Math Challenge #57 ENVELOPES

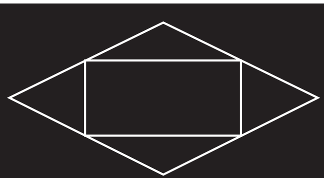
Figure This! Math Challenges for Families

How can you FOLD a sheet of paper to make an envelope?

Figure This! Suppose you wanted to make a business-size envelope from a sheet of paper, with as little waste as possible. What shape would you use for the pattern?

Hint: Unfold a business envelope along its seams, then examine the pattern.

Manufacturers design containers for efficiency. When certain shapes fit together without overlaps or gaps, they form a tessellation, used to minimize waste in making containers. Engineers use tessellations to design heat shields for space shuttles.



Answer:

One common pattern for an envelope is shown. Other shapes also are used.

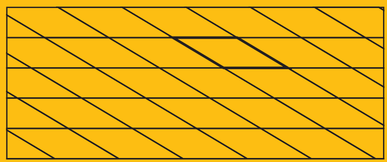
Figure This!

Get Started:

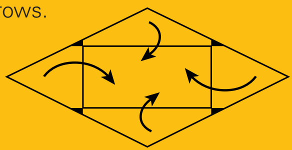
Unfold an envelope at the seams. If you wanted to make many envelopes from this pattern out of a large piece of paper, would there be much waste? Would a different shape reduce the waste?

Complete Solution:

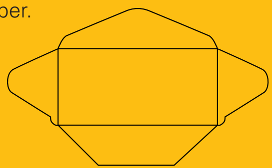
To cover a sheet of paper with many copies of the same shape so that there are no holes or overlaps, the copies have to fit together. A rhombus, with four sides of equal length, as pictured below in bold will do this.



To create tabs for gluing an envelope together, many manufacturers cut notches in the edges of each pattern, as shown in the following diagram. This is the only waste paper in the design. The pattern is then folded in the directions indicated by the arrows.



Other patterns also are used, such as the one shown below, but these may waste slightly more paper.

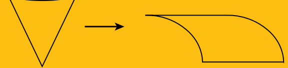


Try This:

- Find several envelopes of various sizes and unfold them at the seams. Can the resulting patterns be cut from a large sheet of paper with little waste?
- Design a pattern that folds into a square envelope.
- Take apart other types of packages or containers, such as paper bags, boxes, or fast food containers. Consider how the design of each minimizes waste in materials, provides strength, and allows for ease of production.

Additional Challenges:
(Answers located in back of booklet)

1. A Baskin-Robbins™ ice cream cone paper holder is unglued and flattened. Will the cone wrapper minimize waste of the material used to make it?



2. Can any four-sided polygon be used to cover a flat surface without gaps or overlaps?
3. What shape is the pattern for the cardboard roll inside a roll of toilet tissue?

Things to Think About:

- Which snack crackers come in shapes that will cover a flat surface with no gaps or overlaps?
- What cookie cutter shapes allow you to make cookies with little wasted dough?
- What other shapes tessellate the plane?

Did You Know That?

- An exhibit at the US Postal Museum chronicles the development of machines that make envelopes.
- Michigan State University offers both bachelors and masters-degree programs in package design.
- The shape of the pattern for a Pillsbury crescent roll will tessellate a plane.
- Paving stones and floor tiles are designed to cover a flat surface with no gaps or overlaps.
- The topic of tiling is an important area of investigation for geometers. There are still unanswered questions in this field, such as finding all the different shapes that can be used to tessellate a plane.
- The Alhambra, a fourteenth century palace in Granada, Spain, contains many decorative majolica tilings and stucco designs that are tessellations. Dutch artist M.C. Escher's designs were influenced by the Alhambra tilings.

Resources:

Books:

- Grünbaum, B., and G. Shephard. *Tilings and Patterns*. New York: W. H. Freeman and Co., 1987.
- Minnesota Educational Computing Corporation. *Tesselmania!* Minneapolis, MN: MECC, 1994.
- Schattschneider, D. *Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M. C. Escher.* New York: W. H. Freeman and Co., 1990.
- Laycock, M. *Bucky for Beginner: Synergetic Geometry.* Hayward, CA: Activity Resources Co., 1984.

Website:

- **web1.si.edu/postal/exhibits/cards4.html**
- **forum.swathmore.edu/sum95/suzanne/tess.intro.html**
- **web.inter.nl.net/hcc/Hans.Kuiper**
- **www.camosun.bc.ca/~jbritton/jbsymteslk.htm**

Math Challenge #58

GONE FISHING

Figure This! Math Challenges for Families

my, my, little fish — how you've GROWN!

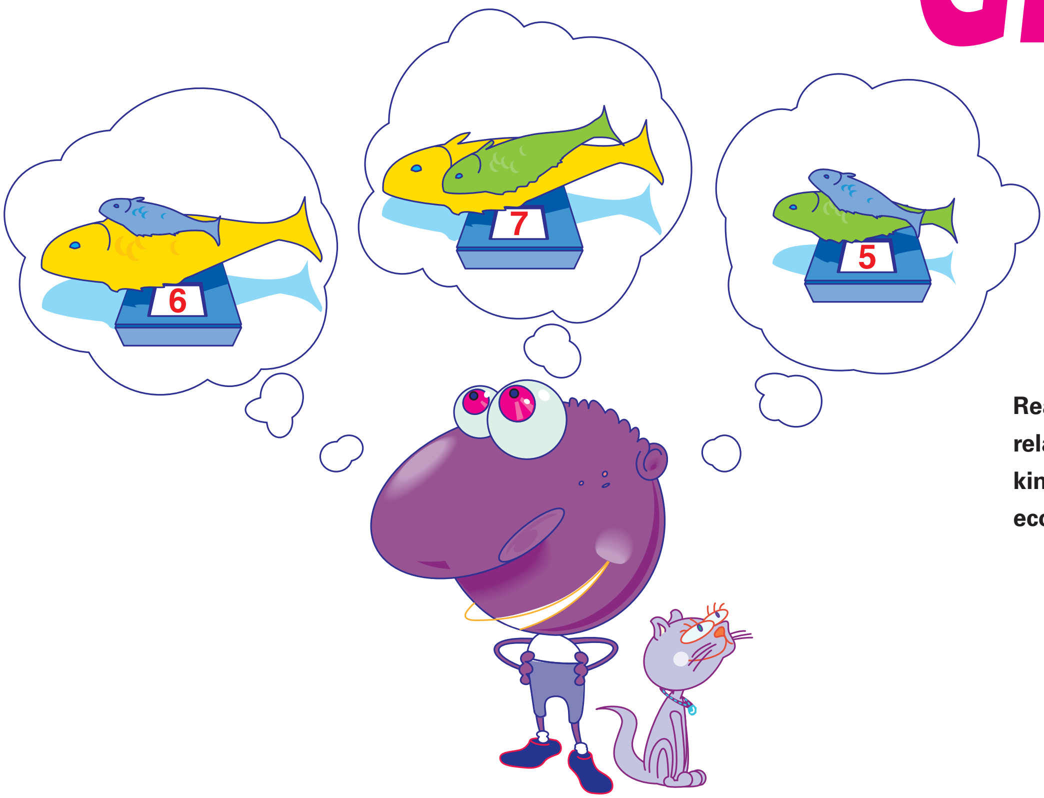


Figure This! How much does each fish weigh?

Hint: How much do the fish weigh all together?

Reasoning about quantities and how they are related is essential in the study of algebra. This kind of reasoning is used by engineers, scientists, economists, statisticians, and psychologists.

Answer:
The little fish weighs 2 pounds; the medium-sized fish weighs 3 pounds, and the big fish weighs 4 pounds.

Figure This!

Get Started:

What happens if you combine the weights on each scale?

Complete Solution:

- There are many ways to solve this problem. One is to add the weights of the fish on all three scales:

  2 big fish + 2 medium fish + 2 little fish = 18 pounds.

  This means that:

  1 big fish + 1 medium fish + 1 little fish = 9 pounds

  But,

  1 big fish + 1 medium fish = 7 pounds

  Subtract and find,

  the little fish weighs 2 pounds.

  If the little fish weighs 2 pounds, and

  1 little fish and 1 medium fish = 5 pounds

  then, 1 medium fish weighs 3 pounds.

  Also, 1 little fish and 1 big fish = 6 pounds

  So, 1 big fish weighs 4 pounds.

- If you put the fish with weights 6 and 7 together on the scales, then

  1 little fish + 1 medium fish + 2 big fish = 13 pounds

  But,

  1 little fish + 1 medium fish = 5 pounds.

  Subtract and find,

  2 big fish = 8 pounds

  So, 1 big fish must weigh 4 pounds.

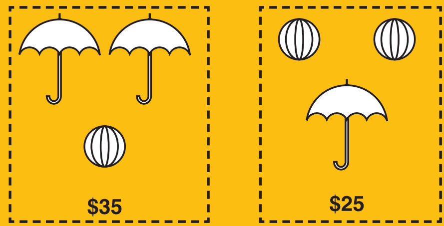
Consider the weights in the two balloons showing 6 and 7. The weights differ by one pound and the big fish is on both scales, so the medium fish weighs 1 pound more than the little one. The third scale shows that the combined weights of the little and medium fish is 5 pounds. Since the medium fish weighs 1 pound more than the little one, the little one must weigh 2 pounds and the medium one 3 pounds.

Try This:

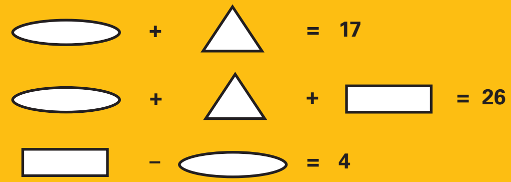
- Look in your cupboard. What combinations of cereal, milk, bread, and peanut butter could you eat to get the minimum daily requirements of vitamins and minerals?
- Check the menu at your favorite fast food restaurant. If you have $10, what can you buy to use as much of your $10 as possible?

Additional Challenges:
(Answers located in back of booklet)

1. How much does the umbrella cost? How much is a beach ball?



2. In the equations shown below, the oval, the triangle, and the rectangle each represent a value. How much is each one worth?



3. Solve:

   The Number of Apples + The Number of Bananas = 170
   The Number of Apples – The Number of Bananas = 90

Things to Think About:

- If three grapefruit cost $1.00, why does one cost $0.34?
- How can pictures help you solve math problems?
- Is it easier to use words or symbols when describing mathematical relationships?
- Why do people use letters, such as FBI or NASA, to identify organizations?
- In math and science, certain letters and symbols are commonly used to refer to specific quantities, such as *d* for distance. What are some other letters and symbols that are used?

Did You Know That?

- Another way to write the problem in the Challenge, where x is the weight of the little fish, y is the weight of the medium-sized fish, and z is the weight of the big fish, is:

$$x + y = 5$$
$$y + z = 7$$
$$x + z = 6$$

- As a young woman, Mary Fairfax Somerville (1780–1872) discovered algebra while reading a ladies' fashion magazine. In its pages, she saw a question with "strange looking lines mixed with letters, chiefly *X*'s and *Y*'s" (Perl, 1978). She continued to study mathematics until she was 92.

- The National Council of Teachers of Mathematics (NCTM) recommends that algebraic thinking start in pre-kindergarten using concrete, pictorial, verbal, and symbolic representations.
- Spreadsheet formulas require algebra.
- Some formulas are found in popular reading materials. For example, the formula for Body Mass Index (BMI) is found in *The Old Farmers' Almanac* (2000). *(See Challenge 21)*

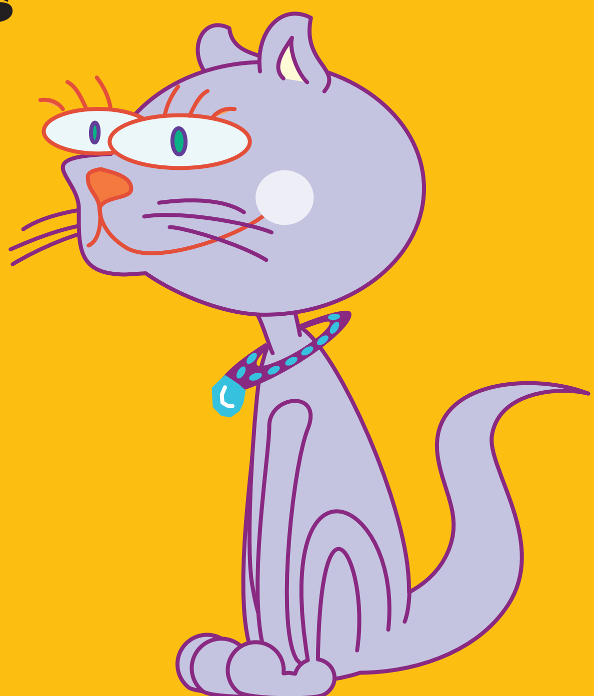
Resources:

Books:

- Greenes, C., and C. Findell. *Groundworks: Algebra Puzzles and Problems, Grades 4-7*. Chicago, IL: Creative Publications, 1998.
- Kindt, M., K. Abels, M. Abels, M. Myers, and M. Pligge. *"Comparing Quantities."* In *Mathematics in Context*. Chicago, IL: Encyclopaedia Britannica Educational Corp., 1998.
- Kroner, L. *In the Balance: Algebra Logic Puzzles.* Alsip, IL: Creative Publications, 1997.
- Perl, T. *Math Equals: Biographies of Women Mathematicians + Related Activities.* Menlo Park, CA: Addison-Wesley, 1978.
- *The Old Farmer's Almanac 2000.* Dublin, NH: Yankee Publishing Incorporated, 1999.

Notes:

Axis



Math Challenge #59

JAPANESE FLOORS

Figure This!

Math Challenges for Families

"How do I cover thee??

Let me *count* the ways."

Figure This! **Tatami mats help define standard sizes of Japanese rooms. Floors are completely covered by these mats which are about 3 feet by 6 feet. In how many different ways can the mats be arranged to cover a 6 foot by 15 foot floor?**

Hint: Sketch some arrangements of tatami mats that could cover the floor.

Arrangements of geometric shapes influence living space, storage space, commercial displays, and the natural environment. The study of shapes is important to architects, pipefitters, construction workers, and biologists.

Answer:

There are eight different ways, each using five mats.

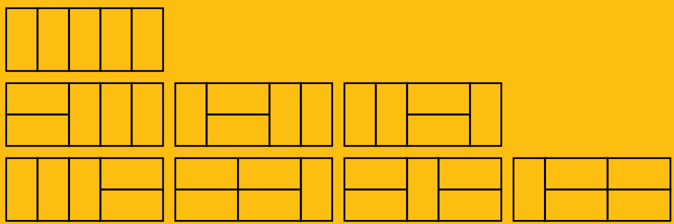
Figure This!

Get Started:

How many mats are needed to cover the floor if they are all lined up in the same way? What are the dimensions of the area covered by two mats? How does that affect their placement?

Complete Solution:

The eight possible arrangements are shown below.



Try This:

- The ratio of the lengths of the sides of a tatami mat is 2 to 1. Dominos have this same ratio. Use dominoes to model different patterns for room sizes.
- Look up tatami mats and their use in Japanese culture in an encyclopedia or on the web.
- In an encyclopedia or on the web, look up bees and how their honeycombs are built.

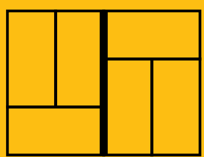
Additional Challenges:
(Answers located in back of booklet)

1. a. Build a table as below to see how many ways you can use tatami mats to cover the floor of the rooms with sizes listed.

Room Size	6x3	6x6	6x9	6x12	6x15	6x18	6x21	6x24
No. of Ways								

   b. Describe the sequence found in part *a*.

2. Japanese homebuilders do not like patterns where four corners come together, believing that is a sign of bad luck. How many of the arrangements that you found in the Challenge have places where four corners meet?
3. When constructing homes from brick, builders try to avoid "fault lines," seams that run the height or length of a wall. For example, the arrangement of bricks shown below has a vertical fault line.



Find a possible fault-free design using more than one brick. In other words, find a rectangle that contains no fault lines, either vertical or horizontal.

Things to Think About:

- In Japan, the size of a room is often described in terms of tatami mats—for example, a four-mat room or a six-mat room. What are some possible dimensions for a ten-mat room?
- Why do you think the size of tatami mats has helped set the standards for room size in Japan?
- Why do bricklayers try to avoid fault lines?

Did You Know That?

- A tatami mat consists of a thin layer of tightly woven rushes, on top of a coarser mat of straw bound with cords. The upper mat is sewn to the lower one with twine. The mat is firm but not hard.
- In the United States, building dimensions often are dictated by the size of a standard sheet of plywood: 4 feet by 8 feet.
- The sequence in Additional Challenge 1b is a Fibonacci sequence. Fibonacci was the nickname for the Italian mathematician Leonardo of Pisa. In his book *Liber abaci*, published in 1202, Fibonacci described the following problem:

  How many pairs of rabbits can be produced from a single pair in a year, if every month each pair bears a new pair which, from the second month on, also becomes productive?

  The resulting sequence of the pairs at the end of each month, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,... is known as the Fibonacci sequence.

Resources:
Books:

- Gardner, M. *New Mathematical Diversions.* Washington, DC: Mathematical Association of America, 1995.
- Garland, T. *Fascinating Fibonaccis: Mystery and Magic in Numbers.* Palo Alto, CA: Creative Publications, 1987.

Websites:

- **www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Fibonacci.html**
- **www.swifty.com/apase/charlotte/@A4.html**

Math Challenge #60

NUMBER TRICKS

Figure This!
Math Challenges for Families

Bet I can guess your color!

MAGIC NUMBERS

Take the number of your birth month.
Add 32.
Add the difference between 12 and the number of your birth month.
Divide by 2.
Add 3.
The result is your special number.

Figure This! Find your special number. If $a = 1$, $b = 2$, and so on, what letter corresponds to your special number? Write the name of a color that begins with this letter. Helix bets you chose yellow. Why?

Hint: Find the special numbers for a few of your family members. How do the numbers compare?

In algebra, symbols are used to generalize arithmetic procedures. Many people use algebraic procedures everyday in working with spreadsheets, monitoring dosages of medicine, and applying formulas.

Answer:
The special number is 25, which corresponds to the letter y.
Yellow is the most common color that begins with y.

Figure This!

Get Started:

Let *B* represent the number of your birth month. What happens as you go through the steps?

Complete Solution:

If B represents the number of the birth month, each step can be written as shown in the table below.

Step	Algebraic Representation
Take the number of your birth month.	B
Add 32.	$B + 32$
Add the difference of 12 and the number of your birth month.	$B + 32 + (12 - B)$
Divide by 2.	$\frac{B + 32 + (12 - B)}{2}$
Add 3.	$\frac{B + 32 + (12 - B)}{2} + 3$

The final step can be simplified as follows:

$$\begin{aligned} \frac{B + 32 + (12 - B)}{2} + 3 &= \frac{(B - B) + (32 + 12)}{2} + 3 \\ &= \frac{0 + 44}{2} + 3 \\ &= 22 + 3 \\ &= 25 \end{aligned}$$

Therefore, no matter what number you choose for *B*, it is added and then subtracted and does not affect the answer. The answer is always 25. The 25th letter of the alphabet is *y*. The most obvious color beginning with *y* is yellow.

Try This:

- Create a number trick of your own. Try the trick on a friend; then explain how the trick works.
- Computer spreadsheets, interest-bearing savings accounts, and many medicine dosages involve algebraic formulas. Ask your friends and family members to describe how they use formulas in their daily lives.

Additional Challenges:
(Answers located in back of booklet)

1. Write a three-digit number, then make a six-digit number by writing the three digits again. Divide the six-digit number by 7, then the result by 11, and then that result by 13. The answer is your original three-digit number. Explain how this trick works.
2. Write a three-digit number in which each digit is different. Reverse the digits. Subtract the larger number from the smaller. If you tell me the first digit of the difference, I'll tell you the result. Why does this trick work?
3. Take any number and add 15 to it. Now multiply that sum by 4. Next subtract 8 and divide the difference by 4. Subtract 12 from the quotient. If you tell me the result, I will tell you the number with which you started. How does this trick work?

Things to think about:

- Number tricks typically depend on performing an arithmetic operation, then "undoing" it. What are some pairs of operations that "do" and "undo"?
- Is it easier to follow a procedure explained in words or using algebra?
- Algebra is often referred to as a form of shorthand for arithmetic.
- Why do you think the letter *x* is used so often in algebra?

Did You Know That?

- Algebra is required for admission to most colleges.
- The word *algebra* comes from the Arabic *al-jabr*, which means "the reduction."
- If n is a whole number, $2n$ is always an even number and $2n + 1$ is always odd.
- The Greek mathematician Diophantus (about 250 AD) is sometimes called the father of algebra.

Resources:
Books:

- Boyer, C. *A History of Mathematics*. Princeton, NJ: Princeton University Press, 1985.
- Greenes, C., and C. Findell. *Groundworks: Algebra Puzzles and Problems,* Grades 6, 7, 8. Chicago, IL: Creative Publications, 1998.
- Perl. T. *Math Equals: Biographies of Women Mathematicians + Related Activities.* Menlo Park, CA: Addison-Wesley, 1978.
- Townsend, C. *World's Most Baffling Puzzles.* New York: Sterling, 1992.

Math Challenge ANSWERS

Figure This! Math Challenges for Families

Looking for answers?

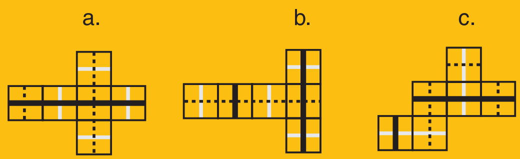
Here are the answers for the **Additional Challenges** section of each Challenge.

Figure This!

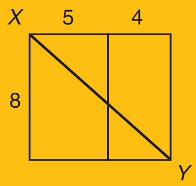
Challenge 55:

1. 40 inches.

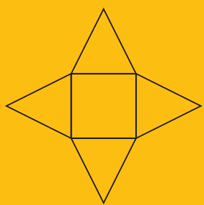
2.



3. About 12 inches. (One way to solve this problem is to form the rectangle below.)



4. One possible pattern is shown here.



Challenge 56:

1. At 20 gallons per week, about 1040 gallons. A small backyard pool contains about 8000 gallons. You could not swim in it.

2. About 21,011,744,000 gallons.

3. The price per liter is more expensive.

Challenge 57:

1. It tessellates the plane so that there is no waste except at the end of paper rolls.

2. Yes.

3. A parallelogram.

Challenge 58:

1. The umbrella costs $15; the beach ball costs $5.

2. The oval is worth 5, the triangle 12, and the rectangle 9.

3. The number of apples is 130, and the number of bananas is 40.

Challenge 59:

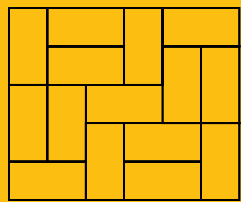
1a. A completed table is shown below.

Room Size	6x3	6x6	6x9	6x12	6x15	6x18	6x21	6x24
No. of Ways	1	2	3	5	8	13	21	34

1b. Each number after the first two is the sum of the previous two numbers.

2. 2.

3. For example: The dimensions of the smallest possible rectangle are 5 by 6, where 1 unit represents the width of a rectangle. One arrangement is shown below.



Challenge 60:

1. Because 7 • 11 • 13 = 1001, dividing by these three numbers is the same as dividing by 1001. Any six-digit number in the form *abcabc* divided by 1001 is *abc*. Another way of looking at this is that writing a three-digit number next to itself is like multiplying the original by 1001.

2. If the ones digit is 9 in a two-digit difference, the result is 99. Otherwise, the sum of the first and third digits is 9; the middle digit is always 9.

3. The net result of the steps is that you have added 1 to the original number.