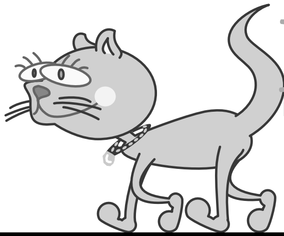
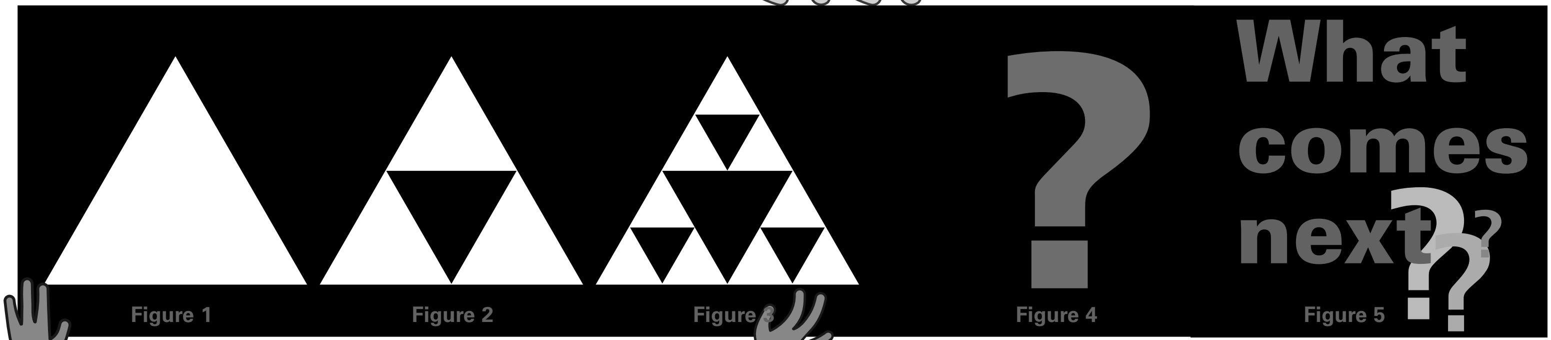
Math Challenge #23 FRACTALS

Figure This!

Math Challenges for Families





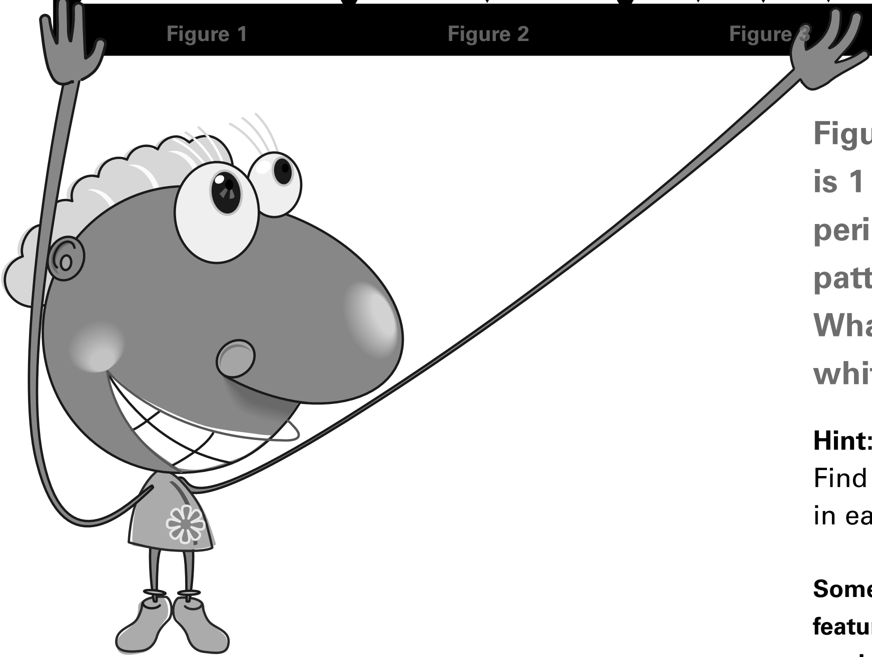


Figure This! If each side of the triangle in Figure 1 is 1 inch long, this means the triangle has a perimeter of 3 inches. Suppose you continued the pattern in the diagram until you reached Figure 5. What is the sum of the perimeters of all the white triangles in Figure 5?

Hint: The perimeter of a shape is the distance around it. Find the sum of the perimeters for all the white triangles in each of the figures above. What pattern do you see?

Some repeating patterns form fractals. Many naturally occurring features, such as ferns, weather patterns, or coastlines, can be modeled by fractals.

Answer:
15 3/16 inches

Figure This!

Get Started:

Figure 1 has only one white triangle. Since each side is 1 inch long, its perimeter is 3 inches. How many white triangles are in Figure 2? What is the perimeter of each of these triangles? What is the sum of the perimeters for Figure 2? Make a table to help answer the questions.

Figure Number	Side Length of Each White Triangle (inches)	Perimeter, *p*, of Each White Triangle = 3x Side Length (inches)	Total Number, *n*, of White Triangles	Sum, *np*, of the perimeters
1	1	3	1	3
2				

Complete Solution:

Complete the table in the hint.

Figure Number	Side Length of Each White Triangle (inches)	Perimeter, *p*, of Each White Triangle = 3x Side Length (inches)	Total Number, *n*, of White Triangles	Sum, *np*, of the perimeters
1	1	3	1	3
2	1/2	3/2	3	9/2
3	1/4	3/4	9	27/4
4	1/8	3/8	27	81/8
5	1/16	3/16	81	243/16

As the figure numbers increase, the side length of each white triangle is halved, and the number of white triangles is tripled. This means the sum of the perimeters in any particular figure is, 3 • 1/2 or 3/2 the sum in the previous figure. The sum of the perimeters in Figure 5 would be 243/16, or 15 3/16 inches.

Try This:

- As the pattern in the challenge continues, smaller copies of the first few figures repeat themselves. A similar effect occurs in some surprising places. For example, look at the front of a Cracker Jack™ box, or stand between two parallel mirrors and look at the images created. What patterns do you notice?
- Design your own fractal pattern.

Additional Challenges:

1. Suppose it takes 1 ounce of paint to cover the triangle in Figure 1, the original white triangle in the challenge. How much paint would you need to cover the white triangles in Figures 2 and 3?
2. Find a general rule for the sum of the perimeters of the white triangles in each figure in the challenge.
3. Each side of the square in Figure 1 below is 1 inch long. Suppose you continued the pattern in this diagram until you reached Figure 5.

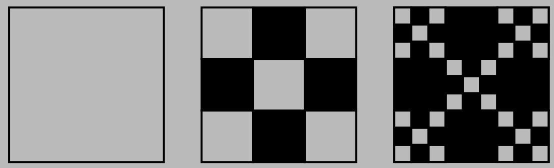


Figure 1 **Figure 2** **Figure 3**

   a. Find the sum of the perimeters of the white squares in Figure 5.
   b. Find the sum of the areas of the white squares in Figure 5.

Things to Think About:

- If the pattern in the challenge continues, what do you think will eventually happen to the perimeter and area of successive figures?
- How do Bart Simpson's comic books provide an illustration of fractals? (See the references.)

Did You Know That?

- The pattern in the challenge results in a figure called Sierpinski's triangle, named after the Polish mathematician Waclaw Sierpinski, who developed it in about 1915.
- Parts of some fractals look like the whole fractal. This property is called self-similarity.
- Figures that can be built with smaller copies of the same figure are sometimes called rep-tiles or clones.
- Some computer illustrations use fractals to model clouds, coastlines and mountains.

Resources:

Books:

- Peitgen, H., H. Jurgens, D. Saupe, E. Maletsky, T. Perciante, and L. Yunker. *Fractals For The Classroom: Strategic Activities Volume One.* New York: Springer-Verlag, 1992.
- *Simpsons Comics Issue 4*. Los Angeles: Bongo Entertainment, Inc., 1994.
- *(Bart Simpson's) Treehouse of Horror Issue 2.* Los Angeles: Bongo Entertainment, Inc., 1996.

Website:

- **math.rice.edu/~lanius/fractals**

Answers to Additional Challenges:

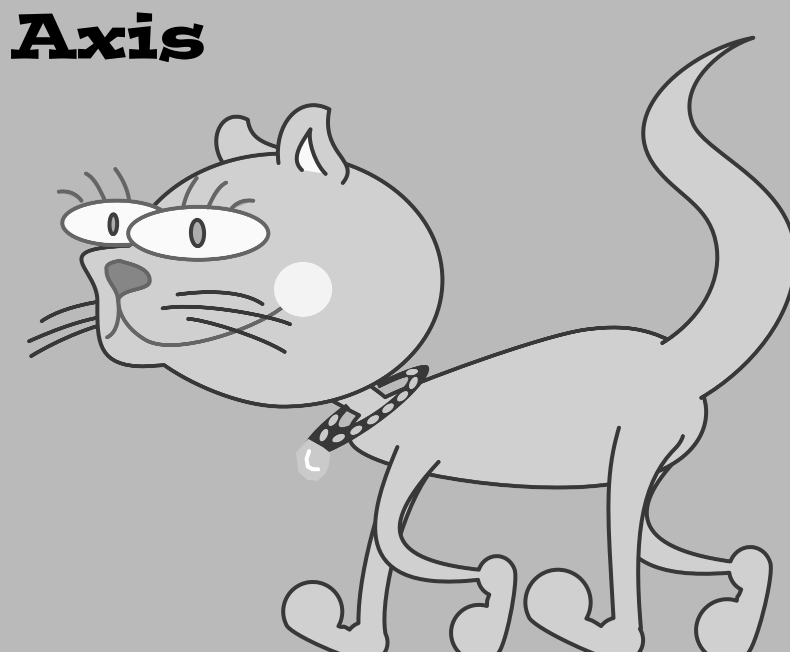
(1.)
If it takes 1 ounce to paint the triangle in Figure 1, then it would take 3/4 of an ounce to paint the white triangles in Figure 2, and 9/16 of an ounce to paint the white triangles in Figure 3.

(2.)
If n is the figure number, and the side length in Figure 1 is 1 inch, then the sum of the perimeters, in inches, is: $\frac{3^n}{2^{n-1}}$ or $3 \cdot \left(\frac{3}{2}\right)^{n-1}$

(3 a.)
The sum of the perimeters for Figure 5 would be 2500/81 or about 30.9 inches.

(3 b.)
The sum of the areas is 625/6561, or about 0.1 square inches.

Notes:



Math Challenge #24 GASOLINE TANKS

Figure This! Math Challenges for Families

How far can you GO on a tank of gas?

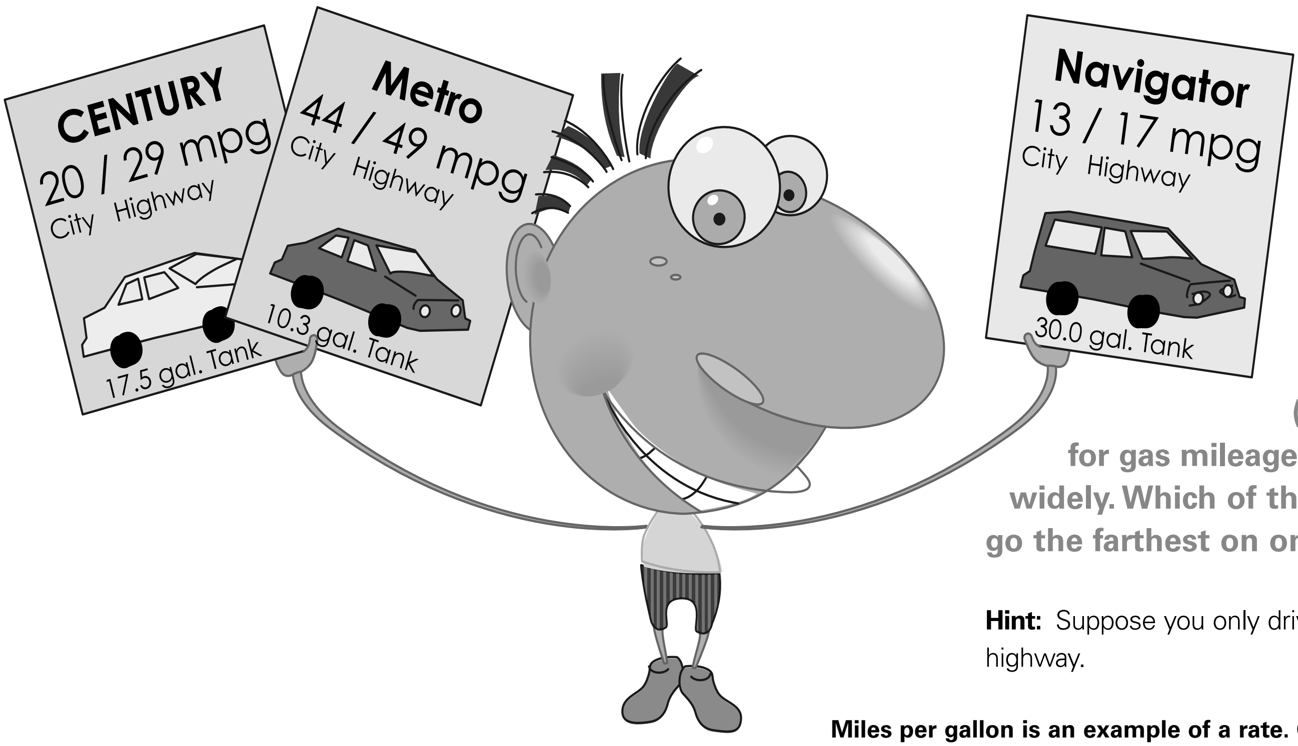


Figure This! The Environmental Protection Agency (EPA) estimates for gas mileage on 1999 cars vary widely. Which of these cars should go the farthest on one tank of gas?

Hint: Suppose you only drive in the city. On the highway.

Miles per gallon is an example of a rate. Grocers, demographers, financiers, actuaries, and economists all use rates in their work.

Answer:

For city driving, the Metro would go the farthest. For highway driving, the Navigator would go farthest although not significantly farther. You might also consider combinations of city and highway driving.

Figure This!

Get Started:

How many miles could each car travel on one tank of gas? You need to know its EPA mileage estimate, as well as its fuel capacity. The number of miles possible can be found using the formula: number of miles traveled = fuel capacity in gallons x the estimated mileage in miles per gallon.

Complete Solution:

Different combinations of city and highway driving are possible. If you assume all driving is done on the highway, the Navigator should go farthest as seen in the chart below, but all three go approximately the same distance.

CAR	Highway Mileage (mpg)	Tank Capacity (gallons)	Approximate Distance (miles)
Century	29	17.5	508
Metro	49	10.3	505
Navigator	17	30	510

If you assume all driving is done in the city, the Metro will go the farthest.

CAR	City Mileage (mpg)	Tank Capacity (gallons)	Approximate Distance (miles)
Century	20	17.5	350
Metro	44	10.3	453
Navigator	13	30	390

You might also consider different combinations of highway and city driving.

Try This:

- Choose a car. Use information from a website, magazine, or car dealer to determine how far this car can travel on one tank of gas.

Additional Challenges:

1. Suppose the following two cars travel 100 miles, half in the city and half on the highway. Which uses the least amount of gas?
   - Ferrari Maranello (9 mpg city / 14 mpg highway)
   - Lamborghini Diablo (10 mpg city / 13 mpg highway)
2. Suppose you wanted to design a car that could travel 600 highway miles on one tank of gas. What are some possible values for this car's fuel capacity and highway mileage?
3. Imagine that you drive a Metro. Your daily commute to work includes 80 highway miles and 10 city miles, each way. If you start the week with a full tank, on what day will you need to buy gas?

Things to Think About:

- How do automobile makers decide how many gallons of gas a tank should hold?
- How do automobile makers decide whether to put the gasoline tank on the left or the right of the car?
- Some experimental cars operate on electric batteries. Do these go farther before the batteries die than another car would go on a tank of gas?

Did You Know That?

- A 1991 Toyota Land Cruiser with a 38.2-gallon gas tank traveled 1691.6 miles without refueling.
- Volkswagen has developed a car that can travel 80 miles on 1 gallon of gas.
- Many trucks have more than one gas tank.
- Because they had no gas gauge, some older-model Volkswagens had a 2-gallon reserve tank.

Resources:

Books:

- *The Guinness Book of Records*. New York: Guinness Book of Records, 1999.
- *Kelley Blue Book Used Car Guide January-June 2000: Consumer Edition*.
- *N.A.D.A. Official Used Car Guide*. McLean, VA: National Automobile Dealers Association.
- *Consumer Reports*. Yonkers, NY: Consumers Union.

Websites:

- **www.edmunds.com**
- **www.consumerreports.org**
- **www.epa.gov/OMSWWW/mpg.htm**
- **www.newcarpoint.com/index.html**

Answers to Additional Challenges:

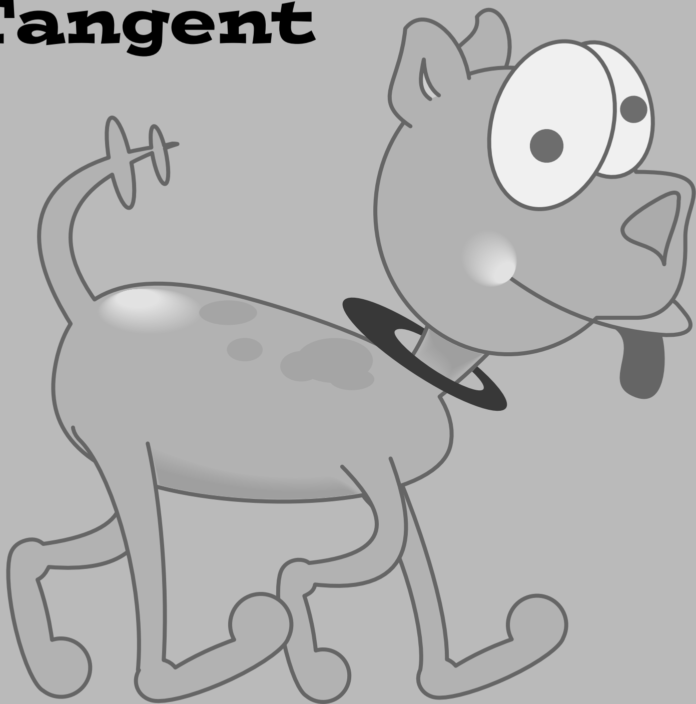
(1.)
The Lamborghini Diablo.

(2.)
There are many combinations possible. For example, a car with a 13.4 – gallon tank and highway mileage of 45 mpg could go about 600 miles.

(3.)
You will need gas on the third day.

Notes:

Tangent



Math Challenge #25

GRAPE JUICE JUNGLE

Figure This! Math Challenges for Families

Which tastes JUICIER?

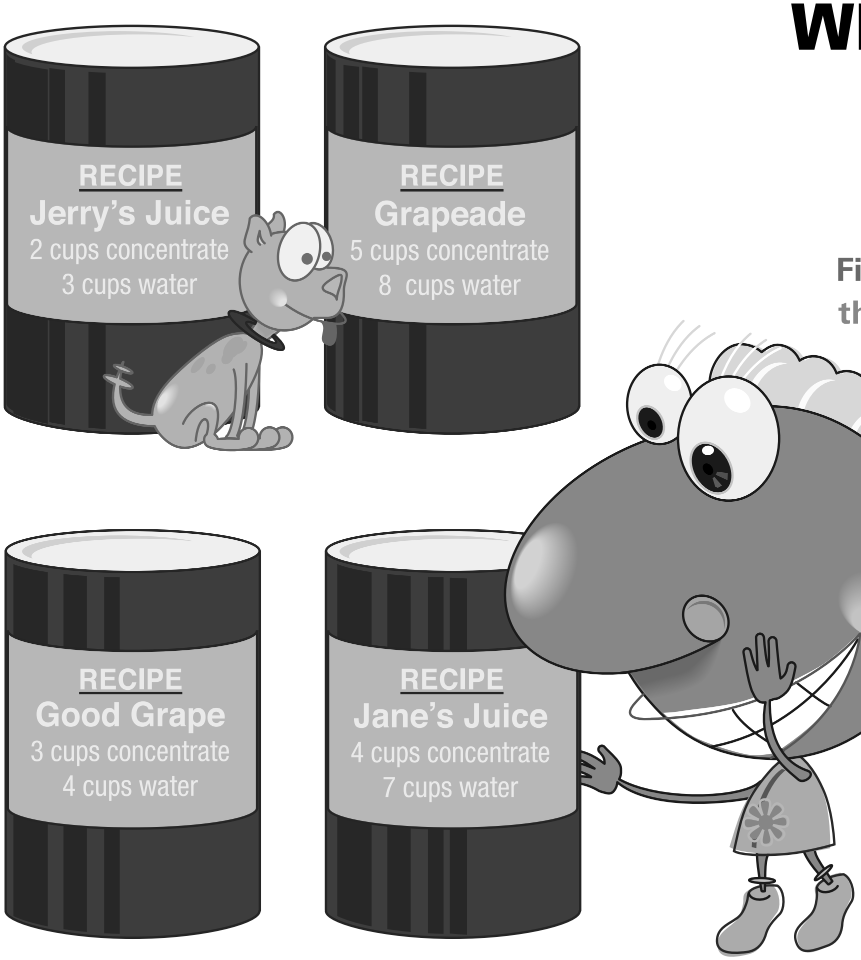


Figure This! If all grape juice concentrates are the same strength, which recipe would you expect to have the strongest grape taste?

Hint: For each recipe think about how much water should be used with 1 cup (c.) of concentrate, or how much concentrate should be used with 1 cup of water.

Ratios are fractions that compare two or more quantities. Shoppers use ratios to compare prices; cooks use them to adjust recipes. Architects and designers use ratios to create scale drawings.

Answer:
Good Grape should have the strongest grape taste.

Figure This!

Get Started:

Answer one of the following questions: Which recipe uses the most water for 1 cup of concentrate? Which recipe uses the most concentrate for 1 cup of water?

Complete Solution:

There are several ways to approach this problem.

- One way is to determine how much concentrate each recipe uses for 1 cup of water. The one that uses the most concentrate should have the strongest grape taste.

RECIPE	Cups of Concentrate per Recipe	Cups of Water per Recipe	Ratio of Concentrate to Water	Ratio of Concentrate to 1 cup of Water
Jerry's Juice	2	3	2/3	$\frac{2 \div 3}{3 \div 3} \approx \frac{0.67}{1}$
Grapeade	5	8	5/8	$\frac{5 \div 8}{8 \div 8} \approx \frac{0.63}{1}$
Good Grape	3	4	3/4	$\frac{3 \div 4}{4 \div 4} = \frac{0.75}{1}$
Jane's Juice	4	7	4/7	$\frac{4 \div 7}{7 \div 7} \approx \frac{0.56}{1}$

Good Grape has the most concentrate (0.75) for 1 cup of water. It should have the strongest grape taste.

- Another way is to find how much water each recipe uses for 1 cup of concentrate. Here, the recipe that uses the least water should have the strongest grape taste.

RECIPE	Cups of Concentrate per Recipe	Cups of Water per Recipe	Ratio of Concentrate to Water	Ratio of 1 Cup of Concentrate to Water
Jerry's Juice	2	3	2/3	$\frac{2 \div 2}{3 \div 2} = \frac{1}{1.5}$
Grapeade	5	8	5/8	$\frac{5 \div 5}{8 \div 5} = \frac{1}{1.6}$
Good Grape	3	4	3/4	$\frac{3 \div 3}{4 \div 3} \approx \frac{1}{1.3}$
Jane's Juice	4	7	4/7	$\frac{4 \div 4}{7 \div 4} \approx \frac{1}{1.8}$

Good Grape has the least amount of water, 1.3 cups, to 1 cup of concentrate and so should have the most grape flavor.

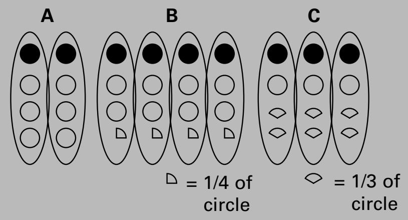
Try This:

- Choose a product such as breakfast cereal, liquid soap, or canned soup. Check several different brands and package sizes to see the differences in the cost per unit of weight or the cost per unit of volume.
- A box of oatmeal usually has recipes for different sized servings. Check a box to see if the proportions of ingredients are the same for the different sized servings.

Additional Challenge:

1. What is the ratio of water to the total amount of liquid in one recipe of Jerry's Juice?

2. A fifth grade student drew this picture to solve a problem like the one in the challenge. The dark circles represent concentrate, and the white circles represent water.



   Describe each recipe. Rank the recipes in order from strongest to weakest.

3. If we mixed the concentrates for Jerry's Juice and Grapeade and made a recipe, is this mixture as strongly grape flavored as Jerry's Juice alone?

Things to Think About:

- What fraction of the human body is water?
- What proportion of fats, carbohydrates, and proteins do you eat in a normal meal?
- Do all lemonade mixes contain lemon?

Did You Know That?

- Grape juice and apple juice provide the base for many flavors of fruit juices.
- If a recipe is doubled, tripled, or quadrupled, the proportion of an ingredient stays the same. This is an example of a direct proportion.
- Direct proportions lead to the study of lines or linearity.

Resources:

Books:

- Keijzer, Ronald, Mieke Abels, Laura J. Brinker, S. R. Cole, and Julia A. Shew. *Ratios and Rates.* In *Mathematics in Context*. National Center for Research in Mathematical Sciences Education and Freudenthal Institute. Chicago: Encyclopaedia Britannica Educational Corporation, 1998.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and R. Phillips. *Connected Mathematics: Comparing and Scaling.* Palo Alto, CA: Dale Seymour Publications, 1996.
- Niven, Ivan. *Numbers: Rational and Irrational*. New Mathematical Library, Vol. 1. Washington, DC: Mathematical Association of America, 1961.

Answers to Additional Challenges:

(1.)
The ratio is 3 to 5.

(2.)
A is 2 cups of concentrate to 6 cups of water. B is 4 cups of concentrate to 9 cups of water. C is 3 cups of concentrate to 5 cups of water. The ranking is C, B, and A from strongest to weakest.

(3.)
Jerry's Juice is stronger than the mixture.

Notes:

Axis