Math Challenge #41 RADAR BILLS

Figure This! Math Challenges for Families

Do you have a radar bill in your pocket?

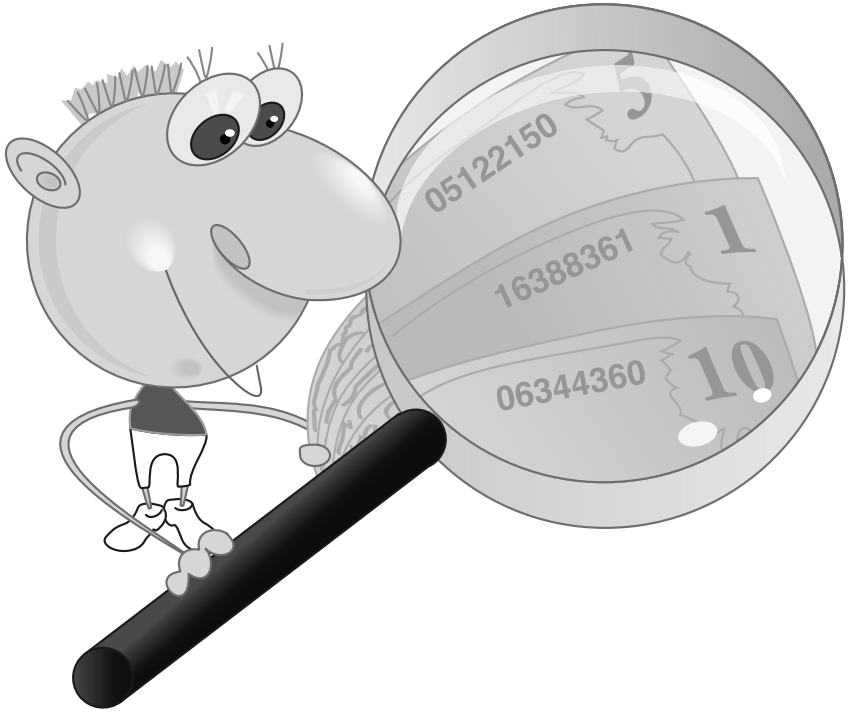


Figure This! **Paper money, such as dollar bills, with serial numbers that read the same backwards as forwards are sometimes called "radar bills." How common are radar bills?**

Hint: Serial numbers on US bills have eight digits. How many different serial numbers are possible?

Symmetry and repeating patterns are crucial to the study of mathematics. Artists, scientists, and designers all use these properties in their work.

Answer:
In the 100,000,000 eight-digit serial numbers from 00000000 to 99999999, there are 10,000 that read the same forwards and backwards. Considering this fact, you might expect 1 in every 10,000 bank notes to be a radar bill.

Figure This!

Get Started:

Begin with a similar, but simpler problem. Suppose there were only two digits in a serial number. In this case, a radar bill would have to have a serial number with two identical digits: 00, 11, 22, 33, 44, and so on. There are a total of 10 of these serial numbers out of the 100 possible 2-digit numbers. If there were four digits, the serial numbers would range from 0000 to 9999. Think of the four digits as two groups of two. If the last two digits were 01, the number of the radar bill would be 1001. How many radar bills would there be altogether for four-digit serial numbers?

Complete Solution:

Since there are eight digits, the serial numbers can range from 00000000 to 99999999. In other words, there are 100,000,000 possible serial numbers. Think of the eight digits as two groups of four. The last four digits of a radar bill serial number determine the entire number. For example, if the last four digits are 0001, then the entire number must be 10000001. Since the four digits that determine the radar bill serial number range from 0000 to 9999, there are 10,000 possible radar bill numbers. Since there are 10,000 possible radar bill numbers out of 100,000,000 serial numbers, you might expect 1 of every 10,000 bills to be a radar bill.

Another way to think about this is to look at the following pattern:

Number of Digits in Bill	Number of Radar Bills
2	10
4	100
6	1000
8	10,000

Following this pattern, there could be 10,000 radar bills that have eight digits.

Try This:

- Look at some dollar bills to see if you can find a radar bill.
- Words or phrases that read the same forwards and backwards are called palindromes. Think of at least five words besides radar that are palindromes.

Additional Challenges:

(Answers located in back of booklet)

1. Suppose that the serial numbers on a bank note contained nine digits. How many radar bills would be possible?
2. In the United States, all telephone numbers in a given area code have seven digits. If there were no restrictions on the digits, how many possible "radar" telephone numbers would there be?
3. Is a serial number more likely to begin with a 0 or a 1?

Things to Think About:

- Does knowing any four digits of a radar serial number let you determine the entire number?
- What is the significance of the capital letters at the beginning and end of each serial number on a US bill?
- Why does a star follow some serial numbers on a US bill?

Did You Know That?

- US currency is printed in sheets of 32 bills organized in an 8 x 4 arrangement. The last two digits of the serial numbers are the same for all 32 bills on the sheet. One hundred of these sheets are stacked, cut to size by a guillotine, then bundled.
- The newer versions of US currency have two letters in front of the serial number and one behind it.
- Some collectors specialize in collecting radar notes, notes with many 7's in the serial number, or notes with the same serial numbers from different Federal Reserve Banks.
- In the year 1999, the US government began issuing paper money that was less likely to be counterfeited. The bigger picture is one of the reasons. There is a hologram behind the picture.

Resources:

Books:

- Blocksma, M. *Reading by the Numbers: A Survival Guide to the Measurements, Numbers, and Sizes Encountered in Everyday Life.* New York: Viking Penguin, 1989.

Websites:

- **www.strawberries.com/home.html**
- **www.ping.be/~ping6758/index.shtml**
- **www.bep.treas.gov/allfacts.htm**
- **www.jakesmp.com/Specials_part_013.htm**
- **www.drbanks.com/currency/glossary.html**

Math Challenge #42 ROSE BOWL

FigureThis! Math Challenges for Families

Can a football team score 11 points in a game??

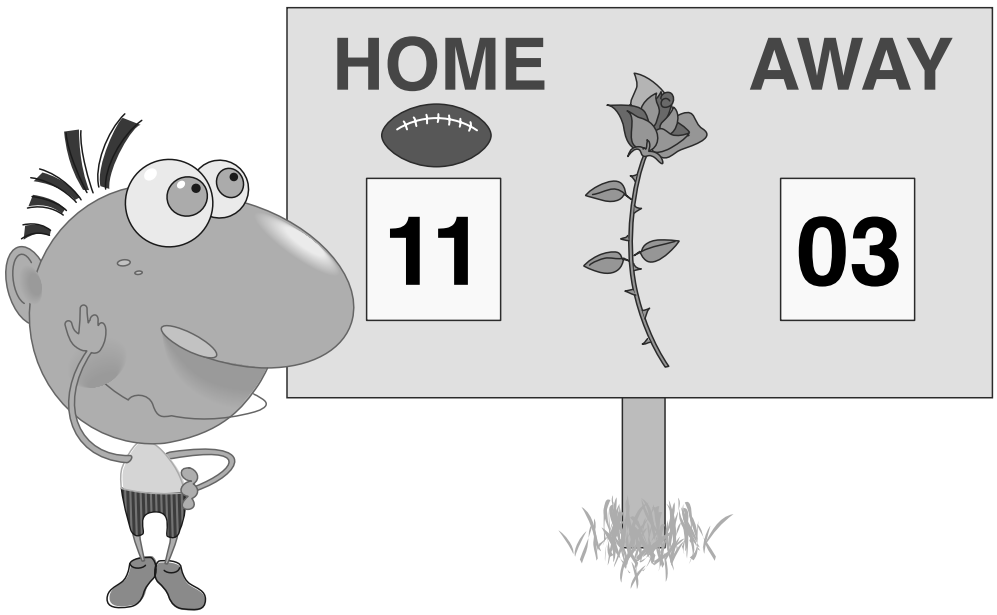


Figure This! **In the history of college football's Rose Bowl, no team's final score has ever been 11 points. How many different ways are there for a team to score 11 points?**

Hint: In American football, a team may score points in the following ways:

8 points (touchdown and 2-point conversion)

7 points (touchdown and 1-point conversion)

6 points (touchdown and no conversion)

3 points (field goal)

2 points (safety)

Making a list or a table is a method of organization used in problem solving and in prioritizing work. People in business and industry use this strategy in their jobs as well as to simplify daily chores.

Answer:
There are 5 different ways to score 11 points.

Figure This!

Get Started:

What would have to be scored with a touchdown and a 2-point conversion (8 points) to produce 11 points? A table can help you make sure that every possibility has been considered, and that no case has been counted more than once.

Possible Scores

	8 pts	7 pts	6 pts	3 pts	2 pts	Total Points
Possible Combinations of Points	1	0	0	1	0	8 +3 = 11

Complete Solution:

The table below shows the five different ways to score exactly 11 points.

Possible Scores

	8 pts	7 pts	6 pts	3 pts	2 pts	Total Points
Ways Score Could Be Made	1	0	0	1	0	8 + 3 = 11
	0	1	0	0	2	7 + 2 x 2 = 11
	0	0	1	1	1	6 + 3 + 2 = 11
	0	0	0	3	1	3 x 3 + 2 = 11
	0	0	0	1	4	3 + 4 x 2 = 11

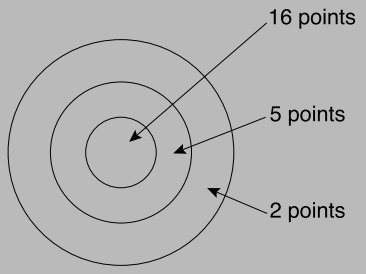
Try This:

- Search newspapers, almanacs, magazines, or websites for the final scores of football games. How often did you find a final score of 11 points?

Additional Challenges:

(Answers located in back of booklet)

1. In the nearly 100-year history of the Rose Bowl, there were six occasions when a team's final score was 10 points. In how many ways can 10 points be scored?
2. Are there any point totals that cannot be made in American football?
3. If you land four darts in this dart board, what scores are possible?



4. Six is called a "perfect number" because the sum of all its factors is twice itself, or 12. What is the next perfect number?

Things to think about:

- What total number of points, other than 11, might be rare in a football game?
- Which is more common, a field goal or a touchdown?
- What final score do you think has occurred most often?

Did You Know That?

- The first Rose Bowl was played in 1902. The final score was Michigan 49 and Stanford 0.
- The most points ever scored in the Rose Bowl was 49 by Michigan in 1902 and again in 1948.
- In 18 Rose Bowl games, one of the teams had a final score of 0. Seventeen of these shutouts occurred prior to 1954.

Resources:

Books:

- *The World Almanac and Book of Facts 2000.* Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.Rosebowl.org**
- **www.ncaa.org**

Math Challenge #43 SEEING STARS

FigureThis! Math Challenges for Families

Can you **draw** a picture of the stars on an American flag?

FigureThis! The American flag has 50 stars, one for each state. The rows are of two different lengths. Each row has one more star or one fewer star than the row next to it. Use these clues to figure out how the stars are arranged.

Hint: Select a number of stars for one row; then use the information given to test some possible patterns.

Mathematics has been defined as the study of patterns. Biologists, geologists, architects, designers, and computer scientists all use patterns in their work.

Answer:
Using the clues, there are six possible arrangements. The actual US flag has 9 rows of stars: 4 rows of 5 stars, and 5 rows of 6 stars.

Figure This!

Get Started:

By studying the clues, you know that if there are 2 stars in the first row, one possibility is that there are 3 stars in the next row, 2 stars in the following row, and so on. That pattern could look like this:

2 + 3 + 2 + 3 + …

Using this pattern, is it possible to reach a sum of 50 stars?
How about 3 + 4 + 3 + 4 + … ?

Complete Solution:

- One way to approach this problem is to test all the possible patterns. Knowing that the numbers of stars in alternating rows differ by 1, determine which patterns allow an arrangement of 50 stars. For rows of 2 and 3 stars, you could have:

  2 + 3 + 2 + 3 +…+ 2 + 3 = 50.

  Ten rows of 2 stars and 10 rows of three stars equal 50 stars.

  With rows of 3 stars and 4 stars, you have

  3 + 4 + 3 + 4 +…+ 3 + 4 = 49

  The closest that you can get to 50 is 49, and 49 plus another row of either 3 or 4 stars will not make 50 stars. So rows of 3 and 4 will not work. Continuing to test possibilities in this way, you should find that there are six solutions that satisfy the clues given in the challenge. The actual American flag has 4 rows of 5 stars and 5 rows of 6 stars.

No. of Stars in Row	No. of Rows	No. of Stars in Next Row	No. of Rows	Total No. of Stars
1	16	2	17	1x16 + 2x17 = 50
2	10	3	10	2x10 + 3x10 = 50
4	5	5	6	4x5 + 5x6 = 50
5	4	6	5	5x4 + 6x5 = 50
12	2	13	2	12x2 +13x2 = 50
16	1	17	2	16x1 + 17x2 = 50

- Another way to think about this problem is to consider the sums of the pairs of numbers in which one number is 1 more than the other. Using 2 and 3, for example, the sum is 5. Since 10 sets of 5 make 50, there could be 10 rows of 2 stars and 10 rows of 3 stars. Using 3 and 4, the sum is 7. Seven sets of 7 is 49, and neither 3 nor 4 can be added to 49 to get 50. Therefore rows of 3 and 4 will not work. This process can be continued to find the rest of the possible solutions.

Try This:

- How would you arrange the stars if the United States included 51 states?
- Look up flags of different countries in a dictionary, an atlas, or on the Internet. What patterns can you find?
- Draw each of the possible flags from the challenge. Which do you like the most?

Additional Challenges:

(Answers located in back of booklet)

1. How could the 50 stars be arranged in 5 rows so that every row had one more star than the one before it?
2. What is the least positive number such that when you divide by 2, the remainder is 1; when you divide by 3, the remainder is 2; when you divide by 4, the remainder is 3; and when you divide by 5, the remainder is 4?
3. A band director found that if the band members lined up two at a time, three at a time, four, five, or even six at a time, there was always one person left over. However, if they lined up seven at a time, no one was left over. If there were fewer than 500 students in the band, how big was the band?

Things to Think About:

- Are there some basic patterns in flags that occur over and over again?
- Who decides what each new flag should look like?
- The original US flag had 13 stars arranged in a circle, along with 13 stripes. In 1795, when Kentucky and Vermont were added to the original 13 US states, the flag featured 15 stars and 15 stripes. When more states were added, however, the designers returned to 13 stripes. Why do you think this happened?

Did You Know That?

- While making the pattern for the first American flag, Betsy Ross was reportedly able to create a 5-pointed star from a single sheet of paper with one cut.
- June 14 is Flag Day, commemorating the adoption of the US flag by the Continental Congress in 1777.
- The first US flag had 13 stars, while the second had 15 stars. Since states sometimes entered the union in groups, no US flags had 14, 16, 17, 18, 19, 22, 39, 40, 41, 42, or 47 stars.
- New US flags can be introduced only on the Fourth of July.
- The "star-spangled banner" described in the US national anthem is the flag with 15 stars and 15 stripes.
- The state flag for Hawaii is the only state flag that includes the flag of a foreign country.
- Not all state flags are rectangular.

Resources:

Books:

- "New Stars for Old Glory." *National Geographic*, July, 1959.
- Olson, A. *Mathematics Through Paper Folding*. Reston, VA: National Council of Teachers of Mathematics, 1987.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.crwflags.com/fotw/flags/us-1777.html**
- **www.ushistory.org/betsy/flagtale.html**
- **www.nationalgeographic.com/ngm/archive/index.html**

Notes:

Axis

Math Challenge #44 TABLE FOR 19

FigureThis! Math Challenges for Families

How much room do you need at a table?

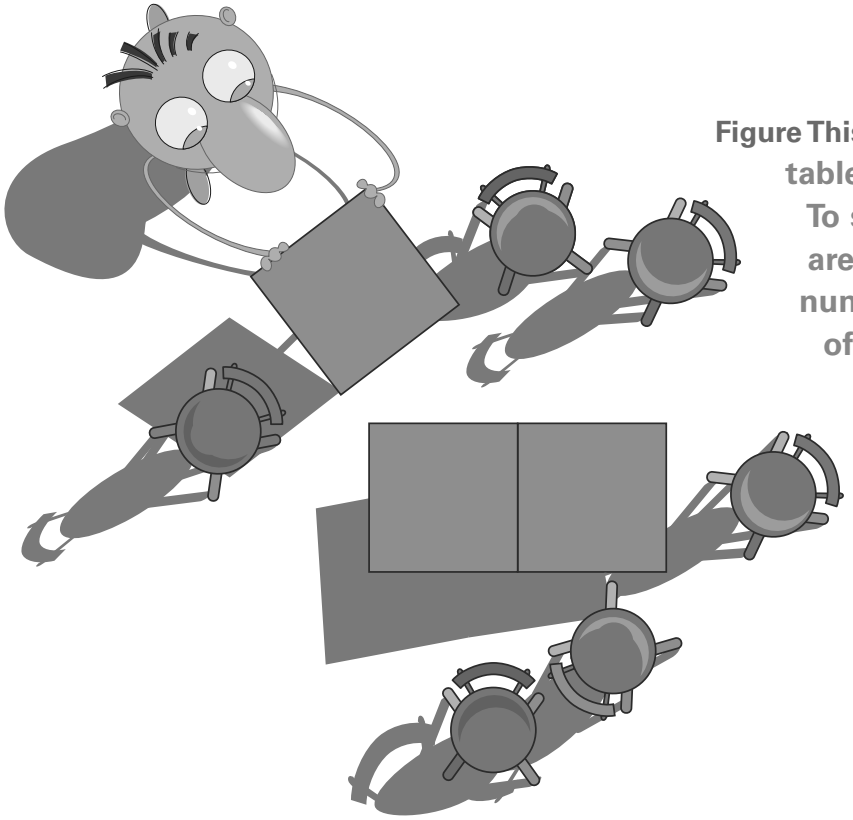


Figure This! Polygon's Restaurant has square tables that seat one person on each side. To seat larger parties, two or more tables are pushed together. What is the least number of tables needed to seat a party of 19 people who want to sit together?

Hint: How many people could sit at two tables pushed together? How many could sit at three tables pushed together?

Finding patterns and arranging geometric shapes are used by architects, landscapers, quiltmakers, and carpet layers in their work.

Answer: 9 tables.

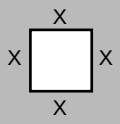
Figure This!

Get Started:

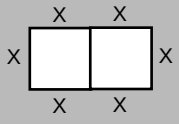
Use squares of paper (or square crackers) to represent the tables and model ways to seat people. Start with one square and see how many people can be seated. If you join two squares together, what happens to the number of seats?

Complete Solution:

- There are several ways to solve this problem. Using the hint, one table can seat four people.



Adding another table takes away one place and adds three places for a net gain of two seats.



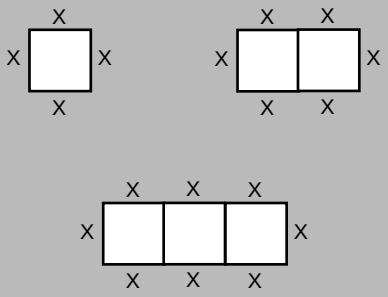
Reasoning in this way, when an additional table is added, one seat is lost and three are gained for a net gain of two. Continuing for three, four, and so on, at least nine tables are required to seat 19 people. There are many different possible arrangements of the tables.

- Another way is to make a chart and look for patterns.

Number of Tables	Number of People Seated
1	4
2	6
3	8
⋮	⋮

The pattern indicates an increase of two seats each time. Continuing the pattern, nine tables will seat 20 people but eight will only seat 18. Thus, nine tables are required for 19 people.

- Thinking geometrically leads to a general rule for the seating pattern. For every table arrangement, you can always seat one "at each end" with as many people on each side as there are tables.
  Examples follow:



With the two people at the ends and twice the number of people as there are tables seated at the sides, a general rule for n tables would allow seating a maximum of $2 + 2n$ people.

For 19 people,

$$2n + 2 \geq 19$$

$$2n \geq 17$$

$$n \geq 8.5$$

This means that at least nine tables must be used.

Additional Challenges:

(Answers located in back of booklet)

1. What is the maximum number of people who can be seated at seven tables put together?
2. If every seat is filled, what is the least number of people that can be seated in an arrangement of nine tables?
3. Think about the picture created by adding tables. Find a different version of the general rule found in the Complete Solution of the Challenge.

Things to Think About:

- If you were a waiter, would it be easier to seat a large group at one big table or two smaller tables? How about serving them?
- Why do some restaurants use round tables?
- How is the computer game Tetris™ related to this challenge?

Did You Know That?

- Arrangements of squares are called polynominos. Dominos are polynominos with two squares.
- According to *The Guinness Book of Records*, the greatest number of people simultaneously participating in a toast was 78,276 on February 27, 1998 in the United States.
- Fred Magel of Chicago, IL dined out 46,000 times in 50 years while rating the quality of restaurants.

Resources:

Books:

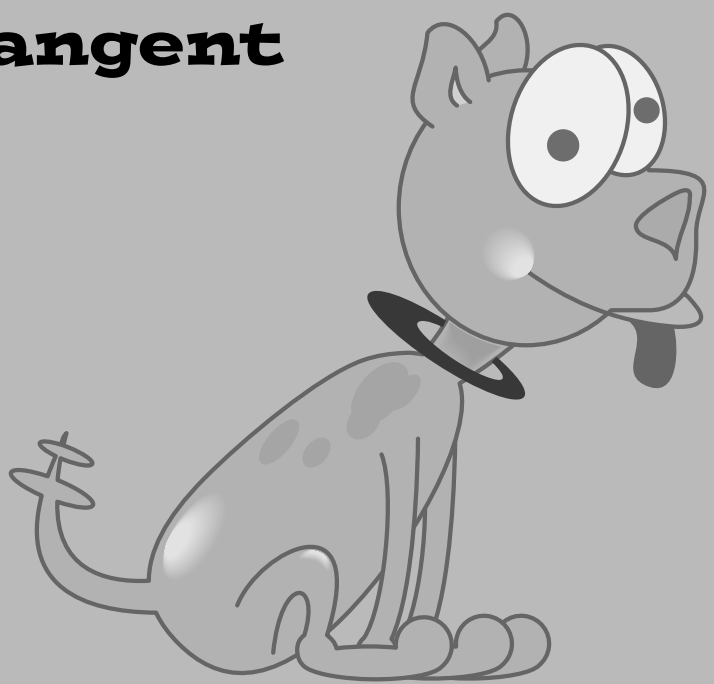
- Burns, M. *Spaghetti and Meatballs for All!* New York: Scholastic, Inc. 1998.
- *The Guinness Book of Records, 2000.* New York: Guinness Publishing, Ltd., 1999.

Websites:

gallery.uunet.be/luxil/2dtetris.htm

Notes:

Tangent



Challenge 41:

1. 100,000.
2. 10,000.
3. Equally likely.

Challenge 42:

1. There are five different ways to score exactly 10 points, the same number of ways as for 11.
2. It is impossible to score 1 point. Any other number of points is possible.
3. 8, 11, 14, 17, 20, 22, 25, 28, 31, 36, 39, 42, 50, 53 or 64.
4. 28.

Challenge 43:

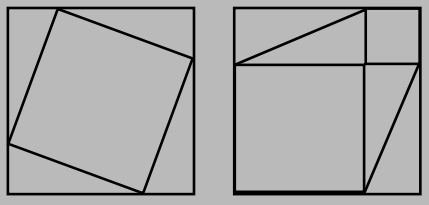
1. The stars could be in rows with 8, 9, 10, 11, and 12 stars.
2. 59
3. 301 students.

Challenge 44:

1. 16.
2. Arranging the tables in a 3 x 3 square leaves only 12 available seats.
3. Among the answers are $2 \bullet 3 + 2(n - 2)$ and $2 (n + 1)$.

Challenge 45:

1. Yes, since $6^2 + 8^2 = 10^2$.
2. 5, 12, 13.
3. 7, 24, 25.
4. No.
5. The two large squares below have the same area.



Taking away the four right triangles from each large square shows that the remaining areas are equal.

Challenge 46:

1. Early in the morning or late at night. For example, 8:00 AM in San Francisco would be 6:00 PM in Cairo; 10:00 PM in San Francisco would be 8:00 AM in Cairo.
2. It is 4:00 AM on January 2.
3. 6:30 AM until 1:00 PM.
4. Yes. For example, the difference in time from Japan to Western Samoa is –20 hours.

Challenge 47:

1. 7 complete shows.
2. Teenagers (12 - 17).
3. Men (18 and older).

Challenge 48:

1. No. The estimated average would be the same in all three cases.
2. 25.
3. 2.87 hours per night.