Math Challenge #49

ANGOLAN SAND PICTURES

Figure This! Math Challenges for Families

WHAT'S missing?

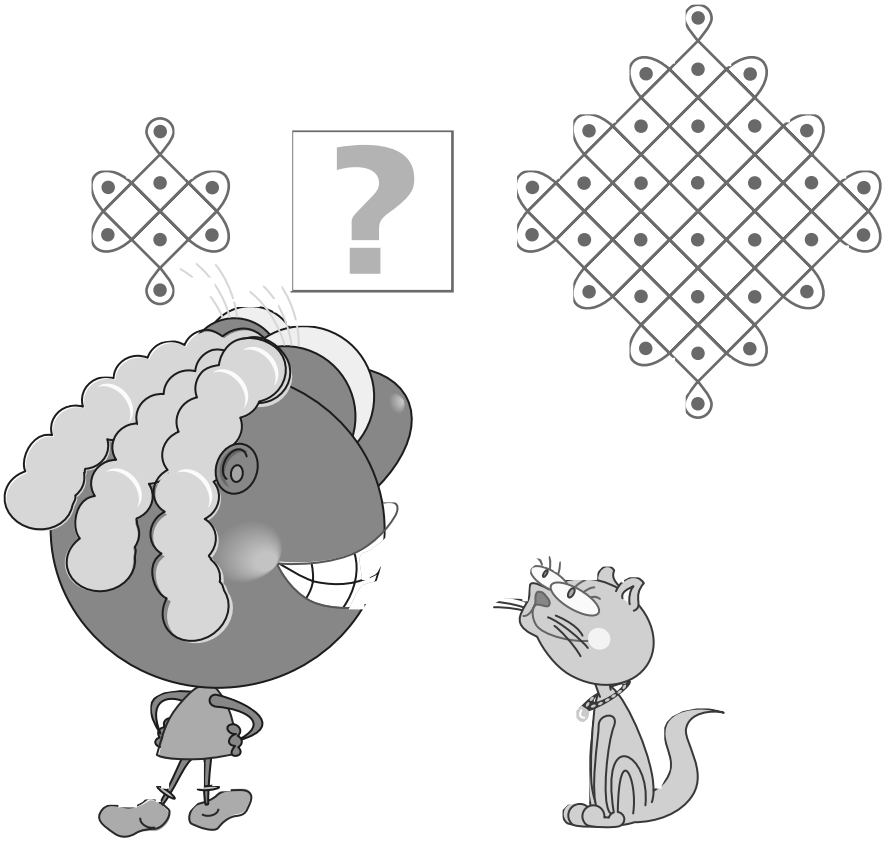


Figure This! **The designs shown here are typical of a *sona*, an African sand tracing. If the missing design is part of a pattern of increasing size, what might the missing design look like?**

Hint: Look for patterns in the tracing, as well as in the number and arrangement of dots in each figure.

Most people throughout history have created designs and patterns to express their cultures. Many such designs also feature an underlying mathematical pattern. Studying mathematical designs and patterns helps archaeologists and anthropologists understand ancient cultures.

Answer:
The missing design could be:

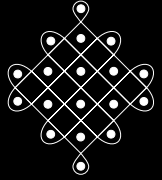


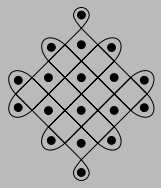
Figure This!

Get Started:

How are the dots arranged? Are they in rows? How are the loops arranged? What are the relationships among the dots, the loops, and the squares? Can you determine the number of dots in the missing figure?

Complete Solution:

- One way to approach this problem is to examine the pattern of dots. In the first figure, there are four rows of horizontal dots in the pattern 1, 3, 3, 1. In the third figure, there are eight rows of dots in the pattern 1, 3, 5, 7, 7, 5, 3, 1. It seems logical, therefore, that the missing figure would include six rows of dots in the pattern 1, 3, 5, 5, 3, 1.



- Another way to look at the pattern is to count the "loops" in the design. The first figure has two loops per side, while the third figure has four loops per side. In this case, it seems reasonable for the missing figure to have three loops on each side.
- A third way to consider the problem involves counting squares. The first design is a two-by-two square; the third is a six-by-six square. Therefore, the second might be a four-by-four square.

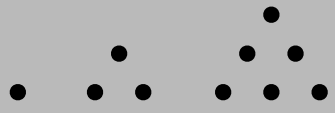
Try This:

- The sona patterns were drawn by tracing in the sand. Trace the outline of the designs in the Challenge without lifting your finger.
- Arrange five rows of dots in the pattern 1, 3, 5, 3, 1. Draw a design that encloses each dot in its own "cell," with no two dots in the same cell.
- Construct your own sequence of patterns of dots and designs. Remove one figure from the sequence. Challenge a family member to draw the missing pattern.
- Research Native American culture looking for patterns and designs in clothing and pottery.

Additional Challenges:

(Answers located in back of booklet)

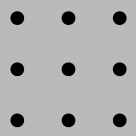
1. If the design shown in the challenge were expanded to a fourth figure, how would the dots be arranged?
2. The diagram below shows three figures in a pattern.



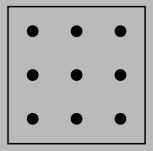
   a. If the pattern continues, what do you think the next figure will look like?

   b. The number of dots, S, in each figure can be found using the formula $S = \frac{n(n+1)}{2}$ where n is the number of the figure. How is this formula related to the pattern?

3. Without lifting your pencil, draw four straight line segments on the pattern below so that every dot lies on at least one of the line segments.



4. Draw two triangles in the square to create nine cells, so that each cell contains exactly one dot.



Things to Think About:

- How are fruits or cans stacked in grocery displays?
- What products use repeating designs in their advertisements or company logos?
- Hopscotch patterns are typically passed from adult to child or child to child. What hopscotch designs do you know?
- Are there other ways to describe the pattern in Additional Challenges 2?

Did You Know That?

- The sand patterns in the Challenge were created by the Tchokwe tribe of northeastern Angola. Members of this tribe used these designs for telling stories.
- Anthropologists Dorothy Washburn and Donald Crowe have studied linking of geometric patterns to cultural identification.
- Some African cultures, such as the Ndebele, make decorations that use line symmetry. These patterns contain no curves, in contrast to the Angolan patterns in the Challenge.
- Jhane Barnes is a designer who uses mathematics to form patterns to weave into cloth for her line of clothing.
- Many entrance examinations for higher education, including the Dental Aptitude Test (DAT), include questions involving the recognition and extension of patterns.

Figure This!

Resources:

Books:

- Gerdes, P. *Geometry from Africa: Mathematical and Educational Explorations.* Washington, DC: The Mathematical Association of America, 1999.
- Nelson, D., G. Joseph, and J. Williams. *Multicultural Mathematics: Teaching Mathematics from a Global Perspective.* Oxford: Oxford University Press, 1993.
- Washburn, D., and D. Crowe. *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis.* Seattle, WA: University of Washington Press, 1988.
- Zaslavsky, C. *Africa Counts: Number and Pattern in African Culture.* Brooklyn, NY: Lawrence Hill Books, 1990.

Website:

- **www.jhanebarnes.com**

Notes:

Tangent

Math Challenge #50 ANIMAL AGES

Figure This! Math Challenges for Families

Do dogs age faster than people??

Large Dog's Age in Years	0	1	2	3	4	5	6	7
Equivalent Human Age	0	12	19	26				

http://petcare.umn.edu/Fun/DogAge.html

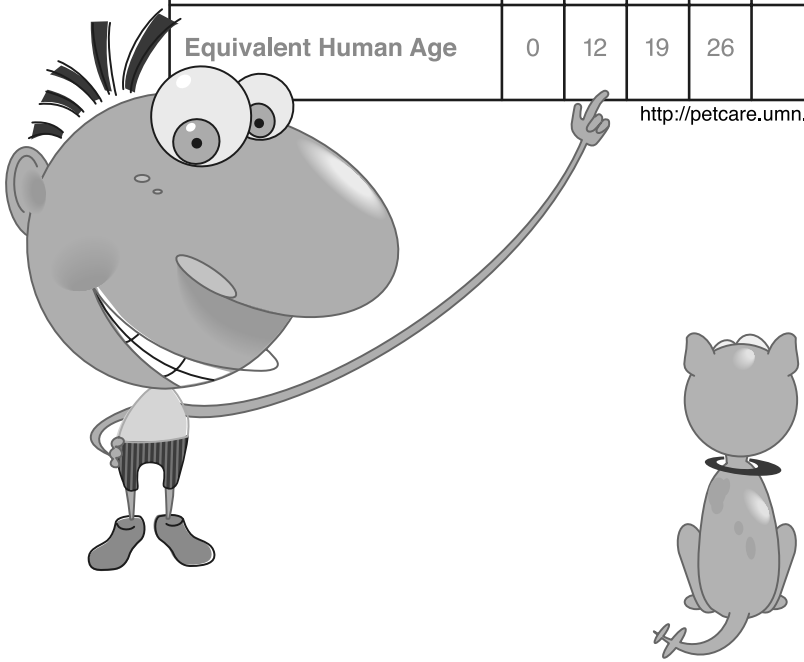


Figure This! Ratio's dad is 35 years old. When will his dad and his newborn Irish setter puppy be the same age in human years?

Hint: The growth rate is different the first year. After the first year, how many human years does a dog age each year?

Information is often converted from one system of measurement to another. Statisticians, scientists, and engineers all use conversion formulas in their work.

Answer:
In 5 years, when Ratio's dad is 40, the dog's "human age" also will be 40.

Figure This!

Get Started:

Add a row to the table that shows Ratio's dad's age with each passing year.

Large Dog's Age in Years	0	1	2	3	4	5	6	7
Equivalent Human Age	0	12	19	26				
Ratio's Dad's Age in Years	35	36	37	38				

Complete Solution:

- After the first year, the "human age" of a large dog such as an Irish setter increases by 7 each year. Use this information to complete the table. When the dog is 5 years old, its human age will be 40. In 5 years, Ratio's dad also will be 40.

Large Dog's Age in Years	0	1	2	3	4	5
Equivalent Human Age	0	12	19	26	33	40
Ratio's Dad's Age in Years	35	36	37	38	39	40

Try This:

- Find the ages of some neighbors and friends, along with the ages of their dogs or cats. How many of them are "older" than their pets? [Note: The age table given in the Challenge is for large dogs. See the Additional Challenges for age tables for small dogs and for cats.]
- Estimate the age (in months) of a large dog whose "human age" is about the same as your own.

Additional Challenges:
(Answers located in back of booklet)

1. Find a formula that shows the relationship between the human age H of a large dog and the dog's actual age d.
2. The table below shows the relationship between actual age and "human age" for small dogs.

Small Dog's Age in Years	0	1	2	3	4	5	6
Equivalent Human Age	0	15	25	30	35		

   a. Based on this information, what type of dogs appear to age faster: large or small?

   b. When Ratio's dad was 35, if he had adopted a small newborn dog, when would the two have been the same age in human years?

   c. If a small dog and a large dog were born on the same day, when will the small dog be 10 human years younger than the large dog?

3. How would a formula for the human age of a small dog differ from the one for large dogs?
4. The following table shows the relationship between actual age and "human age" for cats.

Cat's Age in Years	0	1	2	3	4	5	6
Equivalent Human Age	0	16	24	28			

   The formula for the human age of a cat is $H = 16 + 4c$, for $c \geq 2$. When a cat is 36 in human years, how old is it in cat years?

Things to Think About:

- Given the actual age of a small dog, how could you find the actual age of a cat with the same "human age?"
- How many actual years does it take for a small dog to reach "retirement age?"
- In the Challenge, you related the years in an animal's life to a human age. How could you relate the years in a human's life to an animal's age? For example, if you were 13, what age would you be in cat years?
- How did biologists decide that a 1-year-old small dog is comparable to a 15-year-old human?
- Why do puppies grow faster in their first year of life than in later years? Is this also true for human babies?
- Do larger animals typically live longer than smaller ones?
- Which animals live the longest?

Did You Know That?

- An Asian elephant has an average life span of about 40 years. In 1998, the average life span of a US resident was about 77. The average life span of an opossum is 1 year.
- One familiar conversion formula describes the relationship between degrees Celsius and degrees Fahrenheit: $F = 1.8C + 32$.
- For an age older than 1, the graph of equivalent human age versus a large dog's age in years lies along a straight line.
- According to *The Guinness Book of Records*, the world's oldest dog was an Australian cattle-dog named Bluey. He died at the age of 29 years and 5 months in 1939. The world's oldest cat, a sphinx, was Granpa Rexs Allen, adopted from the Humane Society of Travis County (Texas), who died in 1998 at the age of 34 years, 2 months.

Resources:
Books:

- Burrill, G., M. Clifford, and R. Scheaffer. *Exploring Symbols: Data Driven Mathematics.* White Plains, NY: Dale Seymour Publications, 1987.
- Roizen, M. *Real Age: Are You As Young As You Could Be?* Scranton, PA: Harper Collins, 2000.

Figure This!

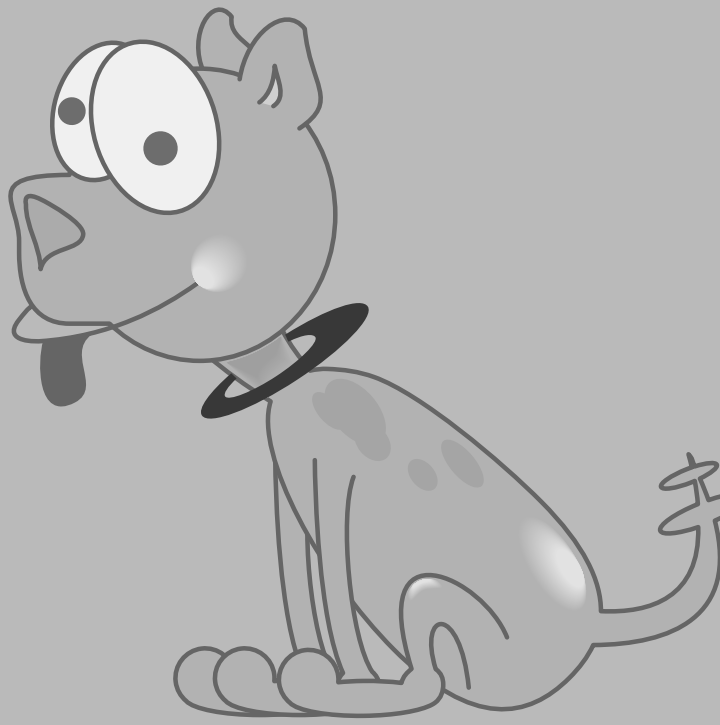
- *The Guinness Book of Records 1999.* New York: Guinness Publishing, 1999.
- *The World Almanac and Book of Facts 2000.* Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **petcare.umn.edu/Fun/CatAge.html**
- **petcare.umn.edu/Fun/DogAge.html**
- **www.ourside.com/age.html**

Notes:

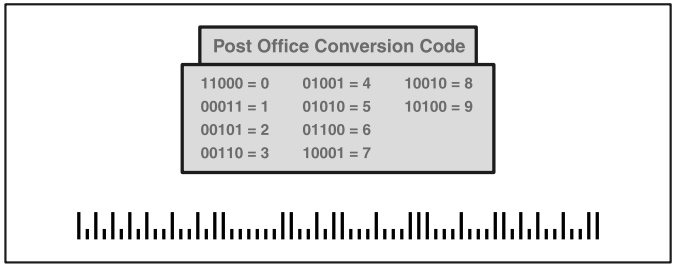
Tangent

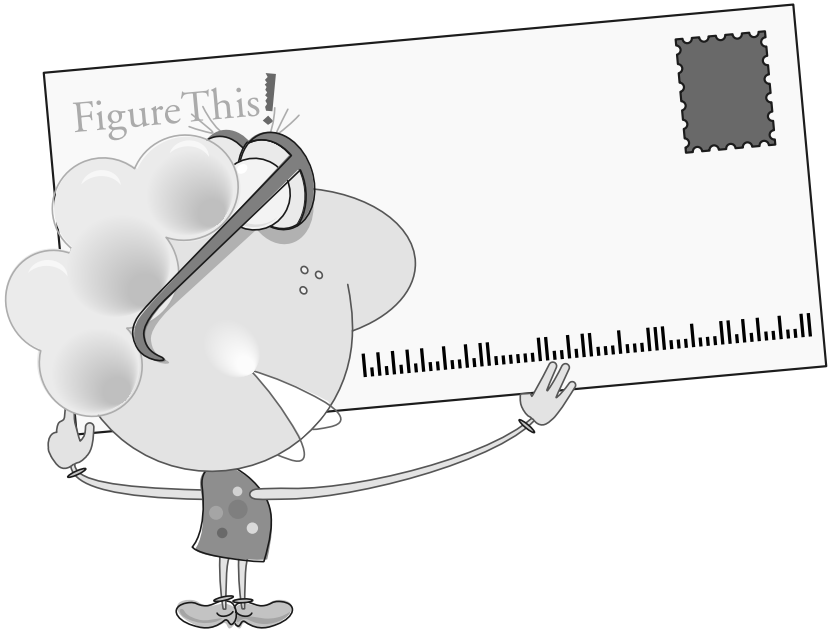


Math Challenge #51 BAR CODES

FigureThis! Math Challenges for Families

Can you DECODE a bar code?





FigureThis! The U. S. Post Office uses short and long bars to represent ZIP Codes. A short bar is 0, and a long bar is 1. The first bar on the left and the last 16 bars on the right are not part of the ZIP Code. What is the 9 digit ZIP Code on Tesselation's envelope?

Hint: Group the bars into sets of five, ignoring the first bar and the last 16 bars that are not part of the ZIP Code. For example, what number is represented by the following bars?

Bar-code systems were devised by scientists and mathematicians to speed the input and flow of data. Their use has enabled many businesses to operate more efficiently, including banks, delivery services, and supermarkets.

Answer: The ZIP Code is 59801-2717.

Figure This!

Get Started:

Convert the long and short bars to 0s and 1s. Then match each group of five to the code chart shown in the Challenge.

Complete Solution:

The first long bar in the US Postal Service bar code is a "start" bar. The next five bars on the envelope represent the sequence 01010 which represents the number 5 on the conversion chart. The following diagram shows the remaining groups of bars. The bars in the box are not a part of the ZIP Code.

The corresponding ZIP Code is 59801-2717.

Try This:

- Use the method described in the Challenge to decode the bar code on a piece of mail delivered to your home.
- The UPC code on supermarket items is another type of bar code. Look in your refrigerator or cupboard for several different products made by the same company. Check their UPC codes. Do you notice similar patterns in their codes?

Additional Challenges:
(Answers located in back of booklet)

1. How many different bar codes are possible using two long bars and three short bars?
2. If no restrictions are placed on the digits, how many different ZIP+4 Codes are possible?
3. The ZIP Code for Honolulu, Hawaii is 96826. Draw the portion of a US Postal Service bar code that would represent this ZIP Code.

Things to Think About:

- The abbreviation UPC stands for Universal Product Code. The figure below shows a typical UPC code. How does this type of bar code differ from the US Postal Service bar code?

- How does the US Postal Service handle pieces of mail that do not have bar codes on them?
- Before UPC codes were invented, how did cashiers know the price of an item?
- UPC labels are never printed in red ink. Why not?

Did You Know That?

- The ZIP in ZIP Code stands for Zone Improvement Plan.
- The bar-code system used by the US Postal Service is called Postnet.
- After the ZIP Code bars, the next two groups of five bars represent the delivery code for the mail on the primary street address. The last set of bars in the code represents a check digit that helps a computer determine if the first digits have been read correctly. The check digit at the end of a US Postal Service code is calculated by adding the 11 previous digits in the code, then identifying the smallest digit that can be added to this sum to obtain a multiple of 10. For example, the check digit for the postal code in the Challenge is 3, since

  $$5 + 9 + 8 + 0 + 1 + 2 + 7 + 1 + 7 + 3 + 4 = 47$$

  and the smallest positive number that can be added to 47 to produce a multiple of 10 is 3.
- The bar code found on most supermarket items is UPC type A. Such codes consist of 12 digits. When space does not allow 12 digits, a UPC type E code—which has only 8 digits—is used.
- The US grocery industry formally adopted the UPC system in 1973.
- The mission of the Uniform Code Council, based in Dayton, Ohio, is to take a global leadership role in establishing and promoting multi-industry standards for product identification and related electronic communication.
- Bar codes are used by libraries, airlines, and blood banks.

Resources:

Book:

- *Designing Letter Mail.* Publication 25. Washington, DC: US Postal Service, August 1995.
- Harmon, C. *Lines of Communication, Bar Code and Data Collection Technology for the 90s.* Peterborough, NH: Helmers Publishing, 1994.
- Masunaga, D. "Zips and Strips." *Student Math Notes.* Reston, VA: National Council of Teachers of Mathematics, January 1994.
- Palmer, R. *The Bar Code Book.* Peterborough, NH: Helmers Publishing, 1995.

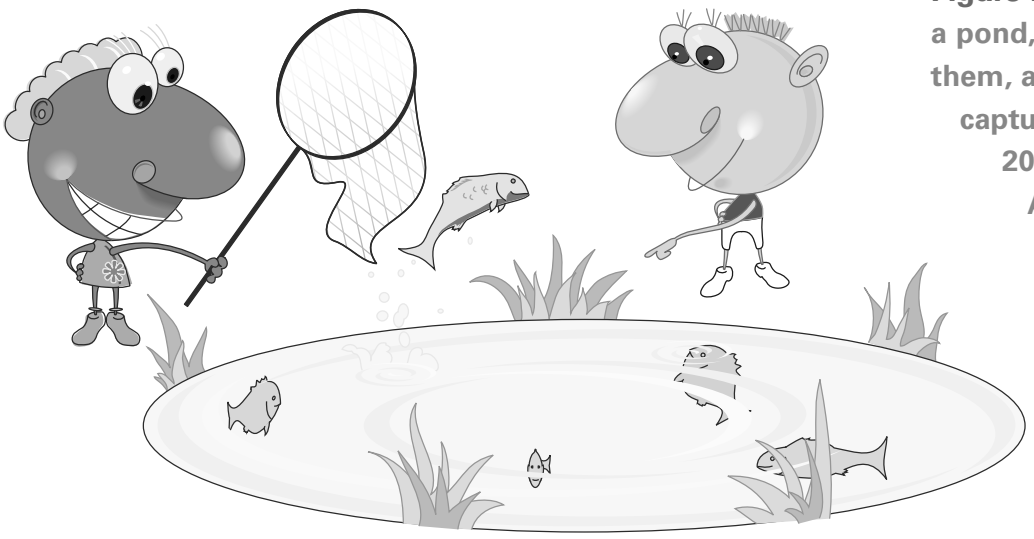
Websites:

- **www.architext.com**
- **www.beakman.com/upc/barcode.html**
- **www.usps.gov/ncsc/lookups/lookup_zip+4.html**
- **www.uc-council.org/main/ID_Numbers_and_Bar_Codes.html**

Math Challenge #52 CAPTURE-RECAPTURE

FigureThis! Math Challenges for Families

HOW MANY fish IN THE POND???



FigureThis! To estimate the number of fish in a pond, scientists captured 150 fish, marked them, and then let them go. The next day, they captured 170 fish from the pond. Of these, 20 had been marked the day before. About how many fish are in the pond?

Hint: What fraction of the fish caught on the second day were marked?

Capture-recapture is a statistical method used to estimate the size of a population. Fish and wildlife management experts, demographers, and scientists use this and other techniques to find the number of people or animals in a region.

Answer:
Based on the samples caught by the scientists, there are about 1275 fish in the pond.

Figure This!

Get Started:

How do you think the fraction of marked fish caught on the second day is related to the fraction of marked fish in the entire pond?

Complete Solution:

To use the capture-recapture method to estimate a population, scientists assume the proportions of the sample of the marked fish to the total fish in the sample is the same as the marked fish to the total fish in the pond.

$$\frac{\text{Marked Fish in Recapture Group}}{\text{Total Fish in Recapture Group}} = \frac{\text{Marked Fish in Pond}}{\text{Total Fish in Pond}}$$

There were 20 marked out of 170 of the sample. There were 150 marked out of the total fish in the pond. Thus, the following relationship is approximately true. Substitute the information given in the Challenge, then solve the equation:

$$\frac{20}{170} = \frac{150}{\text{Total Fish in Pond}}$$

$$20 \bullet (\text{Total Fish in Pond}) = 150 \bullet 170$$

$$\text{Total Fish in Pond} = \frac{25{,}500}{20}$$

$$\text{Total Fish in Pond} = 1275$$

This does not mean that there are exactly 1275 fish in the pond, but it is likely to be a reasonable estimate.

Try This:

- For the following experiment, you can use different flavors of goldfish crackers, or you could use a bag of dry beans that you can mark with a nontoxic marker.

  Take two boxes of different flavored goldfish crackers and a large paper bag.

  1. Pour one flavor of goldfish crackers into the paper bag.

  2. Remove a large handful of crackers from the bag and count them. Replace the crackers taken out with the same number of a different flavored cracker. Mix well.

  3. Remove another large handful of crackers from the bag. Record the total number of crackers in this handful and the number of these that are of the second flavor.

  4. Use the method in the challenge to estimate the total number of crackers in the bag.

  5. Replace the handful of crackers in step three. Mix well. Repeat steps three and four to get 5 estimates in all.

  6. Find the average of the 5 estimates.

  7. How does the average compare to the total number of crackers in the bag?

Additional Challenges:
(Answers located in back of booklet)

1. A biologist set several large traps in a meadow. On the first day, she caught 50 mice. She marked each mouse with non-toxic paint, then released it. The next day she caught 170 mice. Of these, 20 had been marked. About how many mice do you think live in the meadow?

2. Two scientists are trying to estimate the trout population in a mile-long stretch of river. On their first day they catch 150 trout. Each fish is tagged, then released. On their next visit, they catch 340 trout. About how many were marked if their estimate of the trout population in the river is about 2550 trout?

3. Four students are using the capture-recapture method to estimate the number of fish in an experimental pond. Their professor marked 150 fish from the pond, then asked each student to net a sample of fish. Their results are shown in the table below.

Student	Captured	Marked
A	180	30
B	160	20
C	205	38
D	110	16

Which student will report the largest estimate for the population? Which will report the smallest?

Things to Think About:

- When using the capture-recapture method to estimate population size, scientists make several assumptions. For example, they assume that each member of the population has the same chance of being captured and that the marks or tags won't fall off or become unrecognizable. What other assumptions do you think would be reasonable when using this method?
- What sort of problems might occur when using the capture-recapture method?
- How do you think the size of the samples in a capture-recapture experiment affects the accuracy of the estimate?
- What other populations might be estimated using a capture-recapture model?
- How would you mark a fish for a capture-recapture experiment?

Did You Know That?

- The foundations for the capture-recapture method were established in 1812 by French mathematician Pierre Laplace, considered by some to be the father of probability.
- In 1896, Carl George Johannes Petersen, a Danish fisheries scientist, published the results of a study in which he estimated the size of a fish population using the capture-recapture method and brass tags he invented.

- Biologists use a number of different ways to mark individuals in a capture-recapture study. Birds are typically fitted with leg bands. Turtles receive non-corroding metal or plastic tags. Large mammals, such as elk or bear, often are fitted with ear tags or radio collars. Migrating salmon are sometimes equipped with tiny microchips inserted under the skin.
- The US Census Bureau has considered using the principles of the capture-recapture method to help count the homeless population in large urban areas.
- To estimate deer populations, wildlife biologists take a picture from an airplane and then count the number of deer in the photograph.
- In Iowa, the rooster pheasant population is estimated by rural mail carriers, who count the cackles that roosters make early in the day.

Resources:

- Your state department of natural resources.

Book:

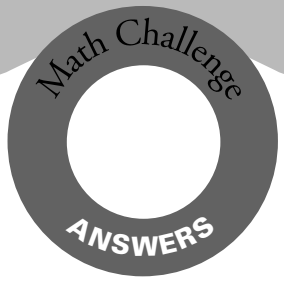
- Landwehr, J., J. Swift, and A. Watkins. *Exploring Surveys and Information from Samples.* Palo Alto, CA: Dale Seymour Publications, 1987.

Website:

- **www2.pitt.edu/~yuc2/cr/history.htm**

Notes:

Tangent



FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

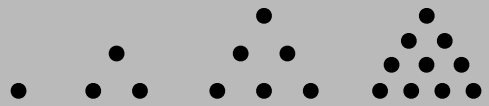
Figure This!

Answers to Additional Challenges:

Challenge 49:

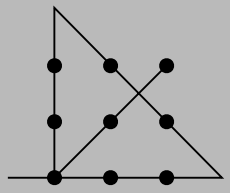
1. There would be 10 rows of dots in the pattern 1, 3, 5, 7, 9, 9, 7, 5, 3, 1.

2a. The next pattern has 10 dots as shown.

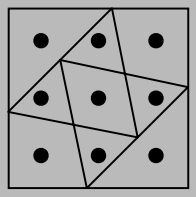


2b. It is a general pattern to give the number of dots in the n th figure. It gives the numbers 1, 3, 6, 10, ...

3. One possible solution is shown in the diagram below.



4. One possible solution is shown in the diagram below.



Challenge 50:

1. $H = 5 + 7d$, for $d \geq 1$

2a. Large dogs age faster after age 2; small dogs age faster initially.

2b. As with a large dog, a small dog will have a human age of 40 after 5 years.

2c. When the dogs are 10 years old.

3. The rate of change with each passing year is 5 not 7. It is constant after year 2, not year 1. The comparable formula is $H = 15 + 5d$, for $d \geq 2$.

4. 5.

Challenge 51:

1. 10.

2. 1,000,000,000.

3. Without the start bar, this portion of the bar code appears as follows:

Challenge 52:

1. About 425 mice.

2. About 20 trout.

3. Student B will report the largest estimate; student C will report the smallest.

Challenge 53:

1. The number of years between doublings has been decreasing.

2. No.

3. Approximately 688,004.

Challenge 54:

1. Divide the area and perimeter of the cake's square top into five equal portions.

2. Infinitely many with one cut, as long as the cut passes through the center of the rectangle.

3. One way is to find the midpoint (M) of the base, $\overline{BC}$. A line from A to M will work, since the height is the same for each new triangle, and the bases are both one-half the original length.