Math Challenge

#29

SALARIES

Figure This! Math Challenges for Families

Will women ever earn as much money as men?

Figure This! A newspaper headline reads:"Women's salaries catching up to men's." Using the information below from the U. S. census, do you think this is true?

Median Salaries for Workers 25 or Over According to Both Year and Gender:

YEAR	MEDIAN SALARY: WOMEN	MEDIAN SALARY: MEN
1991	$11,580	$23,686
1992	$11,922	$23,894
1993	$12,234	$24,605
1994	$12,766	$25,465
1995	$13,821	$26,346
1996	$14,682	$27,248
1997	$15,573	$28,919

Hint: Think of ways to describe and compare the growth in salaries.

Organizing and interpreting information is a critical skill for business, industry, and many other professions. Making claims or decisions based on data is done by advertisers, insurance companies, athletic teams, and manufacturers.

Answer:
Based only on the data given, the statement appears to be untrue, at least in the near future. Over the long run, however, it appears that women's salaries may reach or exceed men's if the trends indicated by these data continue.

Figure This!

Get Started:

Think about either differences or ratios. What are the differences in men's and women's salaries from year to year? Compare women's salaries to men's salaries by making a fraction. What are the differences between men's and women's salaries over the entire time from 1991 to 1997? How do the fractions change? Think about percent increase as well as the difference in dollars.

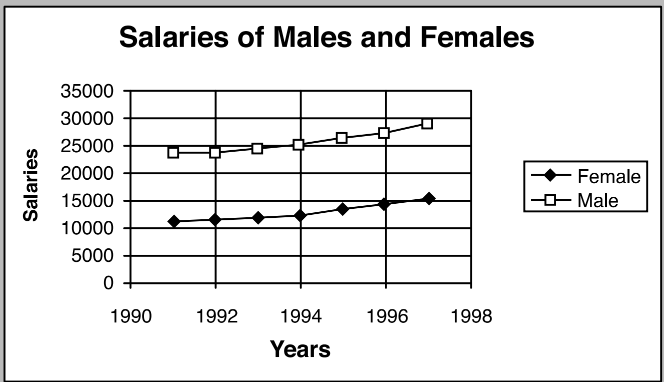
Complete Solution:

There are several ways to think about this problem. Your answer depends on your assumptions.

- Find the difference between men's and women's salaries each year; then look for a trend. Using this method, the gap appears to be growing in favor of men.

YEAR	MEDIAN SALARY: WOMEN	MEDIAN SALARY: MEN	MEN'S SALARIES – WOMEN'S SALARIES
1991	$11,580	$23,686	$12,106
1992	$11,922	$23,894	$11,972
1993	$12,234	$24,605	$12,525
1994	$12,766	$25,465	$12,371
1995	$13,821	$26,346	$12,699
1996	$14,682	$27,248	$12,566
1997	$15,573	$28,919	$13,346

- Another way to think about the data is to draw a graph with the year on the horizontal axis and salaries on the vertical axis. Plot the salary data for women, then connect the data points in order. On the same graph, plot and connect the data points for men's salary. As shown in the following graph, men's salaries are greater than women's for every data point. From the data up to 1998, the two graphs look like they will never cross, which may lead you to predict that men's salaries will remain higher than women's. You may need more information to predict the future.



- A third way to analyze this information is to compare the percent increase in salaries for both men and women. For the period from 1991 to 1997, the percent increases are:

$$\text{Men: } \frac{28{,}919 - 23{,}686}{23{,}686} \approx 22\%$$

$$\text{Women: } \frac{15{,}573 - 11{,}580}{11{,}580} \approx 34.5\%$$

If this trend continues, women's salaries will eventually meet and then exceed men's salaries.

Try This:

- Find information on salaries for a particular career. (Check with your school guidance department, the Internet, or an almanac.) Is there a difference in the salaries for men and women?

Additional Challenges:

1. Using the data in the challenge, find the percent increase in salary for men and for women in every year. What is the average annual percent increase in salary for men and women?
2. The table below shows predicted salaries for men and women using the average annual percent increases from 1991 to 1997. What do these predictions indicate about men's and women's salaries in the future?

YEAR	WOMEN'S PROJECTED SALARIES	MEN'S PROJECTED: SALARIES
2010	$29.365	$44,663
2020	$47,832	$62,395
2030	$77,913	$87,167

Things to Think About:

- What affects the trends in salaries?
- What might cause women's salaries to be lower than men's?
- How do the salaries of male and female athletes compare?
- Are there many women in the list of the world's richest people?
- Why do you think salaries for men and women are usually represented by the median (or middle value), rather than the mean (or average)?

- A 1% salary increase is not necessarily less than a 50% salary increase—it depends on the original salaries involved. (For example, compare 1% of $1,000,000 to 50% of $10,000.)

Did You Know That?

- In 1996, the maximum German hourly pay rate of $31.87 for those in manufacturing was the highest in the world. The corresponding rate in the United States was $17.70.
- In 1996, residents of New York City had the highest average annual salary in the United States: $45,028. Among urban areas, residents of Jacksonville, North Carolina, had the lowest average annual salary: $17,934.
- Mathematicians sometimes use an exponential model to describe rates of growth over time. In some cases, in which the increases level off, a logistic model is used.

Resources:

Books:

- T*he Guinness Book of World Records*. New York: Bantam Books, 1998.
- *The World Almanac and Book of Facts, 1999*. Mahwah, NJ: World Almanac Books, 1998.

Website:

- **www.census.gov/income/p13.txt**

Answers to Additional Challenges:

(1.)

For men, the average annual increase is about 3.4%; for women, the average increase is about 5%.

(2.)

If salaries continue to grow at past rates, women's salaries will be equal to men's in about 2036.

Notes:

Math Challenge #30 SMILES

Figure This! Math Challenges for Families

Which is worth more, a SMILE or a FROWN?

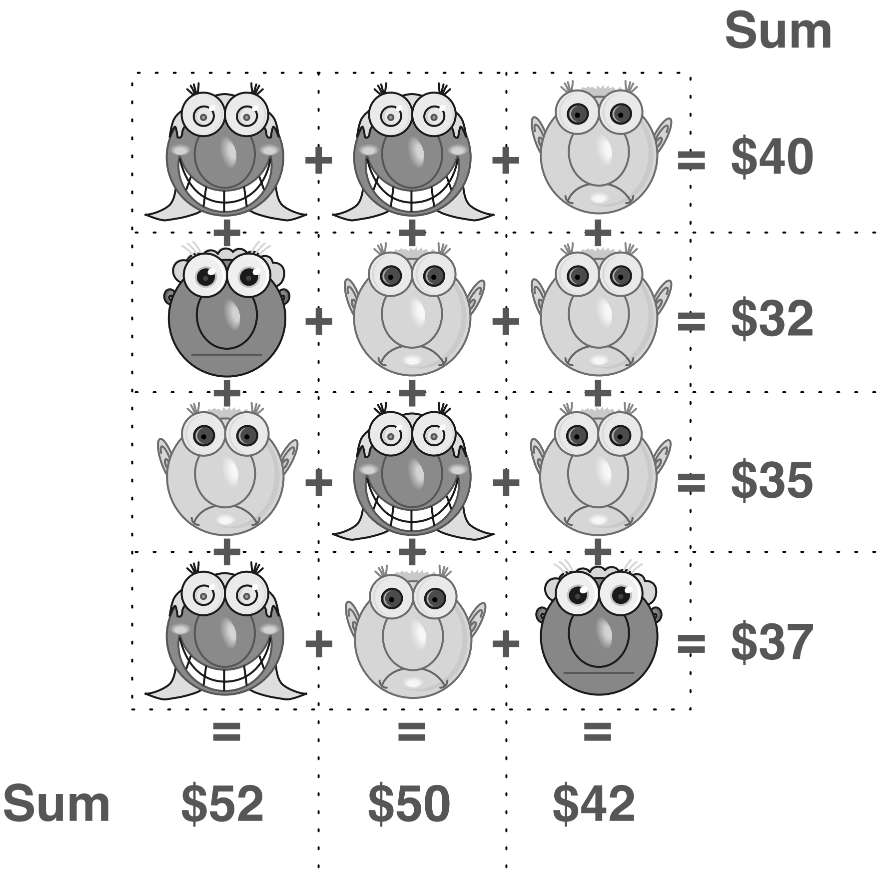


Figure This! The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

Hint: Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.

Answer:
A smile is worth $15. A frown is worth $10.

Figure This!

Get Started:

- Look at the rows and columns. Which rows or columns have only smiles or frowns? What is the same and what is different in these rows and columns? Can you see a pattern?

Complete Solution:

There are many ways to approach this challenge.

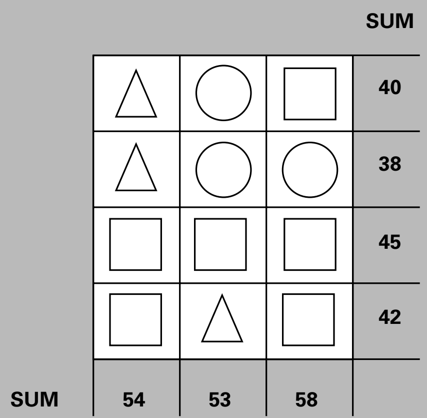
- Top row → 2 smiles and 1 frown = $40.
  Middle column → 2 smiles and 2 frowns = $50.
  The middle column has one more frown and costs $10 more than the top row, so:
  1 frown = $10.
  Since 2 smiles and 1 frown = $40
  and 1 frown = $10,
  Subtracting 2 smiles = $30, so that
  1 smile = $15.
- Another way is to use the first and third rows.
  2 smiles and 1 frown = $40
  1 smile and 2 frowns = $35
  Continue the pattern, lose a smile, and add a frown for $5 less.
  0 smiles and 3 frowns = $30
  1 frown = $10
  Then continue using the the first method to find the cost of a smile.

Try This:

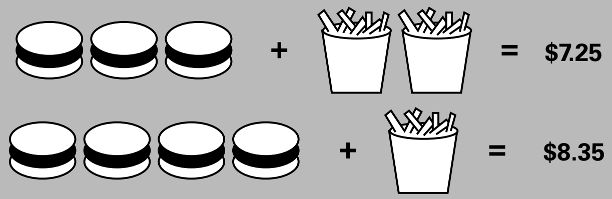
- Choose values for frowns, smiles, and neutral faces, then make a grid of your own. See if a friend can solve it.

Additional Challenges:

1. Find the value of a neutral face in the challenge.
2. Find the value of each shape in the following diagram.



3. The pictures below show the cost of hamburgers and bags of fries. How much does a hamburger, and how much does a bag of fries cost?



Things to Think About:

- Unknowns in math can be represented by pictures, words, or letters.
- Many people use algebra in daily work without thinking about it.
- Are there other combinations in the original challenge that would lead to the solution?
- Does rearranging the rows in the Challenge make a difference? The columns?

Did You Know That?

- Many books and magazines feature puzzles like the one in the challenge.
- Some people earn their livings creating puzzles and logic problems.
- The mathematical study of linear programming, which involves systems of equations, is used by business and industry for decision making.
- Game theory and logic are branches of math where games and puzzles are found.

Resources:

Book:

- Skitt, Gale, Harold Skitt, and Carolyn Skitt. *Mensa Math Games for Kids*. Rocklin, CA: Prima Publishing, 1994.
- Salny, Abbie. *Cranium Crackers*. New York: Dodd, Mead and Co., 1986.
- Sawyer, W.W. *Vision in Elementary Mathematics*. Baltimore, MD: Penguin Books, 1964.

Magazine:

- *Games*. Ambler, PA: Games Publications, Inc.

Answers to Additional Challenges:

(1.) :
$12
(2.) :
Each square is worth 15, each triangle is worth 12, and each circle is worth 13.
(3.) :
One hamburger costs $1.89 and one bag of fries costs $0.79.

Math Challenge #31 TOURNAMENTS

Figure This! Math Challenges for Families

I've got to win how many games?

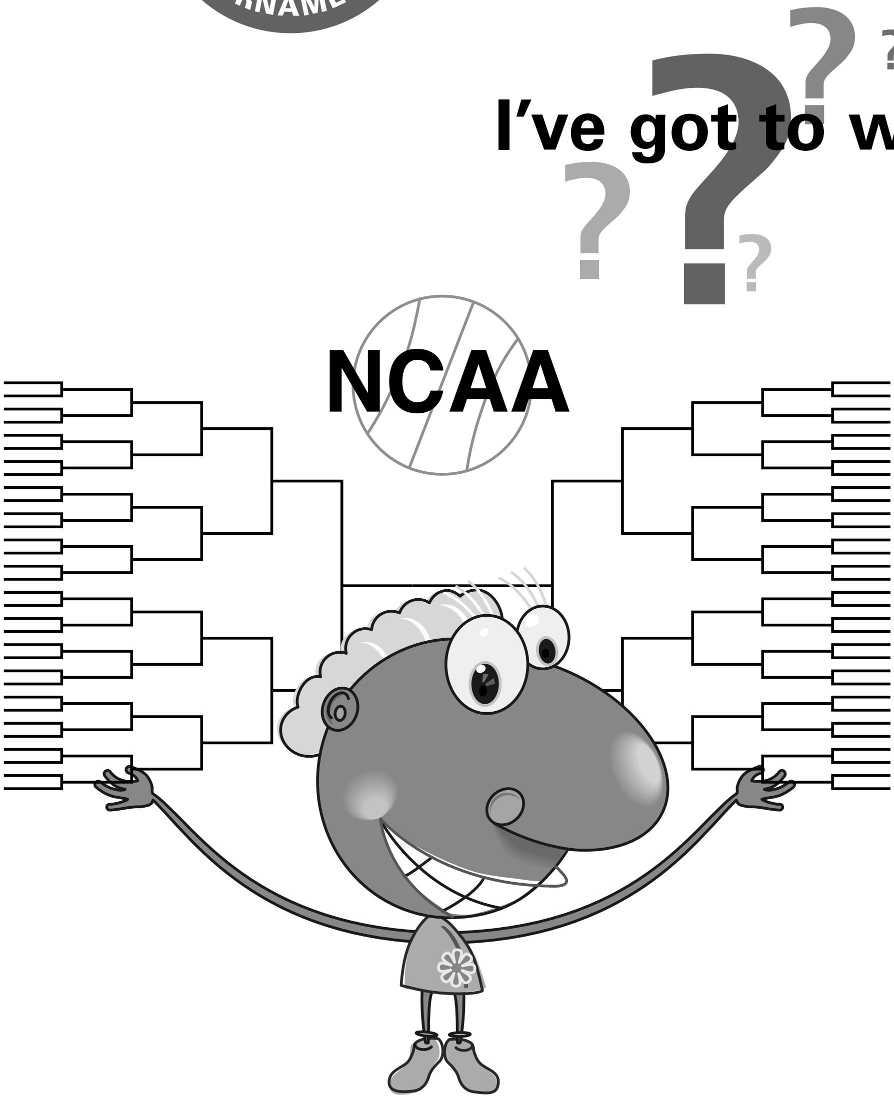


Figure This! In March, the National Collegiate Athletic Association (NCAA) holds a basketball tournament. As soon as a team loses one game, it is out of the tournament. The last unbeaten team is the winner. There were 64 teams in the 1999 men's tournament. In 1985, there were only 32 teams. How many more games were played in the 1999 tournament than in 1985?

Hint: When the final winner is determined, how many teams will have lost?

Thinking about a problem from different perspectives and looking for a pattern are powerful problem-solving strategies. Such strategies are important tools for planning space missions, tournaments, and making business decisions.

Answer: 32 games

Figure This!

Get Started:

Think about the problem in terms of losses as in the hint, or think about how many games there will be and make a schedule. Start with two teams; then three teams. Continue adding more teams and look for patterns. Use the pattern to determine the numbers of games in 1985 and 1999.

Complete Solution:

There is more than one way to do this problem.

- Consider that every team except the winner loses exactly 1 game. If there are 64 teams in the tournament and 1 winner, then there were 63 losing teams. This means there were 63 games. If there are 32 teams in the tournament and 1 winner, there were 31 games. So, 63 – 31 = 32 games.
- Create a table like the one below to determine the number of games required for a 64-team tournament.

ROUND	NO. OF TEAMS AT THE START OF THE ROUND	NO. OF GAMES	NUMBER OF WINNERS
1	64	32	32
2	32	16	16
3	16	8	8
4	8	4	4
5	4	2	2
6	2	1	1
	TOTAL NUMBER OF GAMES		63

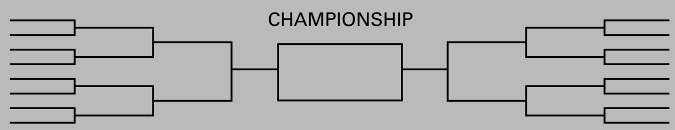
Add the numbers for the games in the table to find the total number, 63, of games for 64 teams. A similar table can be used to find that a 32-team tournament requires 31 games. The difference is 63 – 31 = 32.

- After the first round of 32 games in 1999, the number of teams was the same as in the 1985 tournament. So, the difference in the number of games is the number of games played in the first round, 32.
- Another way is to consider the number of games necessary with 2 teams, then 3, and so on. If you can identify a pattern, you can use that pattern to determine the number of games for 32 and 64 teams.

# of TEAMS	1ST ROUND	2ND ROUND	3RD ROUND	# of GAMES
2 teams (A and B)	A plays B			1 game
3 teams (A, B, C)	A plays B C does not play	B plays C		2 games
4 teams (A, B, C, D)	A plays B C plays D	B plays D		3 games
5 teams (A, B, C, D, E)	A plays B C plays D E doesn't play	B plays D E still doesn't play	B plays E	4 games

As shown in the table, in each case, there is one less game than the number of teams. A 64-team tournament would require 63 games to determine a winner, while a 32-team tournament would require 31 games. The difference is 63 – 31, or 32 games.

- A graphical method to solve the problem uses tournament brackets showing how the teams are scheduled to play. Consider the brackets below.



With this set of brackets, you see that half of the teams are grouped in pairs on the right and half on the left to begin. The first round is played in the outside brackets for a total of 16/2 or 8 games. The next round with teams paired in the second set of brackets consists of 8/2 or 4 games. Continuing this process shows that for 16 teams, there are 8 + 4 + 2 + 1, or 15 total games. Using brackets can be done for any number of teams. For 64 teams, there are 63 games; for 32 teams, 31 games. 63 – 31 = 32.

Try This:

- Find out how tournaments involving your school are scheduled.

Additional Challenges:

1. In 1983, there were 48 teams in the NCAA tournament. The top 16 teams did not have to play in the first round. How many games were played to determine the champion?
2. In a double-elimination tournament, a team is eliminated when it loses two games. If there are 8 teams in a double-elimination tournament, what is the maximum number of games required to determine a champion?
3. Why do you think that the number of teams invited to compete in a tournament is usually a power of 2 (2, 4, 8, 16, and so on)?

Things to Think About:

- How are tennis tournaments scheduled?
- Some tournaments, such as Major League Baseball's World Series, are won by the first team to win four games. How many games are usually played in the World Series?
- Chess competitions often use ladder tournaments. How do you think a ladder tournament might work?

Did You Know That?

- The NCAA Championship game first appeared on television in 1962 when an edited version was shown on ABC's Wide World of Sports.
- The first NCAA Championship for women's basketball was played in 1982.
- Oregon won the first NCAA championship in men's basketball, held in 1939.
- During the first 12 years of the men's tournament, only 8 teams played. As the tournament gained in importance, the field was expanded to 16 teams, then to 32, and then to its present size of 64.

Resources:

Book:

- Halmos, Paul R. *Problems for Mathematicians Young and Old*, Dolciani Mathematical Expositions Number 12. Washington, DC: Mathematical Association of America, 1991.
- Hillstrom, Kevin, Laurie Hillstrom, and Roger Matuz. *The Handy Sports Answer Book*. Farmington Hills, MI: Visible Press, 1998.
- *Sports Illustrated 1999 Sports Almanac*. New York: Little, Brown and Co., 1998.
- Wolf, Alexander. *Sports Illustrated 100 Years of Hoops*. Birmingham, AL: Oxmoor House, 1991.

Website:

- **http://www.ncaa.org/**
- **http://forum.swarthmore.edu/dr.math/problems/topp.8.17.96.html**

Answers to Additional Challenges:

(1.)
47 games.

(2.)
15 games.

(3.)
Using powers of 2, the pairings in each round come out even.
Therefore, no teams must sit out.

Notes:

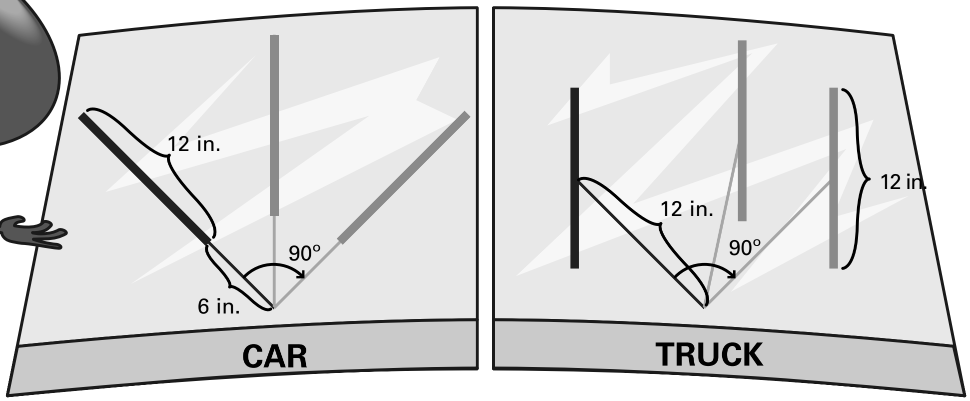
Math Challenge #32 WINDSHIELD WIPERS

Figure This! Math Challenges for Families

IT'S RAINING!!

Who sees more: the driver of the car or the truck?

Figure This! **Which wiper cleans the greater area?**



Hint: Draw models of the areas, then cut them out and compare.

Areas of familiar geometric shapes can be used to find the areas of other shapes. Surveyors, carpet layers, designers, and building contractors all calculate areas in their work.

Answer:
The car wiper cleans a greater area.

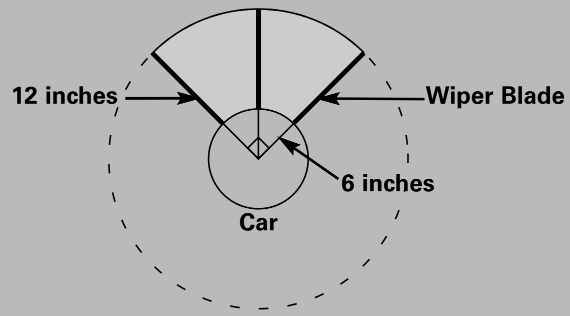
Figure This!

Get Started:

Draw the area cleaned by each wiper. What shape is each area? How could you find the area of each shape?

Complete Solution:

The car wiper rotates through 1/4 (or one quarter) of a circle.The area cleaned is the difference in the areas of two quarter-circles as shown.



The area of a circle can be found using the following formula:

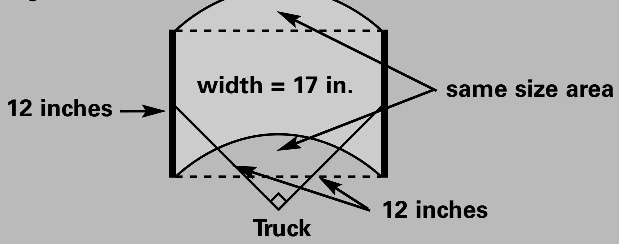
$$\textbf{Area} = \pi \bullet \textbf{radius} \bullet \textbf{radius or } A = \pi r^2$$

The wiper arm swings through an arc of 90˚, or 1/4 of a circle. The area of a quarter-circle can be found by multiplying the area of the corresponding circle by 1/4. In this case, the area cleaned by the wiper is the area of the bigger quarter-circle minus the area of the smaller one. The radius of the big circle is 6 + 12, or 18 inches. The radius of the small circle is 6 inches so that the area cleaned is:

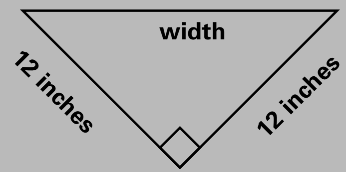
$$1/4\ (\pi \bullet 18^2) - 1/4\ (\pi \bullet 6^2) = 72\pi$$

$$\pi \text{ is about 3.14 so } 72 \bullet \pi \text{ is about 226 square inches.}$$

The area of the shape cleaned by the truck wiper is really the same as a rectangle.



The curved area at the top of the shape is the same size as the part at the bottom that is not cleaned by the wiper. The height of the rectangle is determined by the size of the wiper: 12 inches. The width of the rectangle can be found by drawing the figure to scale, then measuring, or by using the Pythagorean theorem. According to the Pythagorean theorem, the sum of the squares of the two shorter sides of a right triangle equals the square of the longest side.



$$\text{width}^2 = 12^2 + 12^2 \qquad \text{width}^2 = 288$$

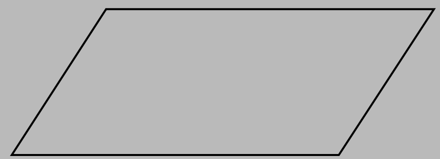
Therefore, the width of the rectangle is the square root of 288, or approximately 17 inches.The area of a rectangle can be found by multiplying its height and its width. In this case, the area is about 17 • 12, or 204 square inches. This is less than the area cleaned by the car wiper.

Try This:

Use straws to make a model of each of the wipers. Move each wiper arm 90° and draw the shape that the wiper blade describes.

Additional Challenges:

1. Cut a parallelogram as shown from a sheet of paper.



   Make a rectangle from the parallelogram using only one cut and rearranging the parts.

2. Cut a triangle from a sheet of paper. Cut the triangle into three pieces and put the pieces together to make a rectangle.

Things to Think About:

- Can you design a 12-inch wiper that would clean more area than either of the two types in the challenge?
- Look at some cars, trucks, and other vehicles. How are their windshield wipers designed? How many different designs can you find?
- Which do you think would clean a windshield better: one large wiper or two smaller wipers that overlap?
- How do rear-window wipers differ from windshield wipers?
- Why do some Mercedes use only one big wiper instead of two?

Did You Know That?

- Most wipers are between 16 and 22 inches long and sweep out an angle between 90° and 120°.
- Robert William Kearns, the developer of the intermittent windshield wiper, has spent much of his life fighting for the rights of inventors. He had to go to court to secure rights to his invention.

Resources:

Website:

- **www.nwb.co.jp/e/encyclo/history.html**

Answers to Additional Challenges:

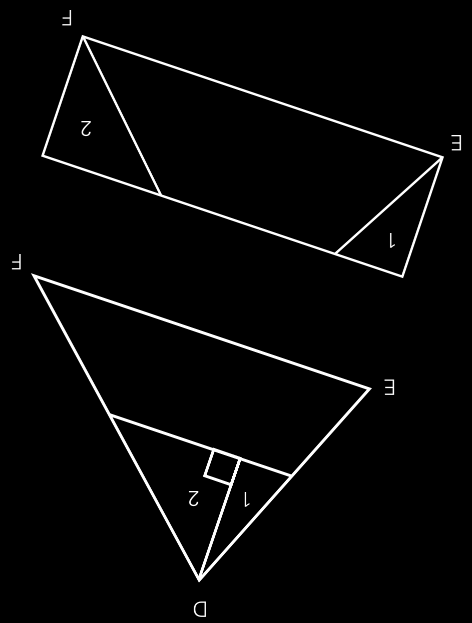
(1) :

One way is to cut a right triangle off one end of the parallelogram, then tape it to the other end of the remaining figure. Both the original parallelogram and the new rectangle have the same area.

(2) :

One way is to cut the triangle as shown below.

- Turn triangle 1 to fit D on E.
- Turn triangle 2 to fit D on F.



Notes: