical factor in heating and cooling. Architects, interior decorators, chemists, and environmental engineers use surface area and volume in their work.

Answer:
The three cubes will melt faster.

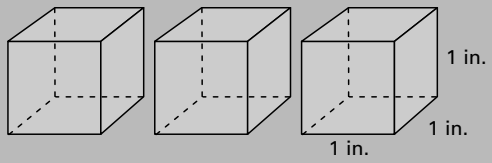
Figure This!

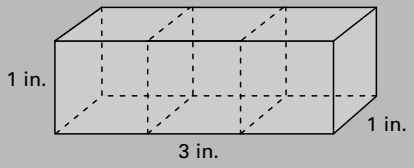
Get Started:

Heat enters an object through its exposed surface. The more surface area exposed, the faster it melts. Ice will melt faster if its exposed surface area is bigger. Compare the surface area of the cubes and the block.

Complete Solution:

Suppose that each of the smaller cubes is 1 in. high. The area of one face of the cube is length times width: 1 x 1 = 1 or 1 sq. in.





On each cube there are six faces, each with an area of 1 sq. in. The total surface area for each of the cubes is 6 sq. in. The surface area of the three cubes is 3 x 6, or 18 sq. in. The big block of ice has four faces, each with an area of 3 x 1 for a total of 12 sq. in. The block has two faces with the surface area of 1 x 1 for a total of 2 sq. in. Thus, the surface area of the block of ice is 12 + 2 or 14 sq. in. The three cubes broken apart will melt more quickly since the surface area of 18 sq. in. is exposed while the block of ice has only 14 sq. in. exposed.

Think what happens when you cut the block into cubes. There is more exposed surface area so the cubes melt faster.

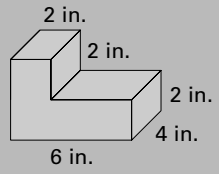
Try This:

- Ask an adult for two Alka-Seltzer™ tablets and two glasses of water. Carefully cut one of the tablets into four pieces. Which tablet do you think will dissolve faster: the whole one, or the one cut into pieces? Use the two glasses of water to test your prediction.

Additional Challenges:
(Answers located in back of booklet)

1. Imagine that you left two identical bottles of cold water in the sun. One bottle contains 1 liter of water; the other contains 2 liters. After 15 minutes, which will be cooler?

2. Find the surface area of this box.



3. Do all boxes with the same surface area have the same volume?

4. Do all boxes with the same volume have the same surface area?

Things to Think About:

- To minimize melting, the best shape to make ice from water is a sphere.
- Does surface area affect how architects design buildings?
- Why are some pills crushed before they are given to a patient?
- When people fall into cold water, the risk of hypothermia (dangerously lowered body temperature) is greater than that of drowning. If you are alone and wearing a life preserver, try the Heat Escape Lessening Posture (HELP; see fig. 1) Keep your arms close to the sides of your chest, cross your legs and pull them up as far as you can.

  If you are in the water with two or more people, try the huddle position shown in Figure 2. Stay close together and keep still to keep colder water out. The huddle can help small children survive longer.

Figure 1

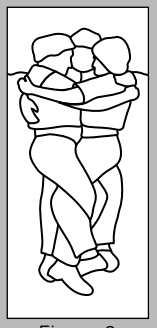


Figure 2

- Does a clove of garlic have more flavor whole or chopped?
- Why is a ring of ice used in a punch bowl while cans of soda are normally put in crushed ice?

Did You Know That?

- The stems of cactus plants are thick and round to minimize surface area and store water. Most species of cactus do not have leaves since leaves would allow too much evaporation in the dry desert air.

- Water is one of the world's strangest substances. Most liquids shrink when they freeze but water expands, making it less dense as a solid than as a liquid. That's why ice floats.
- The length of the time you want something to stay cold should be a determining factor in the type of ice you use to cool it.

Resources:
Books:

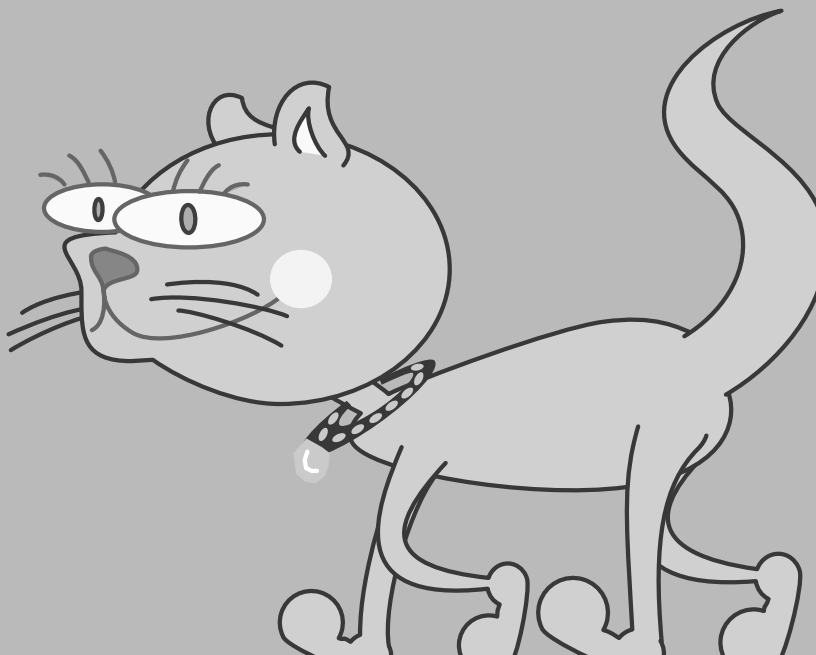
- Souviney, R., M. Britt, S. Gargiulo, and P. Hughes. *Mathematical Investigations, Book One*. White Plains, NY: Dale Seymour, 1990.
- *The Guinness Book of Records 1999*. New York, NY: Guinness Publishing, Ltd., 1999.

Website:

- **www.eecs.umich.edu/mathscience/funexperiments**

Notes:

Axis



Math Challenge #63 MATCHING BIRTHDAYS

Figure This! Math Challenges for Families

Same birth month? No Way!

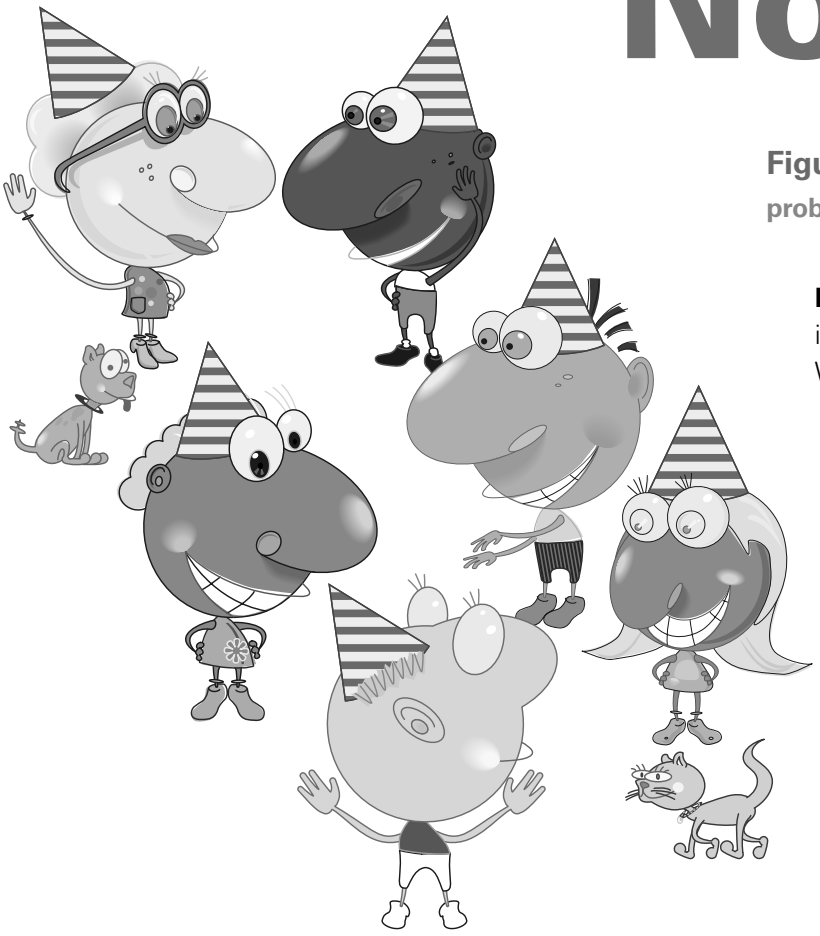


Figure This! In any group of six people, what is the probability that everyone was born in different months?

Hint: If there were only two people in a room, what is the probability they were born in different months? What if there were three people in the room?

The probability of an event is a measure of the chance that it will occur. Insurance companies use probability to set rates, weather forecasters use probability to predict weather patterns, and doctors use probability to determine treatments for disease.

Answer: About 22%

Figure This!

Get Started:

Since there are 12 months in a year, a person's birth month has 12 different possibilities. For this problem, to make it simpler, assume the probability of being born in any given month is 1/12. If there are two people in the room, what is the probability that the second person's birthday occurred in a different month from the first person's? If there are three people in a room, what is the probability that all three people have birthdays in different months?

Complete Solution:

With two people, the probability that the second person's birth month does not match the first person's is 11/12.

A third person could be born in any of the other ten months and not match the first two. The probability that the third person's birth month will not match either of the other two is 10/12. The probability that all three have different birth months is:

$$\frac{11}{12} \times \frac{10}{12} \approx 0.7638 \text{ or about } 76\%$$

Continuing this process, the probability that six people have different birth months is:

$$\frac{11}{12} \times \frac{10}{12} \times \frac{9}{12} \times \frac{8}{12} \times \frac{7}{12} \approx 0.2228 \text{ or about } 22\%$$

Thus, the probability that at least two of the six people do not have the same birth month is about 22%.

Try This:

- Choose a group of people, such as a classroom of students, a group of friends, or the members of a club. Count the number of people in the group and determine the probability that at least two of them were born in the same month.
- Check several classes, or groups of people, of the same size. In how many of the groups do two people share the same birthday? Use the results to estimate the probability that two people in a group that size share the same birthday.

Additional Challenges:
(Answers located in back of booklet)

1. About how many people does it take to have a 99% probability that at least two have the same birth month?
2. What is the probability that if you know the birthday of one person in a room, a second person has the same birthday?
3. What is the least number of people required to make sure that at least two have matching birthdays?

Things to Think About:

- With 30 strangers in a room, the probability of having at least two matching birthdays is about 71%.
- The probability that two people were born on the *same* day is different from the probability that two people were born on a *given* day.
- How many US presidents have had the same birthday? How many of them died on the same day?
- Think of an event that has a 100% probability of occurring.

Did You Know That?

- The probability that an event will occur and the probability that an event will not occur are called "complementary" probabilities. The sum of complementary probabilities is 1 or 100%.
- A probability of 0 means that the event cannot occur.
- The probability of any event is a percentage ranging from 0% to 100%.

Resources:
Books:

- Jacobs, H. *Mathematics, A Human Endeavor*. San Francisco, CA: W. H. Freeman and Co., 1970.

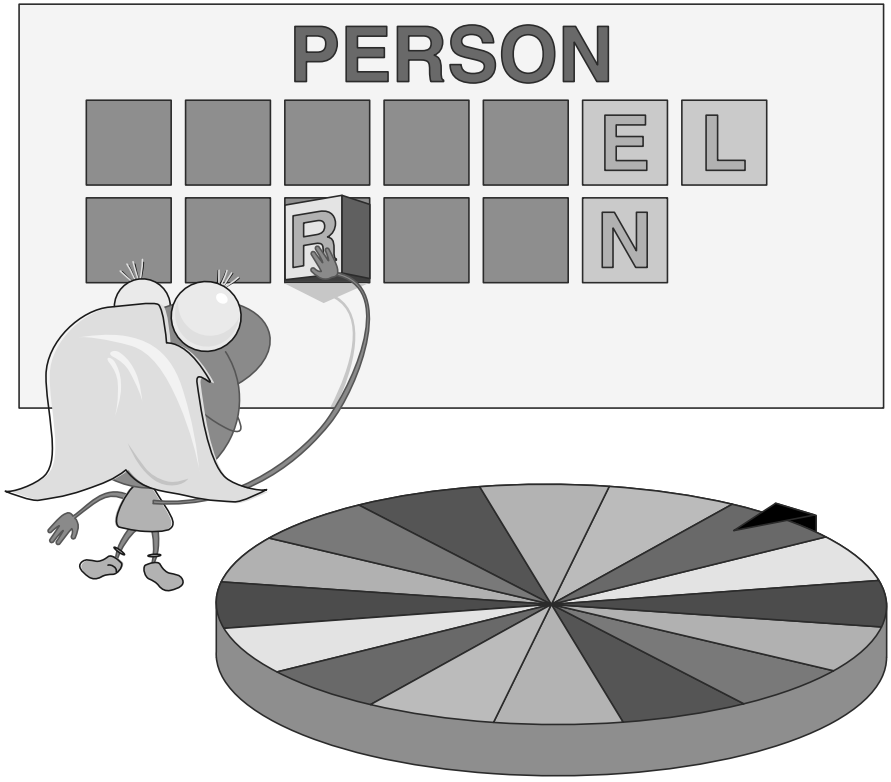
Websites:

- **forum.swarthmore.edu/~isaac/problems/prob1.html**

Math Challenge #64 WHEEL OF FORTUNE

FigureThis! Math Challenges for Families

Who would you guess?



FigureThis! In the game show, Wheel of Fortune™, contestants guess the letters of a word or phrase. The letters E, L, N, R, S, and T are given, if they occur in the puzzle. What three consonants and one vowel would you guess next?

Hint: Which letters are used most often?

Structure of language is important in breaking codes and in voice recognition. Spelling based on frequency of letter usage is fundamental in working with codes.

Answer:
The most commonly used consonants after those listed are D, H and C. The next most common vowels are I and A. You might choose your set of letters from these.

Figure This!

Get Started:
Make a guess; think of letter frequencies and combinations.

Complete Solution:

- The frequency of each letter can vary, depending on the sample. In English, E and T occur the most frequently. The table below shows the number of times each letter occurred in one sample of 1000 letters. In this sample, the letters D and H appear most frequently after those already given, and one of these might be a good choice.

A	B	C	D	E	F	G	H	I	J	K	L	M
73	9	30	44	130	28	16	35	74	2	3	35	25
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
78	74	27	3	77	63	93	27	13	16	5	19	1

Source: Sinkov, A. *Elementary Cryptanalysis.* Washington, DC: Mathematical Association of America, 1968.

Choose H, D, C and either I or O. If I is chosen, then the letters you see are:

	I	C	H		E	L
		R	D		N	

A possible answer is Michael Jordan.

- In a sample of approximately one million words of modern British English, Martin Wynne found that the following letters occur most frequently in the order given: E, T, A, O, I, N, S, H, R, D, L, and U. With this sample, H or D would still be a logical choice.

Try This:

- Choose a sample of 200–300 words from a magazine or newspaper. Work with a partner to count the number of times each letter is used. Make a list of the letters in the English alphabet. Working with a partner, read your words letter by letter and record the number of times each appears.
- Write a short paragraph without using the letters A and N.
- In the game of Scrabble™, players use letter tiles to form words and earn points. The following table shows the number of tiles available in Scrabble™ for each letter in the alphabet. The total number of non-blank tiles is 98; 2 blank tiles can be used as any letter.

A	B	C	D	E	F	G	H	I	J	K	L	M
9	2	2	4	12	2	3	2	9	1	1	4	2
N	**O**	**P**	**Q**	**R**	**S**	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**
6	8	2	1	6	4	6	4	2	2	1	2	1

Is the frequency of each letter in Scrabble™ about the same as its frequency in the sample given in the Complete Solution? What are the biggest differences you notice? How would you construct a "better" set of Scrabble™ tiles? How would you assign point values to each letter that represents letter frequencies?

- Do an Internet search using the following words: "codes and ciphers."
- Play Hangman with a partner. In this game, you take turns guessing letters in each other's words and try to avoid having a hangman consisting of a circle and six segments drawn. A part of the hangman is drawn with each mistake.

Additional Challenges:
(Answers located in back of booklet)

1. In the two examples below, each letter in the alphabet has been replaced by another letter, according to a code. When decoded, each string of letters reveals a famous quotation. What are the quotations?

   a. "WKHVH DUH WKH WLPHV WKDW WUB PHQ'V VRXOV" (by Thomas Paine in "The Crisis", 1976).

   b. "VI VKKGZ V YVT FZZKN OCZ YJXOJM VRVT" (by Benjamin Franklin).

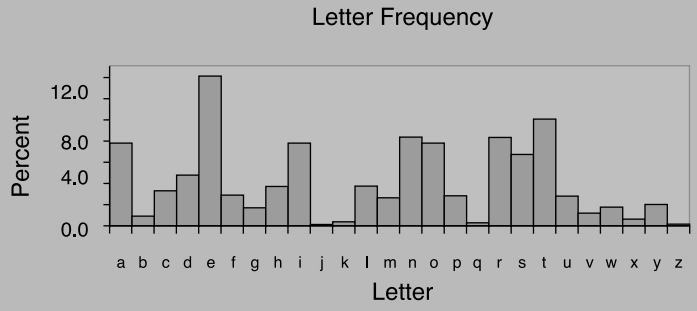
2. What is unusual about each of the following sentences?

   a. "In my opinion, I am saying an awfully abnormal group of words – a bunch of words which is unusually odd, uncommon, and surprising."

   b. "The quick brown fox jumped over the lazy dogs."

Things to Think About:

- Would you expect the frequency of each letter in other languages to be the same as that in English?
- Would you expect the frequency of each letter in a book by Dr. Seuss to be the same as that in a newspaper?
- The graph shows the frequency of each letter as a percentage. Does the graph help you understand the frequency distribution of letters better than the table in the Complete Solution?



Did You Know That?

- Two classic stories in which deciphering a code plays an important role are *The Gold Bug*, by Edgar Allen Poe and *The Adventure of the Dancing Men*, by Arthur Conan Doyle.
- Computer programs available on the Internet can analyze the letter frequencies in any text selection.
- The study of codes and ciphers is called *cryptography*.
- The Enigma Machine, a rotor based cipher machine, was a device for coding and decoding. Originally intended to be used for commercial cryptography, it was adopted by the German Army and Navy prior to World War II for sensitive communications.
- The Navajo Indians' native language was used as a code during World War II, and the code was never broken.
- One of the most popular puzzles of *Games Magazine* is DSZQUPHSBNT! which involves breaking secret codes in which every letter is represented by another letter. Looking at letter frequencies is a productive way to begin to solve the puzzle.

Resources:

Books:

- Doyle, A. C. *The Complete Sherlock Holmes.* New York, NY: Doubleday, 1953.
- Poe, E. A. *Tales of Edgar Allan Poe.* York, PA: Random House. 1944.
- Sinkov, A. *Elementary Cryptanalysis.* Washington, DC: Mathematical Association of America, 1968.

Magazine:

- *Games*. Ambler, PA: Games Publications, Inc.

Websites:

For letter frequencies:

- **www.vindustries.com/letterfreq**

Enigma machine:

- **wordzap.com/enigma**

For breaking codes:

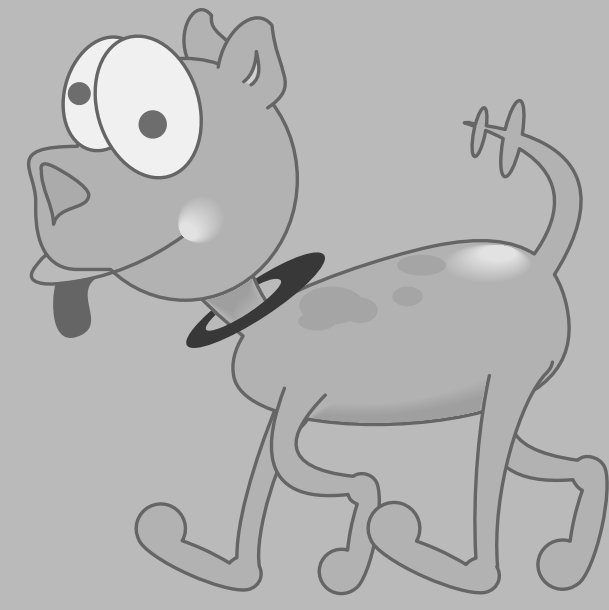
- **www.teachingtools.com/GoFigurePages/Password.htm**

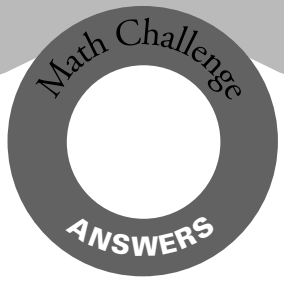
For cryptography:

- **www.arachnaut.org/archive/freq.html**
- **www.kith.org/logos/words/upper2/ZZeus.html**

Notes:

Tangent





FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

Challenge 61:

1. 32 feet.
2. The ratio depends on your height. For example, if you are 5 feet tall, then the ratio is about 1:10.
3. 165 pages.

Challenge 62:

1. The 2-liter bottle.
2. 112 sq. in.
3. No.
4. No.

Challenge 63:

1. 10 people.
2. 1/365.
3. 367 (if you include the leap-year day, February 29).

Challenge 64:

1a. These are the times that try men's souls.

1b. An apple a day keeps the doctor away.

2a. There are no "e's" in the sentence.

2b. All the letters of the English alphabet are used in the sentence.

Notes: