

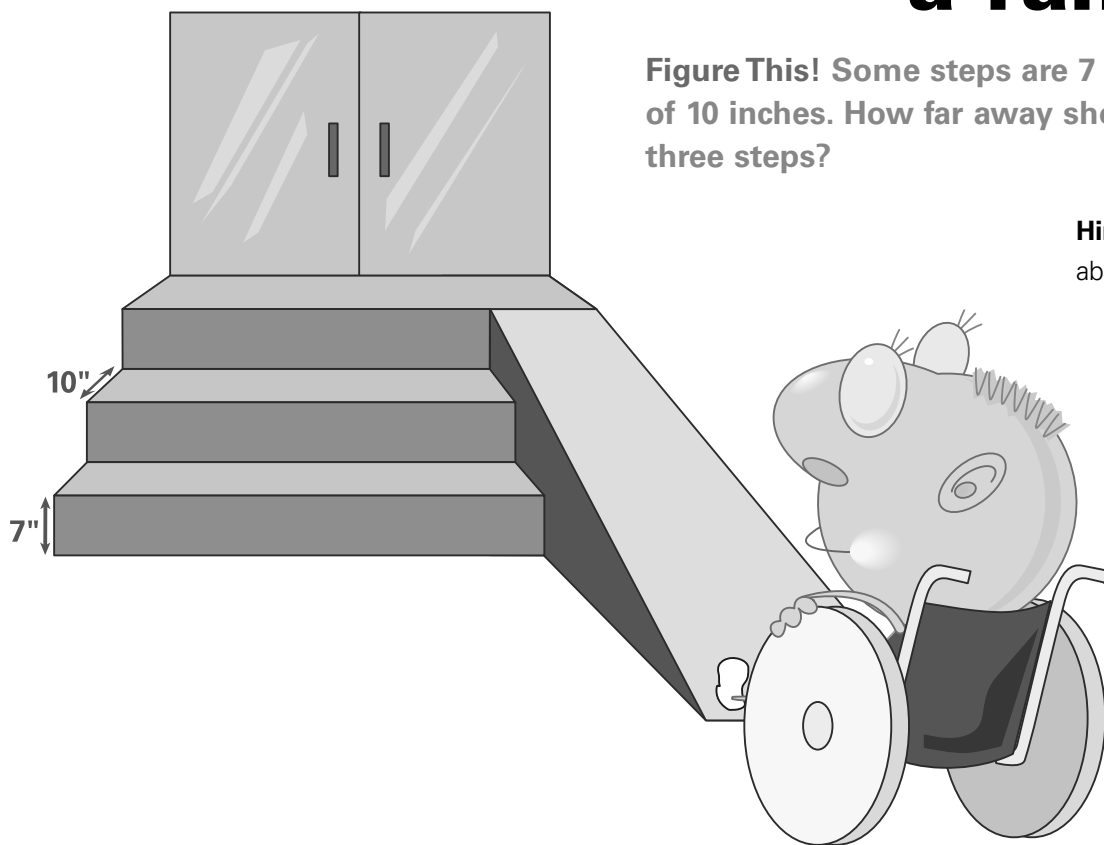


FigureThis!
Math Challenges for Families

How *STEEP* can a ramp be?

Figure This! Some steps are 7 inches high and have a width of 10 inches. How far away should the ramp start to go up three steps?

Hint: Access ramps usually go up about 1 inch for every 12 inches.



Slope is a measure of the steepness of an incline. Slope is used by civil engineers, builders, surveyors, and landscapers in constructing roads through mountains, stairs in houses, and drainage ditches.

Answer:
About 252 inches or 21 feet.

Figure This!

Get Started:

Sketch a diagram of the steps and find their total height. Then use the information given in the hint.

Complete Solution:

Since each step is 7 inches high, the three steps rise a total of 21 inches. The ramp should rise 1 inch for every 12 inches of horizontal distance. This means that the ramp should begin $21 \times 12 = 252$ inches away.

Try This:

- Place a small object, such as a penny, on a sheet of cardboard. Lift one end of the cardboard. How high must you raise the cardboard before the penny begins to slide? If the sheet of cardboard were longer, could you lift the end higher without making the penny slide?
- Measure the height and tread width of one step in your home, school, or business. Find the slope of the stairs containing the step (the ratio of the height to the tread width).
- Is there a skateboard ramp near your home? If so, estimate the slope of the ramp.
- Estimate a distance of 21 feet. Then measure a distance of 21 feet. How accurate was your estimate?

Additional Challenges:

(Answers located in back of booklet)

1. If the building in the challenge had four steps, how would you change the ramp?
2. As a glider travels through the air, it descends 1 foot for every 10 feet of horizontal ground distance it covers. If it descended 3 feet, how far did it actually travel?
3. The steepest parts of intermediate ski hills rise about 4 feet for every 10 feet horizontally, or have slopes of about 4 to 10. Is a hill with this slope steeper than a road with a 6% grade? A 6% grade means that a road ascends 6 feet for every 100 feet of horizontal distance traveled.

Things to Think About:

- Why are there official specifications for the slopes of roads, ramps, and staircases?
- Why do skiers and snowboarders use the switchback (zigzag) technique when descending some slopes?
- What is the typical height of a street curb? If the curb has an access ramp, what is its slope?
- Is the slope of the initial climb of a roller coaster more or less than the slope of the first descent?

- Could a road have a grade (slope) of 100%?

Did You Know That?

- According to US federal building codes, the maximum height for an access ramp is 30 inches. To reach entrances that exceed this limit, lifts or an elevator may be installed.
- Most cars cannot climb hills that have a slope of 30° (or rise about 1 foot for every 1.7 feet horizontally).
- Most local building codes describe a maximum stair rise of 8-1/4 inches and a minimum tread width of 9 inches.
- More than nine out of ten avalanches occur on slopes ranging from 25° to 45° .
- The ratio for slope is independent of units.
- Most black diamond ski slopes are around 30° to 35° , while a 40° slope would be considered the low end of extreme mountaineering. Parts of Tuckerman's Ravine on Mt. Washington have slopes of about 50° .
- Slopes are rates of change, a fundamental idea of calculus.
- Another way to express a slope uses the trigonometry notion of tangent.

Resources:

Books:

- Kleiman, G., et al. *Mathscape: Seeing and Thinking Mathematically. Roads and Ramps: Slopes, Angles, and Ratios*. Alsip, IL: Creative Publications, 1996.
- *Stairs: The Best of Fine Homebuilding*. Newtown, CT: Taunton Press, 1995.
- *The Guinness Book of World Records*, 1999. New York: Guinness Publishing Ltd., 1999.

Websites:

- www.mroudoors.com/columns/2000/0109out.html
- www.tuckerman.org/tuckerman/history.htm
- www.hometime.com/projects/howto.accrss/pc2aces2



FigureThis!
Math Challenges for Families

Can YOU run as fffast as a car?

Figure This! During the 100 meter dash in the 1988 Olympic Games in Seoul, Florence Griffith-Joyner was timed at 0.91 seconds for 10 meters. At that speed, could she pass a car traveling 15 miles per hour in a school zone?



Hint: How many meters in a mile?
How many seconds in an hour?

Conversion between units of measure is required from the kitchen to the construction site to the laboratory. Chefs, carpenters, scientists, and engineers all must convert units of measure in their work.

Her speed would be about 24.6 miles per hour; she could pass the car.

Answer:

Figure This!

Get Started:

There are 2.54 centimeters to an inch and 5280 feet to a mile. How many centimeters are in a meter? How many inches are in a foot; in a mile? What is her rate (distance divided by time)?

Complete Solution:

One method for converting between measures is called dimensional analysis. The conversions between measures are written as fractions so the common units cancel out.

$$\frac{10 \cancel{\text{m}}}{0.91 \cancel{\text{sec}}} \times \frac{100 \cancel{\text{cm}}}{1 \cancel{\text{m}}} \times \frac{1 \cancel{\text{m}}}{2.54 \cancel{\text{cm}}} \times \frac{1 \cancel{\text{ft}}}{12 \cancel{\text{in}}} \times \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} \times \frac{60 \cancel{\text{sec}}}{1 \cancel{\text{min}}} \times \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \approx 24.6 \text{ miles/hour}$$

Her rate is about 24.6 miles per hour, and she could easily pass a car going at a rate of 15 miles per hour.

Try This:

- Many dictionaries contain conversion tables for measures. Find a conversion table and examine it. Are any of the conversions familiar to you?
- Look in a newspaper or website for currency exchange rates. How could you use the information you find to convert Spanish pesetas into Japanese yen?
- Compare the size of a liter and a quart.
- Have someone time how long it takes you to go 10 yards. What is your rate in miles per hour?

Additional Challenges:

(Answers located in back of booklet)

1. A waterbed mattress is 84 inches long, 60 inches wide, and 8 inches deep. There are 231 cu. in. in a gallon. How many gallons of water does it take to fill the mattress?
2. According to *Natural History* magazine, a cheetah is the fastest animal in the world with a speed of 6,160 feet per minute. How many miles per hour is that?
3. If it is 20° Celsius outside, would you need a jacket?

Things to Think About:

- Which measuring system is used in international track and field competitions?
- Roger Bannister, a British physician, broke the four-minute mile in 1954. Will someone break the three-minute mile? Is there a limit to the amount of time required to run a mile? If so, when do you think it will be reached?

- Why are speeds on the 100-meter or 200-meter dashes reported in meters/second instead of kilometers per hour?
- During World War II, American soldiers referred to kilometers as “kiddie miles.” Where do you think this name came from?
- Preying animals, such as the cheetah, lion, and hyena, run faster than most other animals. Why?

Did You Know That?

- Since 1984, running events in the Olympics have been timed in hundredths of seconds because of electronic timing devices.
- The United States is the only developed country that does not use the metric system for its principal units of measurement.
- Almost all scientific measurements are made using metric units. The metric system is based on the decimal system (units of 10) and follows a consistent naming scheme using prefixes.
- The US National Aeronautics and Space Administration (NASA) spent \$125 million on a spacecraft that flew 416 million miles over 9 1/2 months before crashing on Mars. The spacecraft crashed due to a contractor’s error in converting pounds of force into another unit of force called newtons. One newton is the amount of force required to accelerate 1 kilogram of mass 1 meter per second each second.
- Many cars and trucks require metric tools for maintenance.
- One square centimeter (1 cm²) is about the size of your little fingernail.
- The mass of 1 cm³ of water at standard temperature and pressure is 1 gram.
- The US Conventional System of Measurement is a modified version of the British Imperial System, which is no longer in use.
- As of January 1, 2000, it is a criminal offense in Great Britain to sell most packaged and loose products using imperial measures (inches, pounds, and so on). One exception is precious metals.
- One of the few British Imperial units of measure that remains in world-wide use is the barrel, primarily for oil.

Resources:

Books:

- *The Guinness Book of Records 1999*. New York: Guinness Publishing Ltd., 1999.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.



FigureThis!
Math Challenges for Families

What **shape** is at the
very top of a fire hydrant???



Figure This! The water control valve on the cover of a fire hydrant has five sides of equal length and five angles of equal measure. Many common household wrenches will not turn these valves. Why not?

Hint: Think about an ordinary household wrench. Most wrenches have two parallel sides; that is, the sides are everywhere the same distance apart.

The geometric shapes of many objects relate directly to their usefulness. For example, round tires produce a smooth ride, and airplane wings are designed to provide lift.

Most household wrenches will not work on the valves of a fire hydrant because there are no parallel sides on the five-sided (pentagonal) valve.

Answer:

Figure This!

Get Started:

Draw a square. Are any sides parallel? Would a common household wrench open it? Does every shape with four sides the same length have parallel sides? How about a shape with five sides the same length and five angles of the same size?

Complete Solution:

All of the angles of a valve on top of a fire hydrant are the same size. The five sides have equal length. (The valve top is a regular pentagon.) No five-sided figure with these characteristics can have parallel sides. This means that an ordinary household wrench will not fit. This design makes opening hydrants difficult without the special wrenches carried by firefighters.

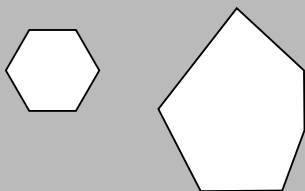
Try This:

- Tie a knot in a long, thin strip of paper. Carefully flatten the knot so the resulting figure has equal length sides. Cut off the excess paper. What shape did you make?
- Look out a window and describe all the sets of parallel lines you see.

Additional Challenges:

(Answers located in back of booklet)

1. What shapes of bolts can be turned with a common household wrench?
2. Would it be easier to use a wrench if the sides of a six-sided bolt are all the same length?



3. If each non-adjacent vertex of a pentagon is connected, diagonals are formed. The diagonals of any regular pentagon form a five-pointed star. How many triangles are there in the star?

Things to Think About:

- If two lines are parallel to a third line, are all three lines parallel?
- Some shapes have sides the same length but angles of different sizes.
- Some shapes have angles of the same size and sides of different lengths.

- A square is a rhombus, but a rhombus doesn't have to be a square.
- Why are pipe wrenches different from ordinary household wrenches?
- Unauthorized opening of fire hydrants frequently causes water pipe damage underground, and the resulting pressure can cause damage to home hot water heaters.

Did You Know That?

- The term "fire plug" dates from the early 1800s, when water mains were made of wood. When responding to an alarm, firefighters had to chop into the main waterline to connect their hoses. When they finished fighting the fire, they would seal the main with a "fire plug."
- The names *pentagon* and *hexagon* describe their respective numbers of angles (or sides). Early Greek mathematicians studied these shapes and gave them their names. In Greek, *penta-* means 5, *hexa-* means 6, *hepta-* means 7, *octa-* means 8, *nona-* means 9 and *deca-* means 10. Also, *gon* comes from a word meaning angle.
- An Allen wrench has a hexagonal cross-section.
- A regular polygon has sides the same length and interior angles the same size.
- Each of the diagonals of a regular pentagon is parallel to one of the sides.

Resources:

Books:

- Jacobs, H. *Mathematics: A Human Endeavor*. San Francisco: W.H. Freeman and Co., 1970.

Websites:

- users.intermediatn.net/hillspainting/wrenches.htm



Figure This!
Math Challenges for Families

What percentage does it take to win a vote?

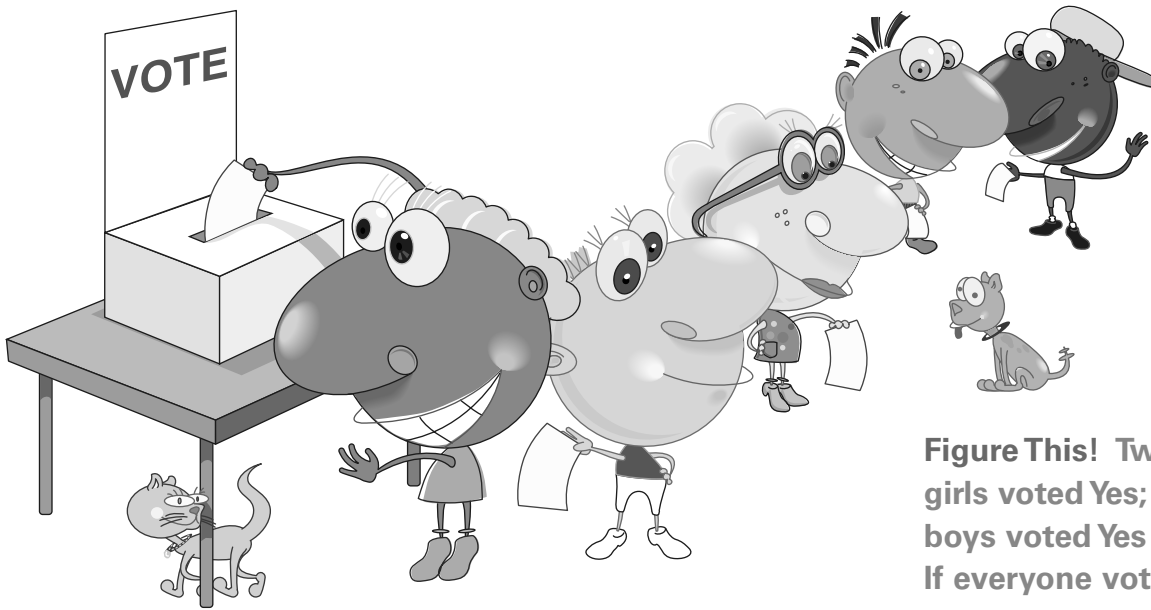


Figure This! Twenty-two percent of the girls voted Yes; thirty percent of the boys voted Yes on the same motion. If everyone voted, did the motion pass?

Hint: Suppose 40 boys and 50 girls voted. How many of each voted Yes?

Understanding percentages is necessary for people to make sense of information in the media, for businesses to summarize work data, for politicians to interpret the results of polls, and for manufacturers to make decisions about marketing.

Answer: Assume winning means a simple majority or over half of the votes. If it takes at least half voting Yes to pass a motion, the motion did not pass.

Answer:

Figure This!

Getting Started:

Suppose the number of boys and the number of girls were the same? Suppose there were many more boys than girls? Many more girls than boys?

Complete Solution:

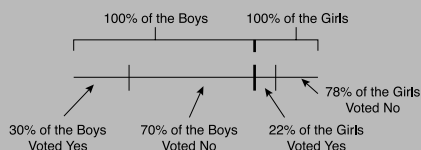
There are different ways to solve the problem.

- Look at different numbers of boys and girls. Think about each of the following:

- (1) A class of 50 boys and 50 girls
- (2) A class of 0 boys and 100 girls
- (3) A class of 100 boys and 0 girls

In neither of the extreme cases nor in any other case will the motion pass.

- Think about a line diagram showing the percentages where the numbers of boys and girls are not the same. Above the line, you see that 100% of the boys are depicted to the left of the bold line with 100% of the girls to the right. Below the line are the voting percentages of Yes and No for both boys and girls.



To determine if the Yes vote wins, then the segments representing the Yes votes together have to be over half the length of the entire segment. The segments representing Yes will never add to half of the total length so the motion cannot pass.

- Think about the percentage who voted No: 78% of the girls voted No; 70% of the boys voted No. There is no way for over half of the total to vote Yes.

Try This:

- Think of a survey question that can be answered either “Yes” or “No.” Ask a group of people your survey question and record the results for males and females. Find the percentage of males and the percentage of females who responded Yes. Then calculate the total percentage of people who said Yes. Does adding the percentages of Yes votes by males and by females result in the total percentage of people who said Yes?

- Suppose a motion passes with a 52% vote. Find some possible ways to break this into percentages for males and percentages for females who voted Yes. Is it possible that only males supported the motion? Only females?

Additional Challenges:

(Answers located in back of booklet)

1. There are 400 boys and 420 girls in a school. If 18% of the boys are left-handed and 10% of the girls are left handed, what percentage of students in school are left-handed?
2. A survey on soda preference had the following results: 25% liked Pepsi™, 30% liked Coca-Cola™, and 10% liked both. How many liked neither soda?
3. A newspaper survey reported that 42% of voters in Western County favored the Democratic candidate, and 25% of the voters in Central County favored this candidate. The newspaper reported a 17% difference in the number of voters favoring the Democratic candidate. Is this conclusion true?
4. Ninety-nine percent of the people in the United States have a television; 40% have 3 or more televisions. Is it valid to conclude that 59% of the people do not have more than 2 televisions?

Things to Think About:

- You cannot add percentages unless they were calculated using the same population and are measuring mutually exclusive events.
- When can you add, subtract, multiply, or divide percentages?
- Washington DC’s population grew –14.5% from April 1, 1990 to July 1, 1999.
- Is it ever possible to have a percentage over 100?

Did You Know That?

- Under US election rules, it is possible for presidential candidates to win with less than 50% of the popular vote.
- Percentages are often used to compare quantities from populations of different sizes.
- A rate of 10 per 1000 is the same as 1%.
- A 50% discount is the same as half price.
- There are many different methods of voting. For example, a simple majority (or plurality) method of voting has as winner the choice receiving the most votes. This method of voting can be unfair if there are more than two choices.

Resources:

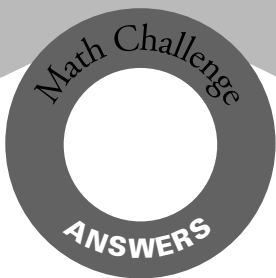
Books:

- Malkevitch, J. *The Mathematical Theory of Voting*. Lexington, MA: COMAP, Inc., 1985.
- *The World Almanac and Book of Facts 2000*. Mahwah, NJ: World Almanac Books, 1999.

Notes:

Tangent





FigureThis!
Math Challenges for Families

Looking for answers?

Here are the answers for the
Additional Challenges section
of each Challenge.

Figure This!

Answers to Additional Challenges:

Challenge 33:

1. The ramp would have to begin 7 feet farther away, or about 28 feet in all.
2. About 30.2 feet.
3. Yes.

Challenge 34:

1. About 175 gallons.
2. 70 miles per hour.
3. No, because the temperature is about 68° Fahrenheit.

Challenge 35:

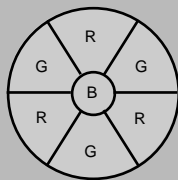
1. Any shape with at least one pair of parallel sides, including squares and regular hexagons (the most common shape).
2. The bolts with the sides the same length would be easier, because you could use the wrench on any of three sets of parallel sides.
3. 10.

Challenge 36:

1. Approximately 14% are left-handed.
2. 55% liked neither soda.
3. Not necessarily.
4. No.

Challenge 37:

1. There are many possibilities. In the example shown here, the regions are colored red (R), blue (B), or green (G).



2. A city with five such neighborhoods will need at least one overpass.
A city with six neighborhoods requires at least three overpasses.

Challenge 38:

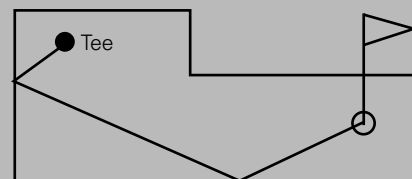
1. Sunday.
2. Yes.

Challenge 39:

1. The perimeters are 58 m and 54 m.
2. The perimeter is 56 m.
3. Since the arrangement of pieces in Helix's patio does not show a relationship between length and width, you cannot figure out the dimensions of a single piece. There are many pairs of numbers that multiply to make 20.
4. A square with a side length of 2 units.

Challenge 40:

1. It wouldn't change.
2. One possible solution is shown here:



3. The sum of the distances is least when angles 1 and 2 in the diagram are equal. In this case, that distance is 20 miles.

