



Figure This!

Math Challenges for Families

WHO PLAYED THE RAPTORS



Figure This! There are four basketball games Saturday night. Three sportswriters predicted the winners in the Saturday morning paper.

- Perimeter picks the Raptors, Pacers, Magic, and 76ers.
- Exponent picks the Hawks, Pistons, Magic, and Raptors.
- Helix picks the Heat, Pacers, Pistons, and Raptors.
- No one picks the Bucks.

WHO PLAYED WHOM?

Hint: Teams chosen by the same sportswriter did not play each other.

Organizing and analyzing information to make logical decisions are important skills in many professions. Company managers, doctors, and scientists all use these skills.

Answer: Raptors played Bucks, Pacers played Hawks, Magic played Heat, and Pistons played 76ers.

Figure This!

Get Started:

Since each game can only have one winner, the four winners picked by each sportswriter are not playing each other. One way to organize the rest of the information is to make a chart like the one shown below. There are eight teams involved in the four games. Each X in the chart shows a match-up that can be ruled out using Perimeter's picks. What match-ups can be ruled out using Exponent's picks?

	Raptors	Pacers	Magic	76ers	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	X				
Pacers	X	X	X	X				
Magic	X	X	X	X				
76ers	X	X	X	X				
Hawks								
Pistons								
Heat								
Bucks								

Complete Solution:

- One way to do this problem is to use each writer's picks to complete the chart described in the Get Started section. If a row or column has an X in all spaces but one, the unmarked space shows two teams that play each other.

	Raptors	Pacers	Magic	76ers	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	X	X	X	X	
Pacers	X	X	X	X		X	X	
Magic	X	X	X	X	X	X		
76ers	X	X	X	X				
Hawks	X		X		X	X		
Pistons	X	X	X		X	X	X	
Heat	X	X				X	X	
Bucks								X

This method shows that the only possible match-up for the Raptors is the Bucks. Put an O in the cells for a Raptors–Bucks game. Fill in the rest of the Bucks' row and column with Xs. The chart now shows that the Pacers must play the Hawks. Continuing to reason in this way, the Magic played the Heat, and the 76ers played the Pistons.

	Raptors	Pacers	Magic	76ers	Hawks	Pistons	Heat	Bucks
Raptors	X	X	X	X	X	X	X	O
Pacers	X	X	X	X	O	X	X	X
Magic	X	X	X	X	X	X	O	X
76ers	X	X	X	X	X	O	X	X
Hawks	X	O	X	X	X	X	X	X
Pistons	X	X	X	O	X	X	X	X
Heat	X	X	O	X	X	X	X	X
Bucks	O	X	X	X	X	X	X	X

- Another way to organize the information is to make a list of the teams chosen by Perimeter on the left side. List the other four teams along the top.

	Hawks	Pistons	Heat	Bucks
Raptors	_____	_____	_____	_____
Pacers	_____	_____	_____	_____
Magic	_____	_____	_____	_____
76ers	_____	_____	_____	_____

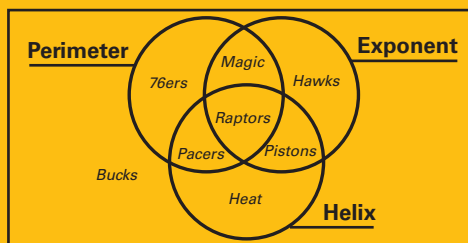
Exponent picked the Hawks, Pistons, Magic, and Raptors. So the Hawks did not play the Pistons, Magic, or Raptors. Mark an X in the blanks pairing the Hawks with the Raptors and Magic. The Pistons cannot play the Raptors or Magic. Helix's picks are the Heat, Pacers, Pistons, and Raptors. So the Heat cannot play the Raptors or Pacers.

	Hawks	Pistons	Heat	Bucks
Raptors	<u> X </u>	<u> X </u>	<u> X </u>	_____
Pacers	_____	<u> X </u>	<u> X </u>	_____
Magic	<u> X </u>	<u> X </u>	_____	_____
76ers	_____	_____	_____	_____

This shows that the Raptors must play the Bucks and the 76ers have to play the Pistons. Fill in the rest of the Bucks column with Xs because no other team can play them. This leaves one empty cell in the Magic row, so the Heat plays the Magic. Finally, the Pacers play the Hawks.

	Hawks	Pistons	Heat	Bucks
Raptors	<u> X </u>	<u> X </u>	<u> X </u>	<u> O </u>
Pacers	<u> O </u>	<u> X </u>	<u> X </u>	<u> X </u>
Magic	<u> X </u>	<u> X </u>	<u> O </u>	<u> X </u>
76ers	<u> X </u>	<u> O </u>	<u> X </u>	<u> X </u>

- A different method to solve this problem is using an arrangement of circles called a Venn diagram. Each sportswriter's picks can be thought of as a set, and two teams that are in the same set cannot play each other. Using the information given in the Challenge, you can draw a Venn diagram like the one below.



All three sportswriters picked the Raptors, so they are included in all three circles. Since the Bucks are the only team outside these three circles, the Raptors must play the Bucks. Perimeter and Helix selected the Pacers. The only remaining team not selected by both of these writers is the Hawks, since it is not in either of those circles. So the Pacers must play the Hawks. Similarly, the 76ers play the Pistons and the Magic play the Heat.

Try This:

- Check the sports page in your local paper and see if it publishes picks for sporting events. Looking only at the picks, can you determine which teams will play each other on a given day?
- Lewis Carroll—the author of *Alice in Wonderland* and other favorites—wrote the following logical arguments in which the last statement is a conclusion based upon the first two.

Every eagle can fly;	All wasps are unfriendly.
Some pigs cannot fly.	No puppies are unfriendly.
Some pigs are not eagles.	Puppies are not wasps.

Make up some logical arguments of your own like the ones Carroll wrote. Try your arguments on a friend to see if they believe them.

Additional Challenges:

- Justin, Aneisha, Steven, and Trish each brought a different pet to their community pet show. Using the following clues, match each pet—a dog, a cat, a horse, and a snake—with its owner.
 - Steven is the brother of the person who owns the snake.
 - Aneisha and Trisha do not like animals that bark.
 - Justin prefers reptiles over mammals.
 - Trisha hopes to ride her pet in an upcoming parade.
- An automobile dealer asked 100 customers if they liked the colors green, white, and black. The results of this survey are shown below.
 - 55 said they liked green
 - 47 said they liked white
 - 15 said they liked both green and white but not black
 - 5 said they liked both white and black but not green

- 20 said they liked both green and black but not white
- 10 said they liked green, white, and black
- 12 said they liked only black

How many customers did not like green, white, or black?

Things to Think About:

- How do census takers sort the thousands of pieces of information needed to report trends in populations, jobs, and salaries?
- In this challenge, there was only one schedule possible, given the sportswriters' picks. Would this always be true?
- How do sportswriters conclude that a team has clinched a title before the season is finished?

Did You Know That?

- Englishman John Venn (1834–1923) developed the Venn diagram to represent sets and their unions and intersections.
- Blood typing can be represented with Venn diagrams.
- Lewis Carroll's *Alice in Wonderland* contains many logic puzzles.
- Sherlock Holmes, created by Arthur Conan Doyle, used logical reasoning to solve mysteries.

Resources:

Books:

- Carroll, Lewis. *Alice's Adventures in Wonderland*. Cambridge, MA: Candlewick Press, 1999.
- Carroll, Lewis. *In The World of Mathematics*, Vol. 4. Newman, James R., ed. New York: Simon and Schuster, 1956. pp. 2397-2415.
- Doyle, Arthur Conan. *The Complete Sherlock Holmes*. New York: Doubleday and Co., 1953.
- Dudeney, H. E. *Amusements in Mathematics*. New York: Dover Publications, 1970.
- Moore, Rosalind, ed. *The Dell Book of Logic Problems*. New York: Dell Publishing Co., 1984.
- Smullyan, Raymond. *The Lady and the Tiger*. New York: Alfred E. Knopf, 1982.
- Smullyan, Raymond. *What Is the Name of This Book?* Englewood Cliffs, NJ: Prentice-Hall, 1978.

Website:

www.inficad.com/~ecollins/logic-prob.htm

Answers to Additional Challenges:

(1.) Justin owns the snake, Aneisha owns the cat, Steven owns the dog, and Trisha owns the horse.

(2.) 11 customers.



Figure This!

Math Challenges for Families

what's your **INDEX** ? ? ?

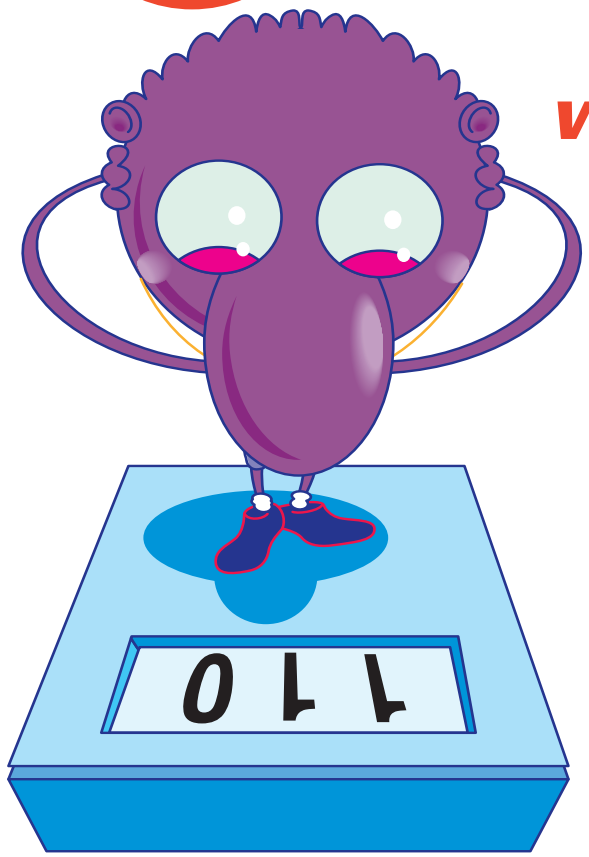


Figure This! Some doctors use body-mass index as an indicator of health risk. According to *The Old Farmer's Almanac 2000*, body-mass index (BMI) can be found using the formula:

$$BMI = \frac{(W \times 705) \div H}{H}$$

where H is height in inches and W is weight in pounds. According to the *Almanac*, an index greater than 27 or less than 19 indicates an increased risk for health problems. Helix is 5 feet, 2 inches tall and weighs 110 pounds.

Is his health at risk?

Hint: Convert Helix's height to inches, then use the formula.

Using and understanding formulas is a critical skill in almost every field, including science, engineering, business, and aviation. Spreadsheets and many computer programs require formulas to analyze situations and predict patterns.

Figure This!

Get Started:

How many inches are in a foot? What is Helix's height in inches? In the formula, replace H with Helix's height in inches and W with his weight in pounds.

Complete Solution:

There are 12 inches per foot. Helix is $5 \times 12 + 2$, or 62 inches tall. He weighs 110 pounds. Using the formula, his body-mass index is approximately 20.

$$\text{BMI} = \frac{(110 \times 705) \div 62}{62} \approx 20$$

Because 20 is greater than 19 and less than 27, Helix's health is not at risk.

Try This:

- Find your own body-mass index. According to the result, should you be concerned about your health?
- Check a website for other indicators of health risk.

Additional Challenges:

1. Another formula for body-mass index is:

$$\text{BMI} = \frac{705W}{H^2}$$

Will Helix have a different body-mass index using this formula?

2. What is a healthy weight range for Helix's height?
3. According to body-mass index, for what heights would a weight of 180 pounds be considered safe?
4. How could you adjust the formula to use weight in kilograms and height in centimeters?

Things to Think About:

- What other factors are involved in determining health risk?
- Why does the formula for body-mass index involve dividing by height twice?

Did You Know That?

- According to *The Old Farmer's Almanac 2000*, a waist measurement of 35 inches or more in women and 41 inches or more in men, regardless of height, suggests a serious risk of weight-related health problems.

- Very muscular people often have a higher body-mass index because muscle weighs more than fat.
- One simple health-risk test is to pinch your side just above your waist. If you can pinch more than an inch, you are at risk of weight-related health problems.
- The amount of skin may be found using the formula $\sqrt{\frac{\text{height} \times \text{weight}}{3600}}$ where height is in centimeters and weight is in kilograms. The result obtained is surface area measured in square meters. Weight is approximately proportional to height times height times height.
- Body-mass index is essentially a ratio of volume to surface area. The solid geometric figure with the greatest ratio of volume to surface area is the sphere.

Resources:

Books:

- "Media Clips." Edited by Dorothy Wood. *The Mathematics Teacher* 92 (March 1999): 234–235.
- Mathematical Sciences Education Board. *High School Mathematics at Work: Essay and Examples for the Education of All Students*. Washington, D. C.: National Academy of Sciences, 1998.
- *The Old Farmer's Almanac 2000*. Dublin, NH: Yankee Publishing, Inc., 1999.

Website:

- www.sirius.on.ca/running/bmi_txt.html

Answers to Additional Challenges:

(1.) No. The formulas are equivalent.

(2.) Using body-mass index, a safe range for weight is about 104 to 147 pounds.

(3.) Between approximately 69 and 82 inches.

(4.) To the nearest whole number, you can use 50 instead of 705 in the formula.



FigureThis!
Math Challenges for Families

I forgot the combination!
I'm ready to cry.
How many combinations
will I have to try?

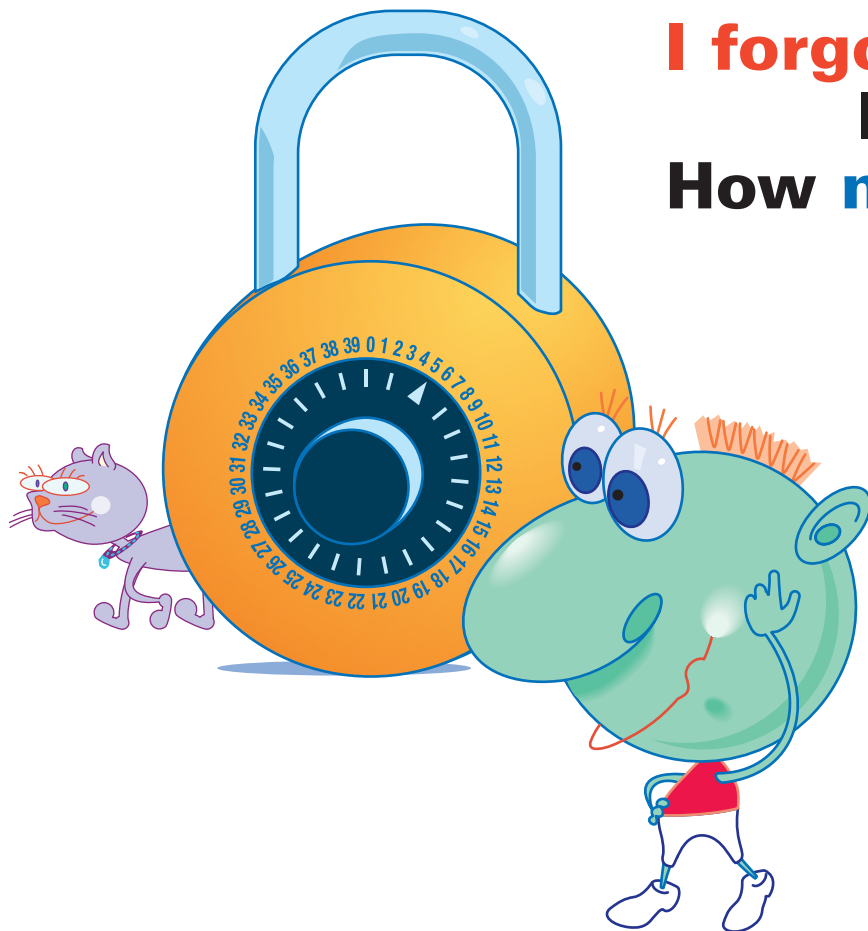


Figure This! This combination lock uses three numbers from 0 to 39. It opens when these numbers are dialed in a particular order: right, left, right. How many possible combinations are there?

Hint: Think about how many choices you have before dialing each number.

Counting the number of possible arrangements in a given situation is an important mathematical skill. Banks, phone companies, and security systems use codes based on such arrangements for Personal Identification Numbers (PIN).

Answer: There are 64,000 different combinations for the lock.

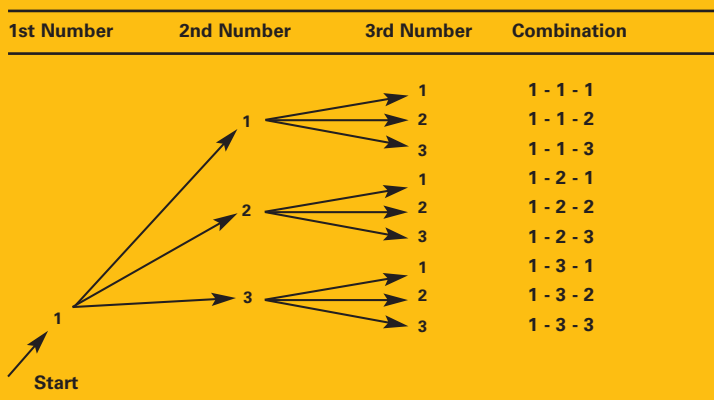
Figure This!

Get Started:

Try an easier problem. If the lock uses only the numbers 1, 2, and 3, how many different three-number combinations are possible? Write out the possible combinations in an ordered list and look for a pattern. What if the lock uses the numbers 1, 2, 3, and 4?

Complete Solution:

In the challenge, the lock uses the numbers 0 to 39. Start with an easier problem using 1, 2, and 3. You can count the possibilities by drawing a tree diagram. A portion of this tree diagram is shown below.



If you start with 1, you get nine different combinations. If you start with 2 instead of 1, you also get 9 different combinations. If you start with 3, you get 9 more possibilities for a total of $9 + 9 + 9$, or 27 different combinations. Think about this in terms of choices. You can choose any of three numbers as a possible first number in the combination, follow that with a choice of any of the three numbers as the second number in the combination, and finally choose any of the three numbers for the final number in the combination for a total of $3 \times 3 \times 3$, or 27 different combinations. If the lock used the numbers 1, 2, 3, and 4, you would have to choose from four numbers, three different times. This would give you $4 \times 4 \times 4$, or 64 choices for the combination. The lock in the challenge requires that you choose from 40 different numbers, three different times. Therefore, there are $40 \times 40 \times 40$, or 64,000 different combinations.

Try This:

- Experiment with a combination lock. Do you have to dial each number exactly?

Additional Challenges:

- Suppose the combination for a particular brand of lock allows each number to appear only once. If the lock uses three numbers from 0 to 39 (as in the challenge), how many combinations are possible?

- Suppose the combination for a bicycle lock is 10-24-32. With this lock, however, the numbers on either side work as well as the actual number. For example, the combinations 9-25-33 and 11-23-33 will also open the lock. How many different combinations will open this particular lock?
- Some keypad locks have 10 keys numbered 0 to 9. You open the lock using 3 keystrokes. A keystroke can consist of pressing 1 key or of pressing 2 keys at the same time. How many combinations are possible on this lock?

Things to Think About:

- Why do you think that telephone companies have to add new area codes for certain regions?
- How many phone numbers do you think are served by each area code?
- Which kind of lock will allow more possible combinations: a keypad lock or a lock with a dial?
- Find all the locks in your house. How are they alike? How are they different?

Did You Know That?

- Mathematicians use the word *combination* differently than it is used in a "combination lock." In a mathematical combination, the order in which an item occurs is not important.
- Some old safes can be opened using more than one combination.
- A disc lock consists of a sequence of discs numbered on their outer edges. To open the lock, you turn the discs (usually three) to the appropriate numbers.
- Many apartment buildings, businesses, and airports—even restrooms—use some form of keypad lock.

Resources:

Websites:

- www.howstuffworks.com/inside-lock.htm
- www.thelockman.com/cl1.htm

Answers to Additional Challenges:

$$\begin{aligned}
 (1.) \quad & 40 \times 39 \times 38 = 59,280. \\
 (2.) \quad & 3_3 = 27 \\
 (3.) \quad & 59 = 166,375.
 \end{aligned}$$