Math Challenge #53 CENSUS

Figure This! Math Challenges for Families

How many PEOPLE live in the United States??

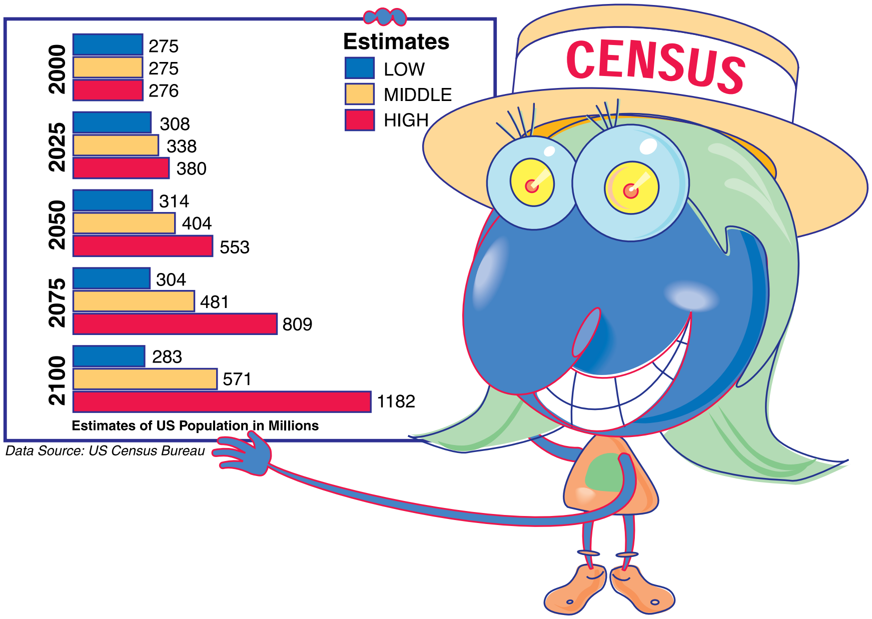


Figure This! The US Census Bureau estimates that the US population may double in the next 100 years. Other experts have suggested that the population may increase by 400% over the same period. Does Polygon's graph support these statements?

Hint: How many people would there be in 2100 if the population in 2000 doubled?

Information is often presented graphically. Politicians, business people, city managers, and government officials read and interpret graphs to predict population growth, plan for schools, and allocate funding for transportation and health programs.

Answer:

Based on the low estimate, the US population will not double by 2100. Based on the middle and high estimates, it will more than double. None of the estimates shows a 400% increase.

Figure This!

Get Started:

What was the estimated US population in 2000? What does it mean to double a value? What is a 100% increase? A 200% increase?

Complete Solution:

The estimated US population in 2000 was about 275 million people. Note that a 100% increase is the same as doubling which gives 550 million people. An increase of 400% would mean the 2000 population plus 4 times the population.

275,000,000 + (4 • 275,000,000) or 5 • 275,000,000, which is 1,375,000,000 people.

Comparing the doubled 2000 population and projected 2100 population, the low estimate is the only one that is not at least doubled. None of the estimates in the graph supports a 400% increase.

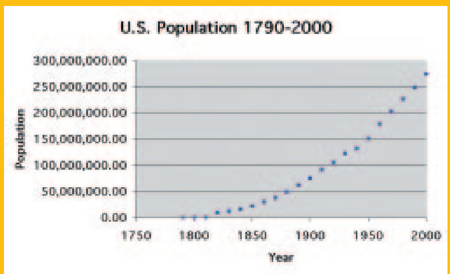
	Low Estimate	Middle Estimate	High Estimate
2000	275,000,000	275,000,000	276,000,000
2100	283,000,000	571,000,000	1,182,000,000
Double the 2000 population	550,000,000	550,000,000	552,000,000
400% increase of the 2000 population	1,375,000,000	1,375,000,000	1,380,000,000

Try This:

- Look in an atlas, almanac, or website for information about the population of your state. Has the population doubled in the past 100 years?
- Look in a newspaper or magazine to find a percentage greater than 100%. How is this value used and what does it mean?
- Find out what questions your family had to answer for the 2000 Census.

Additional Challenges:
(Answers located in back of booklet)

1. The US population has doubled several times over the past 200 years. What does the graph below tell you about the amount of time it has taken for the population to double?



2. Can a 500% increase be found simply by multiplying by 5?
3. The US Census Bureau estimates that about 1 of every 30 people in Prince Georges County, Maryland was uncounted in the 1990 census. If the reported population was 665,071, estimate the actual population.

Things to Think About:

- Why is the spread between the low, middle, and high estimates so small in the year 2000 and so large in the year 2100?
- What factors will affect US population growth in the next 100 years? What factors will affect global population growth? Will these factors be the same?
- The low estimate on the graph predicts a decrease in the US population between 2050 and 2100. What might cause such a decrease?

Did You Know That?

- The United States conducted its first census in 1790. Marshals of US judicial districts visited every home, asking for the name of the head of the family and the number of persons in each household. The reported population was 3.9 million.
- In the late 1990s, the fastest-growing states were concentrated in the South and West.
- By order of the US Constitution, a census must be conducted every 10 years. This has traditionally been done in the years ending in 0.
- Every two years, the Census Bureau conducts a sampling focused on a particular area, such as transportation or housing, to identify trends.
- The results of the US Census determine each state's number of members in the House of Representatives.

Resources:

Books:

- *The World Almanac 2000.* Mahwah, NJ: World Almanac Books, 1999.
- Maloney, B., and D. Stevens. "Census 2000: Answers Behind the Questions," *The Washington Post*, April 26, 2000.
- Nasser, H. "Census Begins Controversial Use of Statistical Procedure," *USA Today*, May 19, 2000.
- Nasser, H. "Census Predicts Ethnic Face of Nation in 100 Years," *USA Today*, January 13, 2000.

Websites:

- **fisher.lib.virginia.edu/census/background**
- **www.gov.state.md.us/gov/press/2000/may/html/census.html**
- **www.census.gov**

Math Challenge #54 CUT THE CAKE

Figure This! Math Challenges for Families

What's a FAIR SHARE??

Figure This! Ratio and five friends want to share a 9-inch square chocolate cake with marshmallow icing. How can Ratio cut the cake so that each person receives an equal share of both cake and icing?

Hint: If six people will share the whole cake, then three people will share half the cake. Don't forget the icing on the sides!

Properties of three-dimensional shapes can be understood by thinking about their two-dimensional faces. Dividing geometric shapes according to certain criteria is critical to the work of city planners, graphic designers, and real-estate developers.

Answer:

Two possible solutions are shown here.

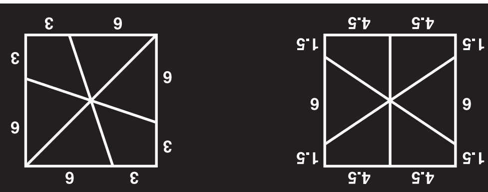


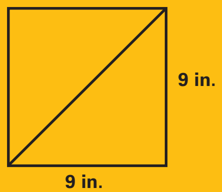
Figure This!

Get Started:

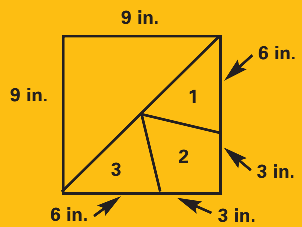
Draw the top of the cake. Divide the square in half, then try to divide one half into three equal shares. How can you make sure that the amount of icing on each piece is the same? How can you make sure that the volume of each piece is the same?

Complete Solution:

- There are many ways to approach this problem. Assuming that all pieces of the cake have the same height, the size (or volume) of each piece depends on the area of its top. Since icing is on both the top and the sides, however, each piece must also have an equal share of the perimeter of the square. If six people share the whole cake, then three people will share half the cake. One way to divide the cake in half is shown here.



Each half of the cake is 18 inches on its two outer edges. So each person should receive 18/3, or 6 inches, of the cake's outer edge. One way to make this division is shown below.



To check the areas of the three pieces, use the formula for the area of a triangle:

Area = (1/2) base • height

The height of triangles 1 and 3 in the diagram is half the width of the cake, or 4.5 inches. Since the length of each base is 6 inches, the area of each of these two triangles is:

$$A = (1/2) \bullet 6 \bullet 4.5 = 13.5 \text{ sq. in.}$$

The area of shape 2 is half the area of the entire cake, minus the area of triangles 1 and 3:

$$\text{Area of half} = (1/2) \bullet 9 \bullet 9 = 81/2 = 40.5, \text{ or } 40.5 \text{ sq. in.}$$

$$\text{Area}_{\text{Triangle 1}} = \text{Area}_{\text{Triangle 3}} = (1/2) \bullet 6 \bullet 4.5 = 13.5, \text{ or } 13.5 \text{ sq.in.}$$

$$\text{Area}_{\text{Region 2}} \text{ is Area of half of the cake} - \text{Area}_{\text{Triangle 1}} - \text{Area}_{\text{Triangle 3}}$$

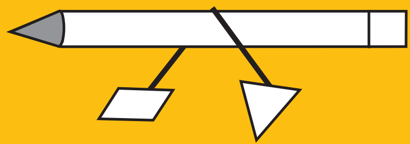
$$\text{Area}_{\text{Region 2}} = (1/2) \bullet 9 \bullet 9 - 13.5 - 13.5 = 13.5, \text{ or } 13.5 \text{ sq.in.}$$

Therefore, these three pieces represent equal shares. The other half of the cake can be divided similarly.

- There are other ways to cut the cake into six equal shares. For example, you could start by dividing the cake into two rectangles of the same size and shape.

Try This:

- Cut a rectangular piece of heavy cardboard into two pieces with different shapes that you think have the same area. One way to determine if the areas are roughly the same is to compare the weights of the two pieces. Assuming that the thickness of the cardboard is constant, if the weights are the same, then the areas must be the same. You can compare weights by taping each piece to one end of a string, marking the midpoint of the string, then placing a pencil under the midpoint, as shown in the following diagram. If the string does not move, the pieces balance.



Additional Challenges:
(Answers located in back of booklet)

1. How would you share a 9-inch square cake among 5 people so that each person receives an equal share of both cake and icing?
2. In how many different ways can you divide a rectangle into two pieces of the same size and shape?
3. Using one cut, how can you divide the triangle below into two pieces with equal areas?

Things to Think About:

- Would the way in which you solved the Challenge have changed if the cake had been rectangular but not square?
- Would you prefer a piece of cake with a regular shape or an irregular shape?
- How do you cut an apple in half?
- Which half of the sandwich below would you choose? Why?

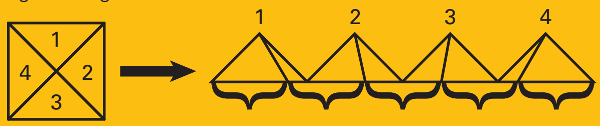
- What factors should be considered when dividing waterfront property into lots?

Figure This!

- Would it have been easier to share a round cake in the Challenge?
- In what other situations is it important to divide something into equal parts?

Did You Know That?

- According to the *1999 Guinness Book of Records*, the world's largest cake was prepared by EarthGrains Bakery to celebrate the centennial of Fort Payne, Alabama in 1989. Shaped like the state, it weighed a little more than 58 tons and used 16,209 pounds of frosting.
- If a cake is square and of the same height everywhere, then you can divide it into five shares of equal area using the idea behind the following drawing.



The square is separated into four triangles and rearranged as shown. If the sum of the bottoms of the triangles on the right is cut into five equal parts, then by connecting the tops of the triangles to the cuts on the bottom, there are 5 equal servings.

- The Italian mathematician Francesco Bonaventura Cavalieri (1598–1647) developed a theorem that states if two three-dimensional solids have the same height and all cross-sections of the solid parallel to the base and at equal distances from the bases have equal areas, then the solids have the same volume.

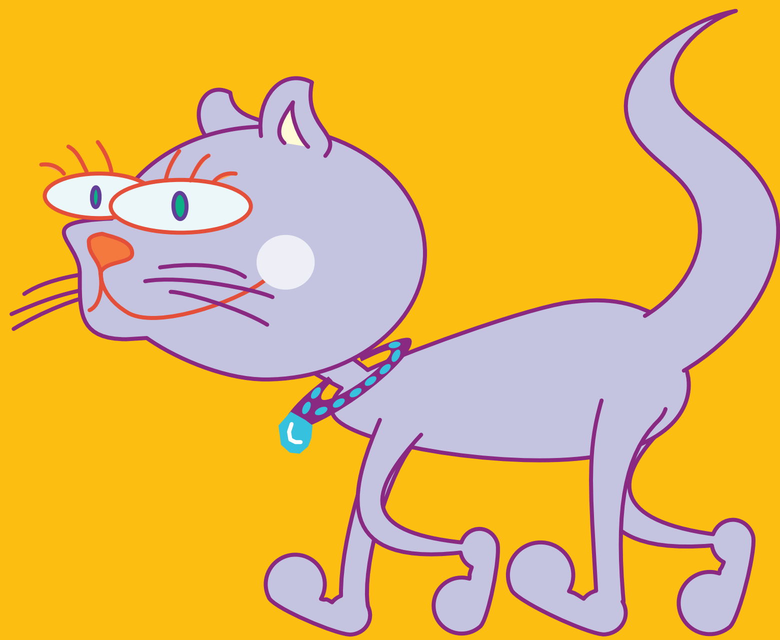
Resources:

Books:

- *The Guinness Book of Records 1999.* New York, NY: Bantam Books, 1999.
- James, R. *Mathematics Dictionary.* New York, NY: Chapman & Hall, 1992.

Notes:

Axis

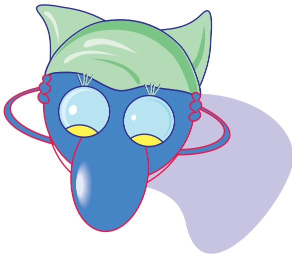


Math Challenge #55

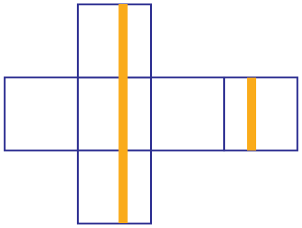
DECORATING BOXES

Figure This! Math Challenges for Families

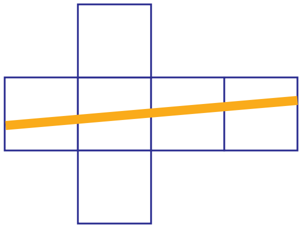
How do you print a ribbon on a box?

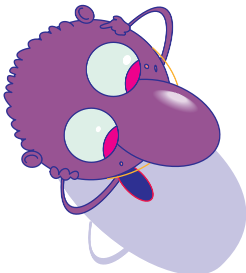


1.

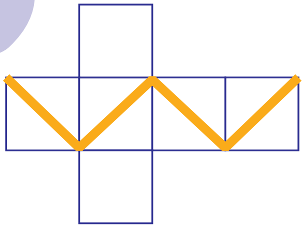


2.





3.



4.

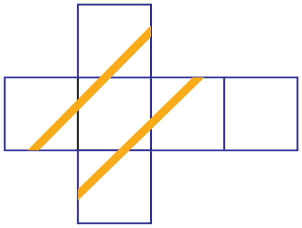


Figure This! **Which of the patterns can be folded into a box with an orange ribbon printed continuously all the way around it?**

Hint: Think about folding each pattern to make a box. Do the ends of the orange ribbon meet?

Two-dimensional patterns are often used to design three-dimensional objects. Visualizing three-dimensional objects from two-dimensional patterns is important for architects, artists, designers, model manufacturers, and doctors.

Answer:
Patterns 1, 3, and 4.

Figure This!

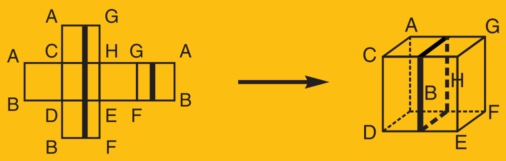
Get Started:

Draw each figure on a sheet of paper, then cut out the pattern. What happens when you fold each pattern into a box?

Complete Solution:

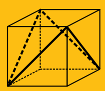
One way to solve the problem is to assign letters to matching corners as shown in the diagram below. Fold the pattern into a box and see if the ribbon matches.

1.

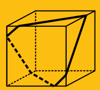


2. In pattern 2, the ends of the ribbon will not meet.

3. Pattern 3 is pictured below.



4. Pattern 4 is pictured below.

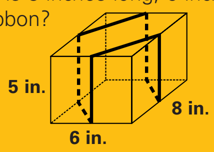


Try This:

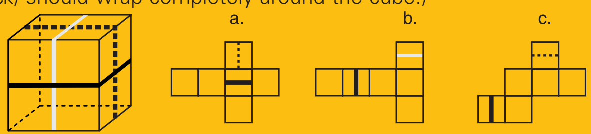
- Find a pattern, not in the challenge, that folds into a box so that a continuous ribbon is formed when the box is put together.
- Determine how to wrap a package using the least amount of ribbon if it has to touch all six faces.
- Find the US Postal Service rules for wrapping and taping boxes for shipping.
- Some products are sold in boxes with unusual shapes. Find some of these boxes, then unfold them and examine their patterns. Examine the patterns used to make them.

Additional Challenges:
(Answers located in back of booklet)

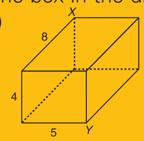
1. A rectangular box is tied with a ribbon so that the ribbon crosses the box at the midpoints of its sides. If the box is 8 inches long, 6 inches wide, and 5 inches high, how long is the ribbon?



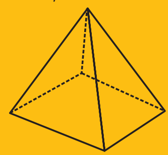
2. Complete each of the following diagrams so that it can be folded into a cube exactly like this one. (Each of the three lines (white, dotted and black) should wrap completely around the cube.)



3. What is the length of the shortest string that can stretch from point X to point Y along the outside of the box in the diagram below? (The box's dimensions are given in inches.)



4. Draw a two-dimensional pattern that, when folded, will make a square pyramid as shown below.



Things to Think About:

- Which patterns that fold to make a box might a manufacturer use? Why?
- Why are parallel lines important when drawing two-dimensional pictures to represent three-dimensional shapes?
- How do painters create two-dimensional pictures that appear to have three dimensions?
- What two-dimensional shape is used to make a quart milk carton?
- The images you see at your local movie theater are stored on flat strips of film. Why do they appear to be three-dimensional on the screen?
- Why are parallelograms used in making cardboard tubes for rolls of paper towels?

Did You Know That?

- The shortest air route from New York to London passes over Iceland. It is part of a great circle, not a straight line.
- A CAT scan is based on the x-ray principle: as x-rays pass through the body they are absorbed or weakened at differing levels creating a matrix or profile of x-ray beams of different strength. This x-ray profile is registered on film, thus creating an image. (CAT stands for Computerized Axial Tomography.)
- Hospitals often use a continuous colored line on the floor to direct patients from place to place in the building.
- Some historical sites use lines on the pavement to indicate the path of self-guided tours. One example is the Freedom Trail in Boston, Massachusetts.

Resources:
Books:

- School Mathematics Project. *Shape Review Book*. Cambridge, England: Cambridge University Press, 1991.
- Winter, M., G. Lappan, E. Phillips, and W. Fitzgerald. *Middle Grades Mathematics Project: Spatial Visualization*. Menlo Park, CA: Addison-Wesley Publishing Company, 1986.

Website:

- **www.imaginis.com/ct-scan/how_ct.asp**

Math Challenge #56
DRIP DROPS

FigureThis!
Math Challenges for Families

How much water do you waste?

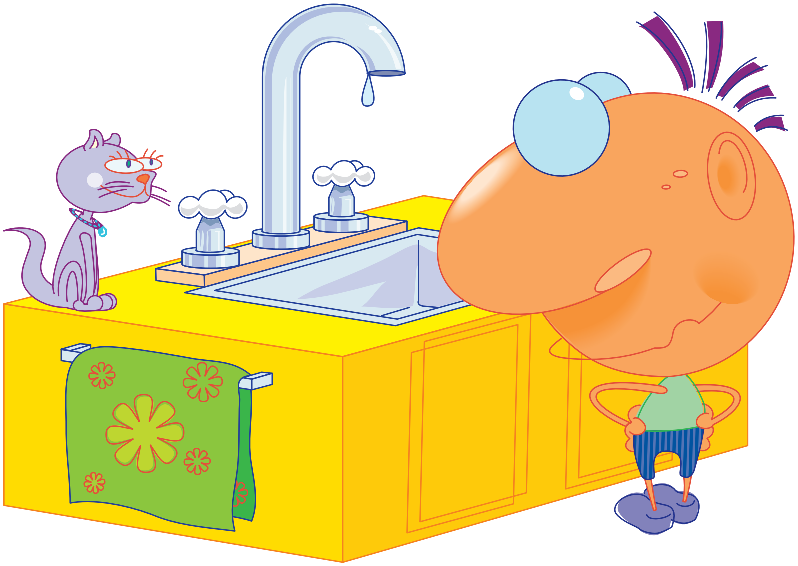


Figure This! **A faucet drips every 2 seconds. In 1 week, how much water goes to waste — enough to fill a glass, a sink, or a tub?**

Hint: One teaspoon holds about 20 drops. There are 96 teaspoons in a pint, and 8 pints in a gallon.

Mathematics can be used to describe, estimate, and measure environmental factors and to communicate this information to the public. This information is used by those interested in the environment, consumers, and governmental agencies.

Answer:
In 1 week, this leaky faucet could more than fill a kitchen sink, but not a standard tub.

Figure This!

Get Started:

How much water drips in an hour? How long would it take for the drips to fill a 1-gallon container?

Complete Solution:

- One drop every 2 seconds is equivalent to 30 drops per minute, or about 1 1/2 teaspoons per minute. This corresponds to 90 teaspoons per hour (about 15 oz. or a bit less than 2 cups.). As shown in the following equation, the faucet would leak about 20 gallons per week.

$$\frac{1\ \cancel{\text{drop}}}{2\ \cancel{\text{sec}}} \cdot \frac{60\ \cancel{\text{sec}}}{1\ \cancel{\text{min}}} \cdot \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \cdot \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{dy}}} \cdot \frac{7\ \cancel{\text{dy}}}{1\ \text{wk}} \cdot \frac{1\ \cancel{\text{tsp}}}{20\ \cancel{\text{drops}}} \cdot \frac{1\ \cancel{\text{pt}}}{96\ \cancel{\text{tsp}}} \cdot \frac{1\ \text{gal}}{8\ \cancel{\text{pts}}} \approx 20\ \text{gal/wk}$$

- Another way to approach this problem is to create a table like the one shown below:

1 drop per 2 seconds	60 seconds per minute	30 drops per minute
30 drops per minute	60 minutes per hour	1800 drops per hour
1800 drops per hour	24 hours per day	43,200 drops per day
43,200 drops per day	7 days per week	302,400 drops per week
302,400 drops per week	20 drops per teaspoon	15,120 teaspoons per week
15,120 teaspoons per week	96 teaspoons per pint	157.5 pints per week
157.5 pints per week	8 pints per gallon	About 20 gallons per week

Try This:

- The following table shows the amount of water used by a typical American for some basic tasks. Use this information to estimate how much water you use in one day.

USE	AVERAGE AMOUNT *(in liters)*	AVERAGE AMOUNT *(in gallons)*
taking a bath	110	29
taking a shower	75	20
flushing a toilet	22	6
washing hands, face	7	2
getting a drink	1	0.3
brushing teeth	1	0.3
doing dishes (one meal)	30	8
cooking (one meal)	18	5

Table from Billstein, R., et al. *A Problem Solving Approach to Mathematics for Elementary Teachers.* Menlo Park: Benjamin Cummings Publishing Co. 1981.

Based on the information given in the table above, how much water does your family use in one day?

- Call your local water company or the city government to find the cost of water per gallon in your community.

Additional Challenges:
(Answers located in back of booklet)

1. If a faucet drips once every 2 seconds, how much water would it waste in 1 year? Could you swim in it?
2. There are about 101,018,000 households in the United States. If 1 of every 5 households has a leaking faucet that drips once every 2 seconds, how much water is wasted by leaking faucets each year?
3. Which is more expensive, gas at $1.56 per gallon or at $0.42 per liter?

Things to Think About:

- What could you do to conserve water at home?
- Where does your community get its water? Is this a renewable source?
- What happens to the water that goes down your drain?
- How much water does it take to wash a car?
- According to the US Environmental Protection Agency, 21% of watersheds in the nation had serious pollution problems in 1997.

Did You Know That?

- Approximately 74% of the earth's surface is covered with water. However, only 3% of this is fresh water. Of the fresh water, only about 1% is contained in lakes, rivers, and wetlands.
- If one in every five households has one dripping faucet, for one year, this is equivalent to nearly four hours of flow of Niagara Falls.
- A cloudburst has about 113 drops per square foot per second; a moderate rain has about 46; and a drizzle has about 14.
- The city of Los Angeles offers its residents free low-flush toilets to help them conserve water and save money. The toilets use only 1.6 gallons per flush.
- In some rural areas, people obtain their water from private wells. In others, water is trucked to a central cistern for distribution.

Resources:
Books:

- *The World Almanac 2000.* Mahweh, NJ: World Almanac Books, 1999.
- *The Old Farmer's Almanac 2000.* Dublin, NH: Yankee Publishing, 1999.

Websites:

- **www.theplumber.com/h_index.html**
- **www.stemnet.nf.ca/CITE/water.htm#General**
- **freespace.virgin.net/john.cletheroe/usa_can/ny/niagara.htm**
- **www.monolake.org/socalwater/ultralow.htm**

Math Challenge

ANSWERS

Figure This!
Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

Figure This!

Answers to Additional Challenges:

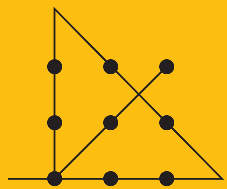
Challenge 49:

1. There would be 10 rows of dots in the pattern 1, 3, 5, 7, 9, 9, 7, 5, 3, 1.

2a. The next pattern has 10 dots as shown.

2b. It is a general pattern to give the number of dots in the n th figure. It gives the numbers 1, 3, 6, 10, ...

3. One possible solution is shown in the diagram below.



4. One possible solution is shown in the diagram below.

Challenge 50:

1. $H = 5 + 7d$, for $d \geq 1$

2a. Large dogs age faster after age 2; small dogs age faster initially.

2b. As with a large dog, a small dog will have a human age of 40 after 5 years.

2c. When the dogs are 10 years old.

3. The rate of change with each passing year is 5 not 7. It is constant after year 2, not year 1. The comparable formula is $H = 15 + 5d$, for $d \geq 2$.

4. 5.

Challenge 51:

1. 10.

2. 1,000,000,000.

3. Without the start bar, this portion of the bar code appears as follows:

Challenge 52:

1. About 425 mice.

2. About 20 trout.

3. Student B will report the largest estimate; student C will report the smallest.

Challenge 53:

1. The number of years between doublings has been decreasing.

2. No.

3. Approximately 688,004.

Challenge 54:

1. Divide the area and perimeter of the cake's square top into five equal portions.

2. Infinitely many with one cut, as long as the cut passes through the center of the rectangle.

3. One way is to find the midpoint (M) of the base, $\overline{BC}$. A line from A to M will work, since the height is the same for each new triangle, and the bases are both one-half the original length.

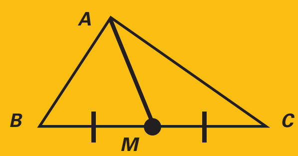
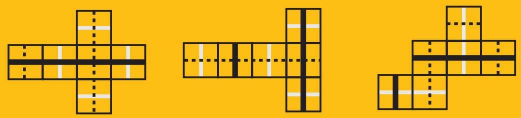


Figure This!

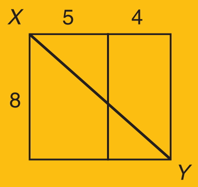
Challenge 55:

1. 40 inches.

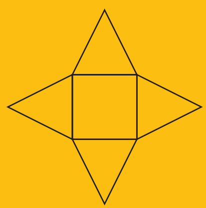
2. a. b. c.



3. About 12 inches. (One way to solve this problem is to form the rectangle below.)



4. One possible pattern is shown here.



Challenge 56:

1. At 20 gallons per week, about 1040 gallons. A small backyard pool contains about 8000 gallons. You could not swim in it.

2. About 21,011,744,000 gallons.

3. The price per liter is more expensive.

Challenge 57:

1. It tessellates the plane so that there is no waste except at the end of paper rolls.

2. Yes.

3. A parallelogram.

Challenge 58:

1. The umbrella costs $15; the beach ball costs $5.

2. The oval is worth 5, the triangle 12, and the rectangle 9.

3. The number of apples is 130, and the number of bananas is 40.

Challenge 59:

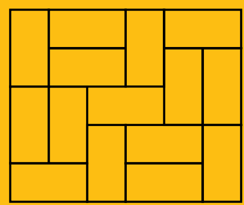
1a. A completed table is shown below.

Room Size	6x3	6x6	6x9	6x12	6x15	6x18	6x21	6x24
No. of Ways	1	2	3	5	8	13	21	34

1b. Each number after the first two is the sum of the previous two numbers.

2. 2.

3. For example: The dimensions of the smallest possible rectangle are 5 by 6, where 1 unit represents the width of a rectangle. One arrangement is shown below.



Challenge 60:

1. Because 7 • 11 • 13 = 1001, dividing by these three numbers is the same as dividing by 1001. Any six-digit number in the form *abcabc* divided by 1001 is *abc*. Another way of looking at this is that writing a three-digit number next to itself is like multiplying the original by 1001.

2. If the ones digit is 9 in a two-digit difference, the result is 99. Otherwise, the sum of the first and third digits is 9; the middle digit is always 9.

3. The net result of the steps is that you have added 1 to the original number.