Math Challenge

#37

MAP COLORING

Figure This!

Math Challenges for Families

How many colors of states are on a map of the US?

Figure This! Mapmakers use different colors on states that share a border. What is the least number of colors needed to color a map of the states west of the Mississippi River in this way?

Hint: Try to fill in the map with as few colors as possible. Then try to show why using fewer colors would not work.

Assigning different colors to objects or decisions is a useful technique for analyzing complex situations. Similar methods can be applied to scheduling meetings, routing air traffic, and designing computer circuit boards.

Answer:
Four colors.

Figure This!

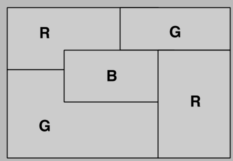
Get Started:

You can color a state or write a symbol for the name of a color in a state. Start in one corner of the map. Then begin coloring neighboring states, adding another color only when necessary.

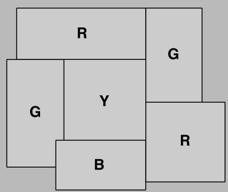
Complete Solution:

The least number of colors needed for this map is four. One possible solution uses the colors red (R), green (G), yellow (Y), and blue (B).

It is not possible to use fewer than four colors. If a state is surrounded by an even number of states, alternate two colors as you move around the states. The surrounded state will require a third color. See the diagram.



If a state is surrounded by an odd number of states, the surrounding states require three colors, and the surrounded state requires the fourth color.



Try This:

- Get a map of your state showing all counties. Color the map so that neighboring counties have different colors. How many colors did you need?
- Find a map of the US where states are colored. Do neighboring states get different colors? How many colors does the map use?
- Make a map that requires only three colors and that is different from the one pictured.
- Find maps of the US in 1790, when there were only the 13 original colonies. How many colors does that map need? How about a map of the US in 1810, or 1850?

Additional Challenges:

(Answers located in back of booklet)

1. Draw a map that has at least seven regions requiring only three colors.
2. Imagine that you are planning an ideal city. The city has five neighborhoods, and highways connect each neighborhood with every other neighborhood. To save driving time, none of these highways must intersect. What is the fewest number of overpasses you will need to build? How would your answer change if the city had six neighborhoods?

Things to Think About:

- On some surfaces, a map can be drawn requiring more than four colors.
- How many colors would you need to color the "four corners" region of the United States (Arizona, New Mexico, Utah, and Colorado)?
- In the western United States, Utah and Nevada are surrounded by an odd number of states. Are there any eastern states that have this feature?

Did You Know That?

- In 1852 British student Frederick Guthrie asked whether any map drawn on a piece of paper can be colored with no more than four colors. Guthrie's question became the "Four-Color Problem."
- A. B. Kempe, a lawyer, published a proof of the Four-Color Problem in 1879, but P. J. Heawood found an error in Kempe's proof in 1890. The problem remained unsolved until 1976 when mathematicians Kenneth Appel and Wolfgang Haken of the University of Illinois gave a proof based on more than 1000 hours of computer calculations.
- In 1890, Heawood determined that it requires at most seven colors to color any map on a doughnut.
- The number of colors needed for a map drawn on other than flat surfaces was determined in 1968, eight years before the Four-Color Problem was solved.

Resources:

Books:

- Bergamini, D. *Mathematics, Life Science Library*. New York: Time Incorporated, 1963.
- Francis, R. *The Mathematician's Coloring Book, HIMAP Module 13.* Arlington, MA: COMAP, Inc., 1989.
- Montana Council of Teachers of Mathematics. *Integrated Mathematics: A Modeling Approach Using Technology*, Level 4, Vol. 1. Needham Heights, MA: Simon & Schuster Custom Publishing, 1997.

Websites:

- **www.mathispower.org**
- **www.math.utah.edu/~alfeld/talks/S13/4CMP**
- **www.math.ucalgary.ca/~laf/colorful/4colors**

Math Challenge #38 MONDAY'S CHILD

FigureThis! Math Challenges for Families

Were you born on a Monday???

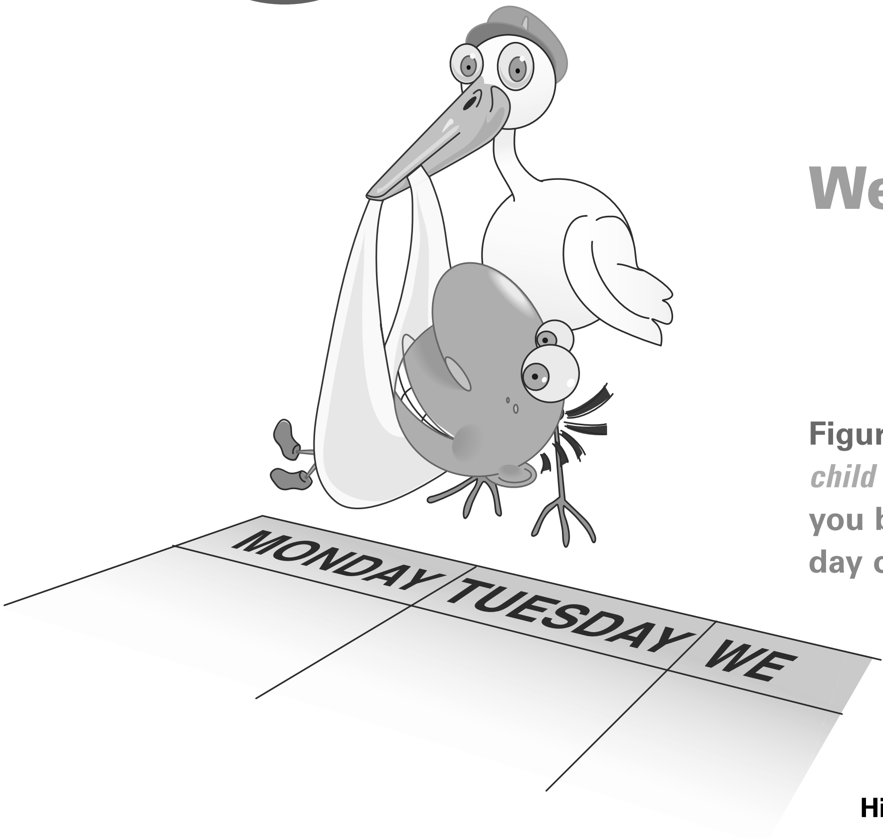


Figure This! ***Monday's child is fair of face; Tuesday's child is full of grace.*** **On what day of the week were you born? Can you devise a method to find the day of the week for any date?**

Hint: January 1, 2000 was a Saturday; January 1, 1999 was a Friday. (But don't forget leap years. The year 2000 is a leap year.)

An algorithm is a step-by-step process for completing a task. Algorithms are used by those who follow routines or recipes in their work: computer scientists, factory workers, statisticians, and chefs, among others.

Answer:

Your answer depends on your date of birth. For example, a student born on December 31, 1986, was born on Wednesday.

Figure This!

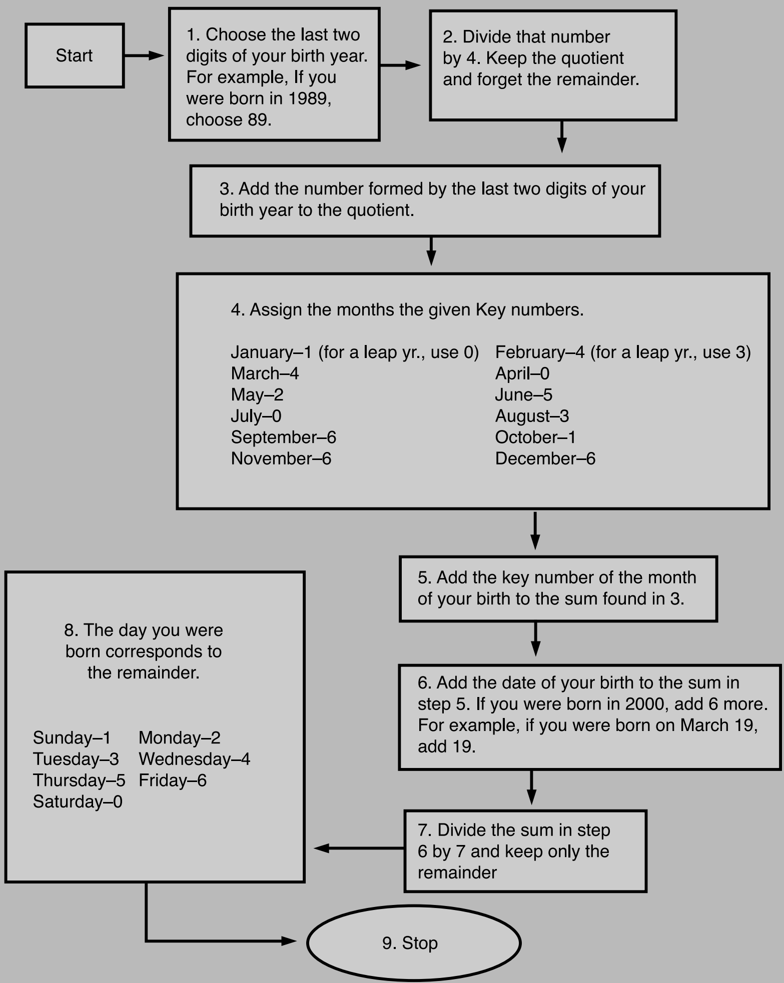
Get Started:

List the number of years between this year and the year of your birth. Then count the number of leap years that are included.

Complete Solution:

There are many ways to approach this problem.

- A general approach to this problem involves creating a step-by-step procedure for determining the day of the week for a past date. One way to describe such a procedure is with a flowchart. The flowchart given here finds the day of the week for any date from 1900 to 2000 (*The Farmer's Almanac*, p. 113).



- Another way to find the day of the week of your birth follows: Consider the number of days in a year. Non-leap years have 365 days. Since $365 \div 7$ gives a remainder of 1, then any particular date moves ahead one day of the week in the following year. Going back a year, each date moves back one day of the week. Since leap years have 366 days, and $366 \div 7$ gives a remainder of 2, any date moves ahead two days of the week in the year following a leap year.

  This means that when going back in time, each date moves back two days of the week for each leap year. (Note: If the current year is a leap year, then only dates after February 29 will have moved two days ahead.)

  Suppose that you were born on December 31, 1986. From the hint, you know that January 1, 2000 was a Saturday. That means that December 31, 1999 was a Friday. The year 1986 was 13 years before 1999. The years 1988, 1992, and 1996 were all leap years. That means that the day of the week moves backward 13 + 3, or 16 days. Since $16 \div 7$ gives a remainder of 2, you should count back two days from Friday to get Wednesday.

Try This:

- Use one or both of the methods described in the complete solution to find the day of the week on which another member of your family was born.
- To use the flowchart for birth years from 2000 to 2099, subtract one from the sum in step seven. Find the day of the week for your 100th birthday.
- Create a flowchart for some task or chore that you do at home. See if a family member can follow your flowchart.

Additional Challenges:

(Answers located in back of booklet)

1. On July 4, 1976, the United States of America celebrated its bicentennial. What day of the week was it?
2. Below are two algorithms for finding the cost with tax of an item. At original cost c dollars and a tax rate of t per dollar,

$$c + ct$$
$$c \bullet (1 + t)$$

do the two expressions represent the same amount?

Things to Think About:

- Why might it be important to know the day of the week for certain dates?
- Compare the two methods given in the complete solution to find the day of the week. How do you think the key numbers in the flowchart are determined?
- A formula is an algorithm.
- The process of doing long division is an algorithm.

Did You Know That?

- One of the first recorded mathematical algorithms was developed by Eratosthenes (about 230 BC) for finding prime numbers.
- Our present calendar is the Gregorian calendar, named for Pope Gregory XIII in 1582. Great Britain adopted the Gregorian calendar in 1752 and forced its use in the American colonies. In 1923, Greece also adopted the Gregorian calendar. Since then, it has become the standard calendar for nearly all the countries in the world.
- Although Thanksgiving day is usually observed on the fourth Thursday in November, its actual date in any year must be decided by proclamation of the sitting president of the United States.
- The first man to walk on the moon, Neil Armstrong, landed there on a Sunday, July 20, 1969.
- When developing software programs, computer scientists often use flowcharts to organize their work.
- To change the flowchart to handle years before 1900, add 2 to the sum in step 6. For years from 1753 to 1800, add 4 to the sum in step 6.

Resources:

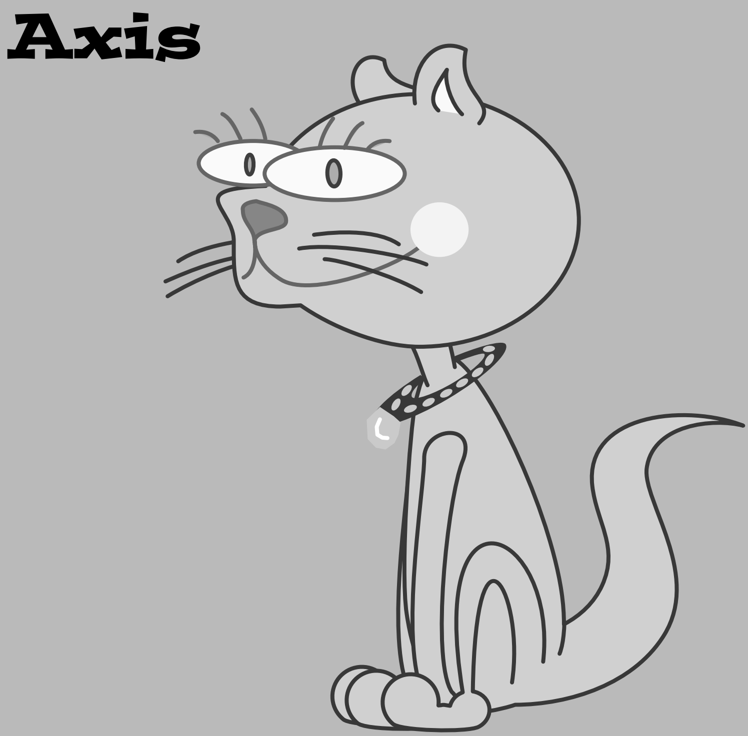
Books:

- Glenn, W., and D. Johnson. *Fun with Mathematics*. Sacramento, CA: California State Department of Education, 1960.
- Rand McNally & Company. *The Real Mother Goose*. Eau Claire, WI: E. M. Hale and Co., 1953.
- *The Old Farmers Almanac*. Dublin, NH: Yankee Publishing Inc., 1999.

Websites:

- **www.geceventures.com/year2000/about_calendars.htm**

Notes:



Math Challenge #39 PATIOS

Figure This! Math Challenges for Families

Does bigger perimeter mean bigger area?

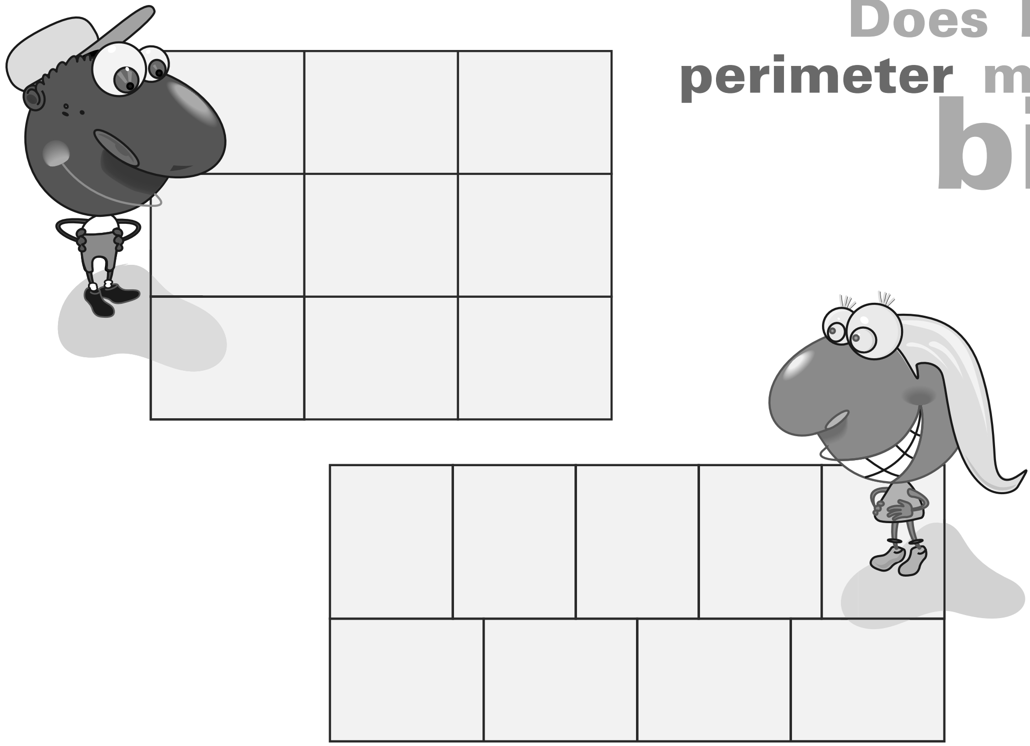


Figure This! Helix and Polygon both used the same number of identical concrete pieces to make their patios. The area of each patio is the same: 180 square meters. What are the dimensions of a single piece of concrete?

Hint: Notice how the pieces fit together on Polygon's patio. What is different about the way the pieces fit on Helix's patio?

Area is an important mathematical concept. Architects, real estate agents, artists, and surveyors all use area in their work.

Answer:
A tile is 4 m by 5 m.

Figure This!

Get Started:

Find the area of one of the concrete pieces. How many of the short sides equals a long side of Polygon's patio?

Complete Solution:

The area of each patio is 180 square meters (m2), and each is made from nine identical rectangular concrete pieces. This means that the area of one piece is 180 ÷ 9, or 20 m2. Since the area of the rectangle equals length (L) times width (W), you know that L • W = 20. From the way in which the pieces are arranged in Polygon's patio, you can see that four lengths is the same as five widths. In other words, the ratio of length to width is 5 to 4. As it happens, 5 • 4 = 20. This means that the length is 5 m, while the width is 4 m.

- Another way to look at this is as follows. You know from Polygon's patio that four lengths equals five widths. Since 4L = 5W, then:

  L = 5/4 • W

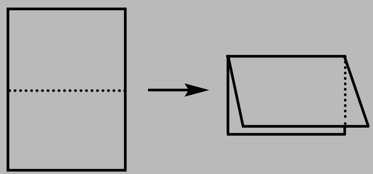
  You also know the area of the piece: L • W = 20.
  Substituting for L,

  (5/4 • W) • W = 20
  5/4 W^2 = 20
  W^2 = 4/5 • 20
  W^2 = 16
  W = 4 (Widths cannot be negative.)

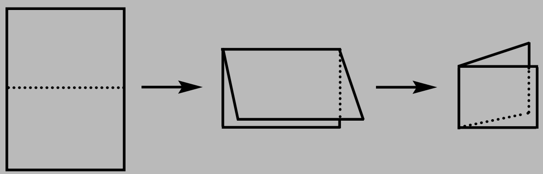
  Substituting 4 for W, you can then determine that L = 5.
  So a single concrete piece has dimensions 4 m by 5 m.

Try This:

- Take two 8 1/2 in. by 11 in. sheets of paper. Fold one of them horizontally as shown.



  Continue folding this sheet horizontally four more times. Take the other sheet of paper and make the first fold exactly as you started above. Then make the second fold vertically as shown.



  Continue folding this piece of paper three more times alternating horizontally and vertically. Compare the areas of the rectangles folded with the two different sheets.

- Estimate the perimeter and area of the top of your bed. Then measure your bed and calculate the perimeter and area. How close were your estimates?
- Draw two rectangles with the same area but different perimeters.
- Tie the ends of a piece of string together to form a circle. Use the string to form various rectangles. The perimeter doesn't change. Does the area?

Additional Challenges:

(Answers located in back of booklet)

1. The distance around a figure is its perimeter. What are the perimeters of the patios in the challenge?
2. The patio in the diagram is made of identical tiles and has an area of 180 m^2. What is its perimeter?

3. If given only Helix's patio in the challenge, you would not get a unique answer. If you were given only Polygon's patio, you would find a unique answer. Explain why.
4. Use squares with a side length of 1 unit to create different shapes that have a perimeter of 8 units. Which of the shapes has the greatest area?

Things to Think About:

- There are rectangles with the same perimeter but different areas.
- The patterns in many wood or tile floors are made with rectangles. Why do you think this is so?
- As the perimeter of a rectangle increases, the area may either increase or decrease.

Did You Know That?

- Guess, check, and revise is sometimes the most efficient way to solve a problem.
- For a rectangle with a given perimeter, the shape with maximum area is a square.

- All squares are rectangles but not all rectangles are squares.
- The least common multiple of two whole numbers *a* and *b* can be found by making a square from rectangles with dimensions of *a* x *b*.
- A baseball diamond is a square 90 feet on a side.

Resources:

Books:

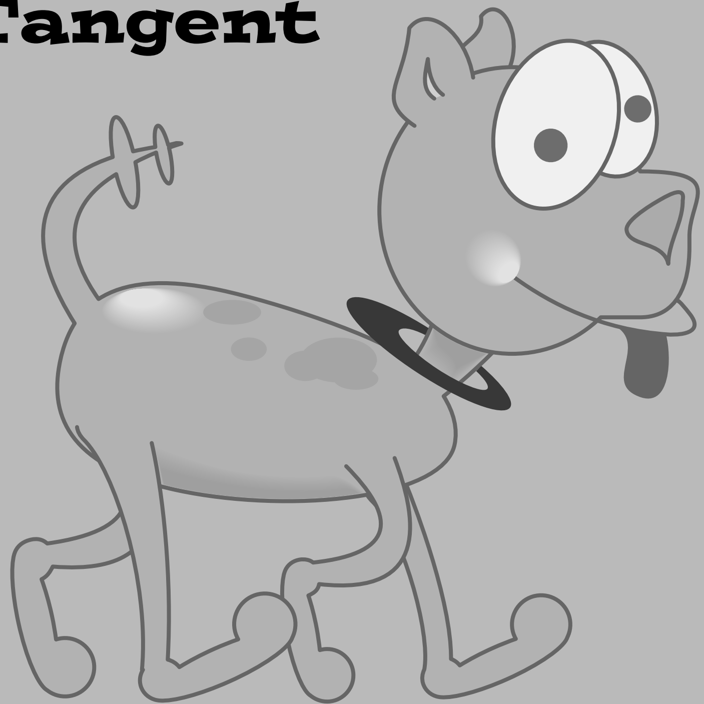
- Conrad, S., and D. Flegler. *Math Contests for High School, Volume 1*. New York: Math League Press, 1992.
- Mumme, J. *"Investigating Perimeter and Area."* In *Teaching with Student Math Notes*, Volume 2 (ed. E. Maletsky). Reston, VA: National Council of Teachers of Mathematics, 1993.

Websites:

- **web.singnet.com.sg/~raynerko/index.htm**
- **www.forum.swarthmore.edu/dr.math/problems/cducote2.16.96**

Notes:

Tangent



Math Challenge #40

PUTT PUTT

FigureThis! Math Challenges for Families

Can you make a hole-in-one?

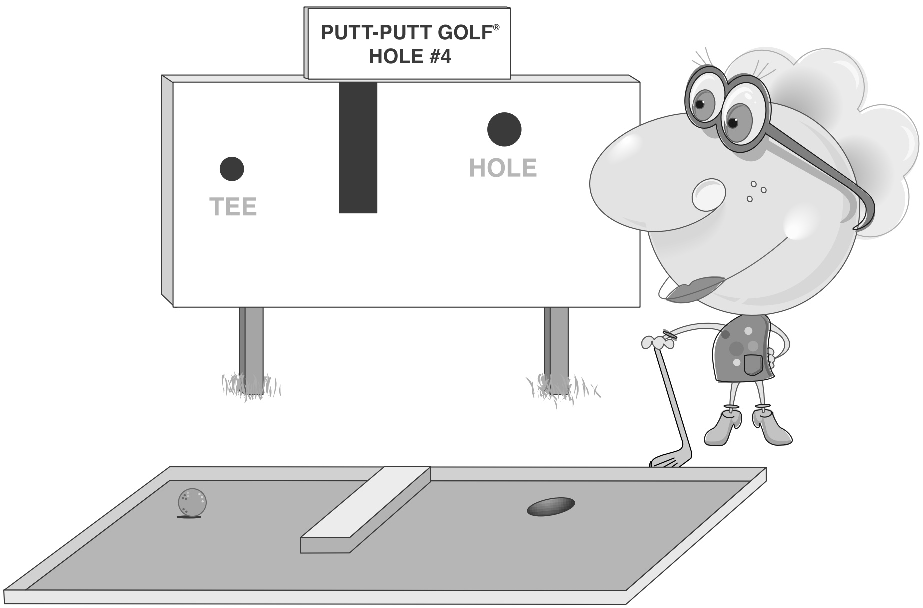


Figure This! How would Tessellation hit a golf ball from the tee to make a hole-in-one?

Hint: The angle at which the ball bounces off the wall will have the same measure as the angle at which it hit the wall.

A golf ball bounces off the side of a miniature course or a pool ball bounces off the edge of a pool table with a hard cushion, the same way light bounces "off" a mirror. Opticians who make lenses, biologists using microscopes, and astronomers using telescopes, as well as makers of satellite dishes and periscopes, are concerned with this principle in their work.

Answer:
The lines in the drawing show a path of a hole-in-one.
The marked angles, 1 and 2, have equal size. You should aim for point P.

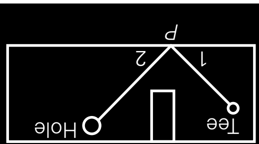
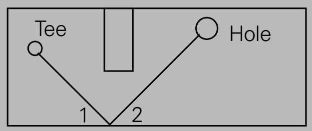


Figure This!

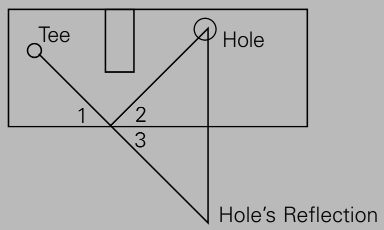
Get Started:

How can you decide where to aim? Since a direct path from the tee to the hole does not exist, you must bounce the ball off the wall. Draw lines from the tee to the edge and the hole to the edge to find a point that makes the bounce angles, 1 and 2, equal.



Complete Solution:

One way to locate the proper place to aim is to think of the sidewall as a mirror. If you looked into this mirror from the tee, the hole's reflection would appear to be in the location shown in the diagram.

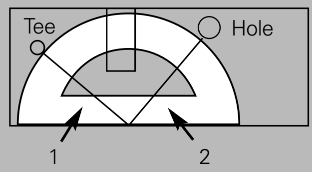


As mentioned in the hint, the angle at which the ball leaves the wall will be the same as the angle at which it hit the wall. If you aim the ball at the point where the line from the tee to the hole's image intersects the wall, then it will bounce into the hole.

This can be proven mathematically as follows: Because the two triangles on either side of the wall in the diagram are mirror images, they are exactly the same size and shape (congruent). This means that angle 2 is the same size as angle 3. Angle 1 and angle 3 are also the same size, because they are vertical angles. (Vertical angles are angles formed by two intersecting lines.) Angle 1 is the same size as angle 2 because both are the same size as angle 3. This means that the path to the hole shown on the diagram is the same path the ball will take as it bounces off the wall.

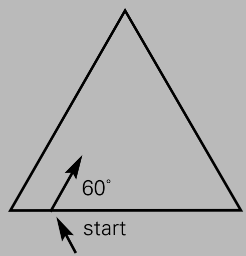
Try This:

- Design a hole for a miniature golf course; then play a game with a friend to show how the ball can bounce off the sides and into the hole. (Try this on a computer.)
- Measure the distance from the ball to the hole along the path the ball travels in the challenge. Suppose the ball hits the boundary anywhere else on the way to the hole. Measure this other path and compare it to the first. What do you find?
- Trace the diagram in the challenge on a sheet of paper. Insert thumb-tacks at the tee and the hole. Stretch a rubber band around the thumb-tacks. Place a protractor on the diagram and pull the rubber band to the center of the protractor as shown.



Slide the protractor until the angles marked 1 and 2 are the same size. Now measure the length of the stretched rubber band. If the angles are not the same size, how do you think the length of the rubber band will change?

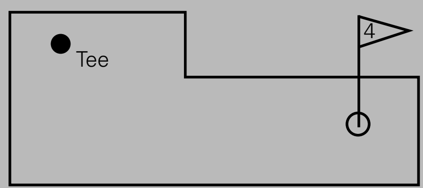
- The diagram below shows a beam of light entering a triangular box with mirrored walls. Trace the path of the light as it bounces off the walls. No matter where you start, what happens to the path?



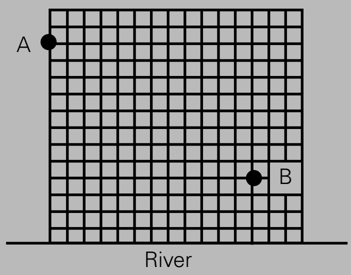
Additional Challenges:

(Answers located in back of booklet)

1. In the Complete Solution to the Challenge, the hole was reflected. How would the solution change if the tee were reflected?
2. Show how to make a hole-in-one on this miniature golf course by bouncing the ball off two walls.



3. Amesburg (A) is 12 miles from the Straight River, while Belleville (B) is 4 miles from the river. The county wants to build a water pumping station on the river to serve both towns. To reduce costs, engineers would like to locate the station so that the sum of the distances from the pumping station to the towns is as small as possible. Where should they build the station? (This happens when the "bounce" angles are equal.)



Things to Think About:

- Is there any way to place a hole on a miniature golf course so that no matter where you aim the ball from the tee, you can make a hole-in-one every time?
- Is there any way to place a hole on a miniature golf course so that no matter where you aim the ball, you can never make a hole-in-one?

Did You Know That?

- Although the properties of reflected light were known to other ancient philosophers and scientists, it was Heron of Alexandria (about AD 75) who developed the related mathematics.
- The longest recorded hole-in-one was 447 yards, made by Robert Mitera in Omaha, Nebraska in 1965.
- Kenneth Schreiber, a legally blind golfer, shot a hole-in-one in 1997 in Bayonet Point, Florida.
- The method of reflecting is sometimes used to find distances that can not be measured directly.

Resources:

Books:

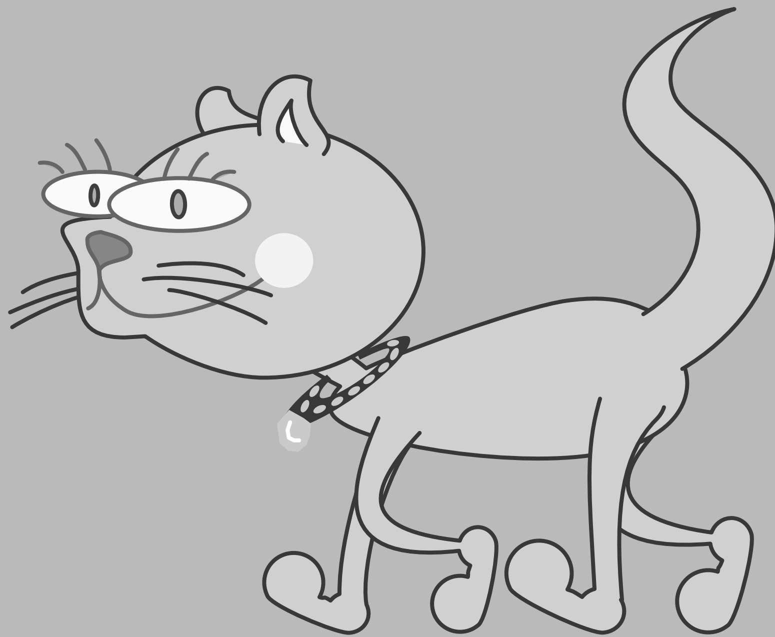
- Jacobs, H. *Mathematics*: *A Human Endeavor*. San Francisco: W. H. Freeman Co., 1970.
- *The Guinness Book of Records 1999*. New York: Guinness Publishing Ltd., 1999.

Websites:

- **www.tenet.edu/teks/math/clarifying/cageoc.html**
- **www.illuminations.nctm.org**

Notes:

Axis



Math Challenge ANSWERS

FigureThis! Math Challenges for Families

Looking for answers?

Here are the answers for the **Additional Challenges** section of each Challenge.

Figure This!

Answers to Additional Challenges:

Challenge 33:

1. The ramp would have to begin 7 feet farther away, or about 28 feet in all.
2. About 30.2 feet.
3. Yes.

Challenge 34:

1. About 175 gallons.
2. 70 miles per hour.
3. No, because the temperature is about 68° Fahrenheit.

Challenge 35:

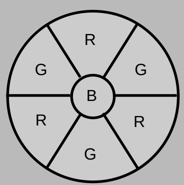
1. Any shape with at least one pair of parallel sides, including squares and regular hexagons (the most common shape).
2. The bolts with the sides the same length would be easier, because you could use the wrench on any of three sets of parallel sides.
3. 10.

Challenge 36:

1. Approximately 14% are left-handed.
2. 55% liked neither soda.
3. Not necessarily.
4. No.

Challenge 37:

1. There are many possibilities. In the example shown here, the regions are colored red (R), blue (B), or green (G).



2. A city with five such neighborhoods will need at least one overpass. A city with six neighborhoods requires at least three overpasses.

Challenge 38:

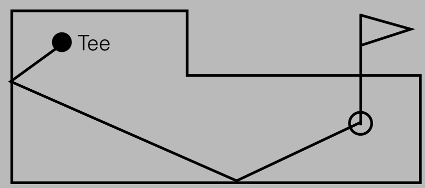
1. Sunday.
2. Yes.

Challenge 39:

1. The perimeters are 58 m and 54 m.
2. The perimeter is 56 m.
3. Since the arrangement of pieces in Helix's patio does not show a relationship between length and width, you cannot figure out the dimensions of a single piece. There are many pairs of numbers that multiply to make 20.
4. A square with a side length of 2 units.

Challenge 40:

1. It wouldn't change.
2. One possible solution is shown here:



3. The sum of the distances is least when angles 1 and 2 in the diagram are equal. In this case, that distance is 20 miles.